# Identification of Biased Beliefs in Games of Incomplete Information Using Experimental Data

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# Background

- In games with incomplete information, a player's behavior depends on her preferences and her beliefs about the uncertain actions of other players
- This paper studies separate identification and estimation of player's preferences and beliefs
- We relax common assumptions on preferences and/or beliefs in experimental games
- Our approach is fully nonparametric and identify a player's preferences and beliefs based on a particular design of monetary payoff matrices

# Common Assumptions

| Assumption/Restriction | Examples | Pros. of Our Method |
|---|---|---|
| Equilibrium/Unbiased Belief | Goeree, Holt and Palfrey (2003) | Understand Player's Expectation |
| Money Reward is True Payoff | Cheung and Friedman (1997) Nyarko and Schotter (2002) | Infer Risk Preference and Loss Aversion |
| Preference/Belief Elicitation | Schotter and Trevino (2014) Schlag et al. (2015) | Not Require Elicit Data; Robust |

# Our Contribution

**Methodological Contribution**

- Equilibrium belief assumption is testable
- Player's preferences and beliefs are separately identified under weak assumptions

**Empirical Application**

- Do inference on player's preferences and beliefs using data from Goeree and Holt (2001) and Heinemann, Nagel and Ockenfels (2009)
- Players' beliefs are out of equilibrium in a subset of games
- Utility function of monetary payoff is "S" shaped

# Example of Experiments

Consider an experiment of $T$ games/treatments indexed by $t$

- Subjects are randomly assigned to one of these $T$ games, as either player 1 or 2

Table: Monetary Payoff Matrix

| | | Player 2 | |
|---|---|---|---|
| | | $X$ | $Y$ |
| Player 1 | $X$ | $10, m_t$ | $0, 5$ |
| | $Y$ | $5, 0$ | $5, 5$ |

- Monetary payoff matrix is public information
- Only player 2's money reward $m_t$ varies across each treatment $t$

# Model: Discrete Game of Incomplete Info.

- Utility of subject $i$ is a function of her own monetary payoff and an additive idiosyncratic component that depends on her choice

$$u(m(a_1, a_2)) + \epsilon_i(a_1)$$

- The additive component $\epsilon_i$ is not known by the opponent
- Subject $i$ forms a belief about other player's behavior: $B_{it}$; such belief is allowed to be off-equilibrium
- Given her preferences and beliefs, a subject $i$ makes her choice to maximize expected utility. Choice of alternative $X$ iff:

$$u(10)B_{it} + u(0)(1 - B_{it}) + \epsilon_{it}(X) \geq u(5)B_{it} + u(5)(1 - B_{it}) + \epsilon_{it}(Y)$$

# Data and Identification Objective

- A large number of subjects randomly chosen from a population participate in the experiment
- Both players' choice probabilities: $P_{1t} = Pr(a_{1it} = X|t)$ and $P_{2t} = Pr(a_{2it} = X|t)$ can be consistently estimated for each treatment $t$
- Our objective is to identify player's utility function $u(\cdot)$ and average beliefs $\bar{B}_t$ based on choice probabilities

# Identification Intuition

| | | Player 2 | |
|---|---|---|---|
| | | $X$ | $Y$ |
| Player 1 | $X$ | 10 , $m_t$ | 0 , 5 |
| | $Y$ | 5 , 0 | 5 , 5 |

- Player 2 will adjust her behaviors as $m_t$ varies
- Player 1 who anticipates this will adjust her beliefs; and consequently alters her behaviors
- As player 1's utility holds constant, the variations of her choice probabilities reflect how she adjusts her beliefs

# Main Identification Result

For any three treatment $t$, $t'$ and $t''$

- A function of player 1's belief is identified

$$\frac{\bar{B}_{t'} - \bar{B}_t}{\bar{B}_{t''} - \bar{B}_t} = \frac{F(P_{1t'}) - F(P_{1t})}{F(P_{1t''}) - F(P_{1t})}$$

- Such function can be used to test equilibrium belief assumption

$$\frac{F(P_{1t'}) - F(P_{1t})}{F(P_{1t''}) - F(P_{1t})} = \frac{P_{2t'} - P_{2t}}{P_{2t''} - P_{2t}}$$

- When player 1 has equilibrium beliefs in 2 of $T$ treatments, her utility and beliefs in other treatments are identified

# Goeree and Holt (2001)

Table: Monetary Payoff of Matching Pennies

| | | $X$ | $Y$ |
|---|---|---|---|
| Treatment 1 | $X$ | 40 , 44 | 80 , 40 |
| | $Y$ | 80 , 40 | 40 , 80 |

| | | $X$ | $Y$ |
|---|---|---|---|
| Treatment 2 | $X$ | 40 , 80 | 80 , 40 |
| | $Y$ | 80 , 40 | 40 , 80 |

| | | $X$ | $Y$ |
|---|---|---|---|
| Treatment 3 | $X$ | 40 , 320 | 80 , 40 |
| | $Y$ | 80 , 40 | 40 , 80 |

# Summary Statistics

Table: Empirical Choice Probability of $X$

| | Player 1 | Player 2 |
|---|---|---|
| Treatment 1 | 0.8<br>(0.0800) | 0.08<br>(0.0543) |
| Treatment 2 | 0.48<br>(0.0999) | 0.48<br>(0.0999) |
| Treatment 3 | 0.16<br>(0.0733) | 0.96<br>(0.0392) |

Note: Standard error in parenthesis

# Unbiased Belief Test

Table: Test of Unbiased Belief

| | Test Statistics |
|---|---|
| Probit | 0.0503<br>(0.3753) |
| Logit | 0.0726<br>(0.6479) |
| Exponential | -0.1942<br>(0.5426) |
| Note: Standard error in parenthesis | |

Given these data, we cannot reject the null:

- Player 1 is able to correctly predict the change of player 2's behavior as player 2's monetary payoff varies

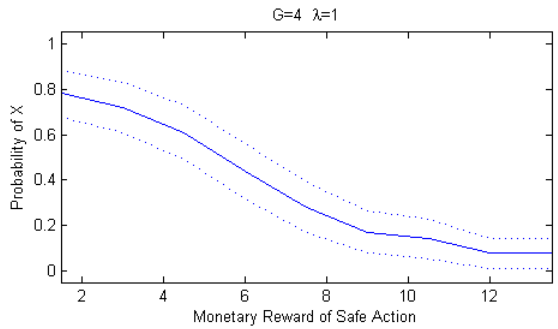
# Heinemann, Nagel and Ockenfels(2009)

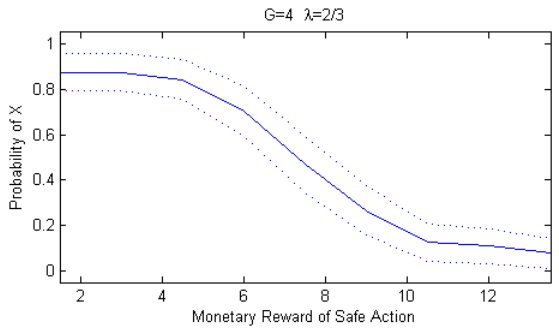
Table: Player $i$'s Monetary Payoff

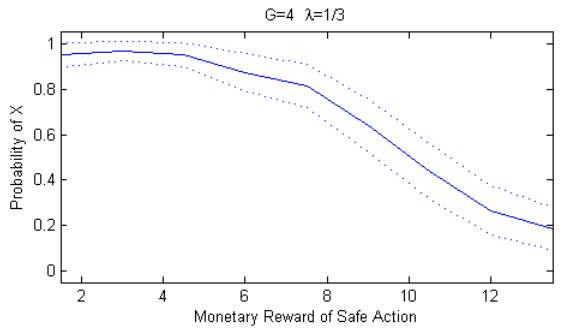
| | | Other Players | |
|---|---|---|---|
| | | $F_{-i}^X \geq \lambda$ | $F_{-i}^X < \lambda$ |
| Player $i$ | $X$ | 15 | 0 |
| | $Y$ | $m_0$ | $m_0$ |

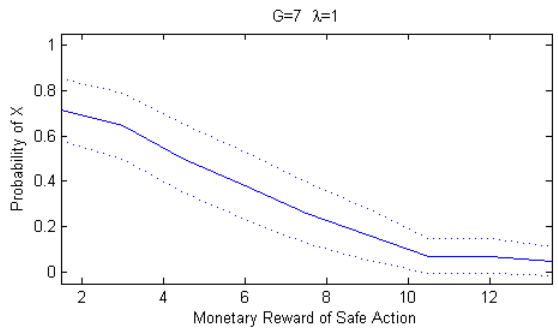
- Coordination game with $G$ players
- $F_{-i}^X$ is the fraction of other players who choose $X$
- Treatments: $\lambda \in \{\frac{1}{3}, \frac{2}{3}, 1\}$, $G \in \{4, 7, 10\}$ and $m_0$ varies from 1.5 to 15
- Variations of $\lambda$ and $G$ provide identification power

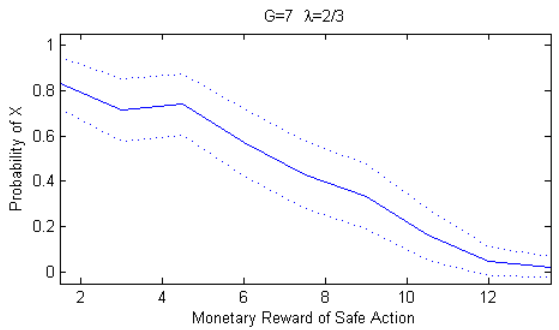
# Empirical Choice Probability of $X$

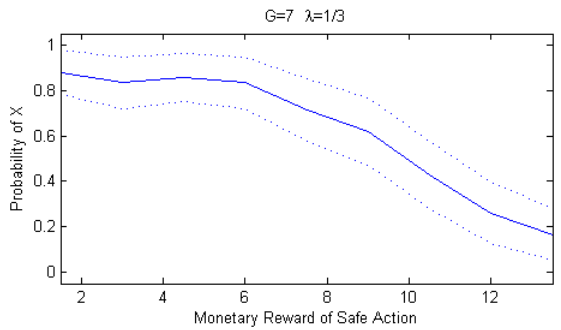


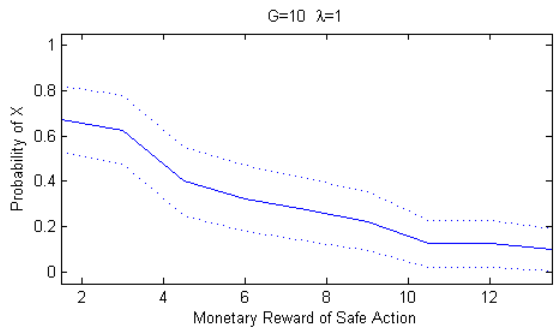



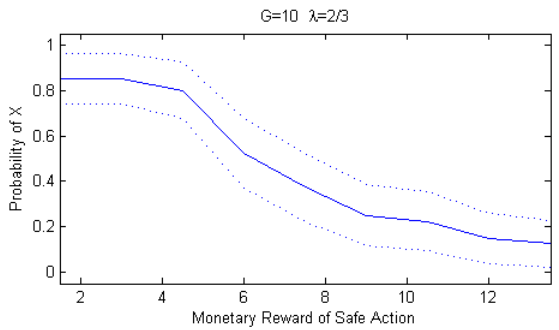



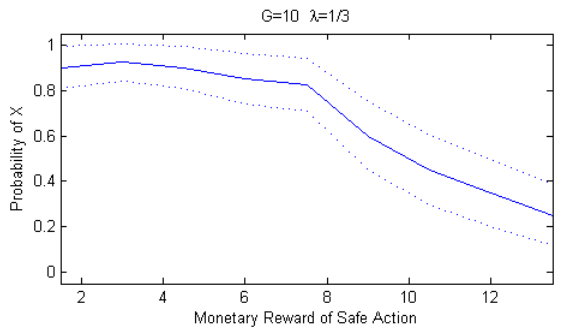



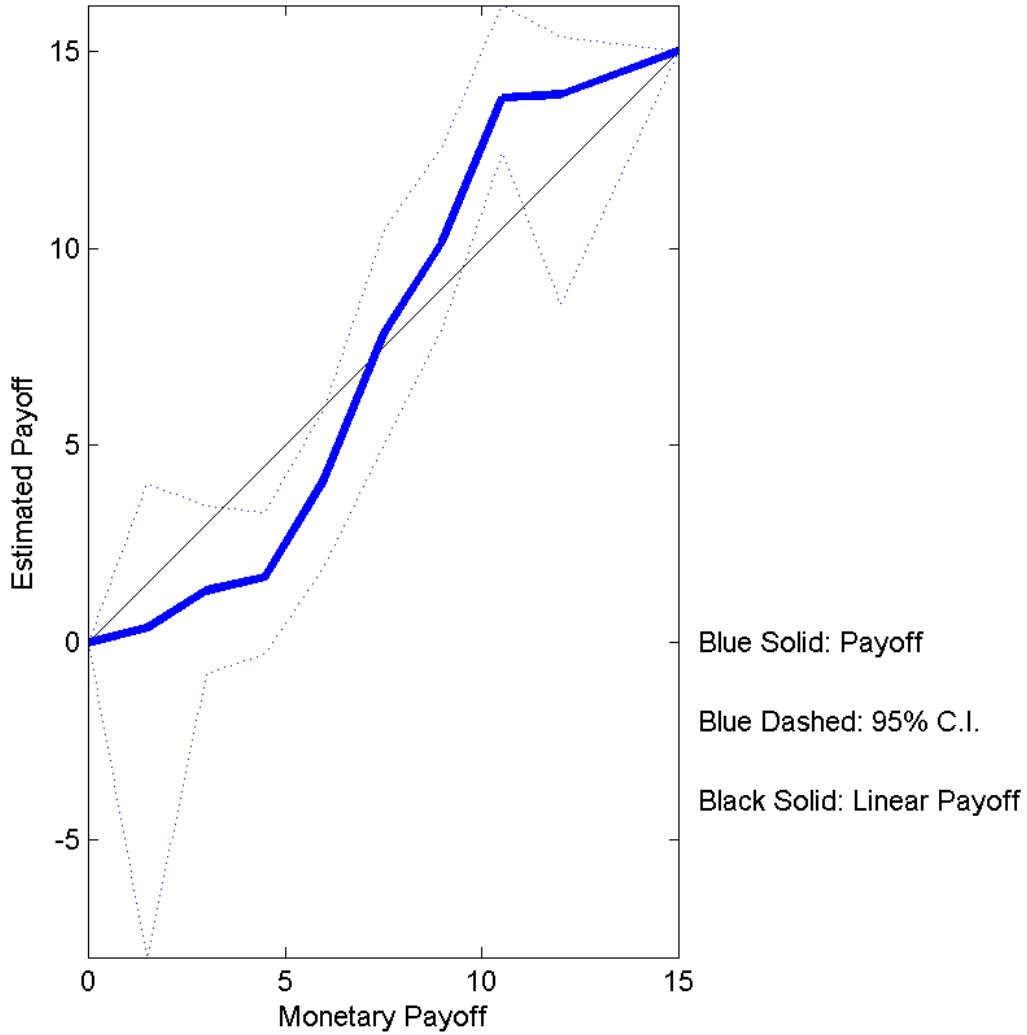



# Estimation Procedure

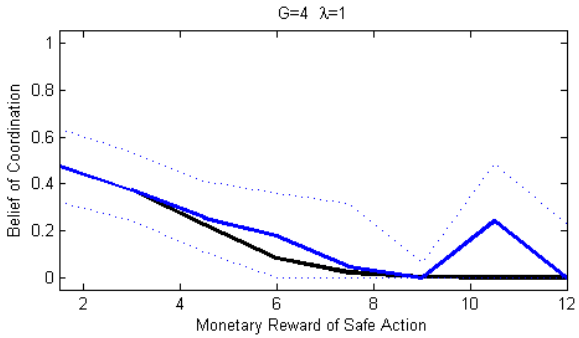
- Run unbiased belief test for every combination of treatments
- Choose the combination yields the highest p-value and assume player forms unbiased belief in such combination
- Estimate player's utility function
- Estimate player's beliefs in other treatments using estimated utility function

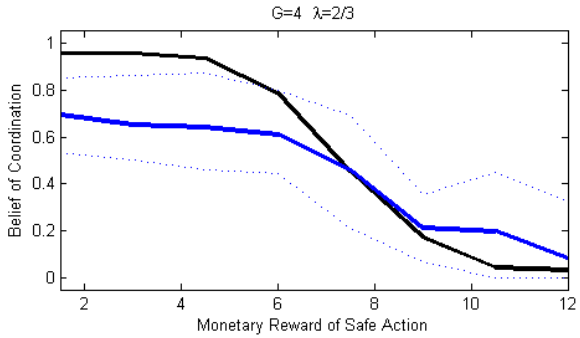
# Estimation Result: Utility Function

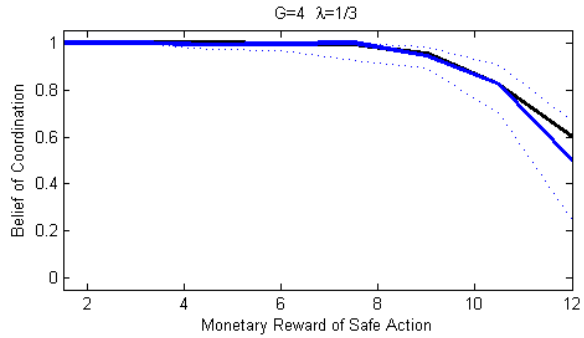
Figure: Subject's Utility Function

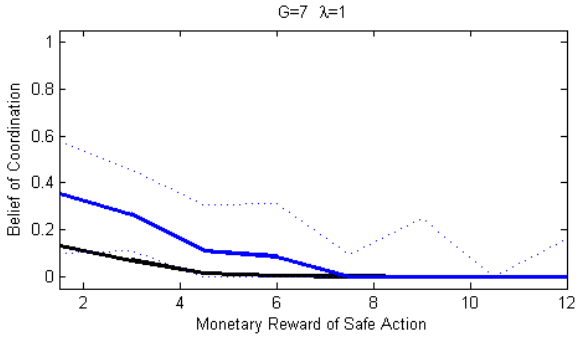


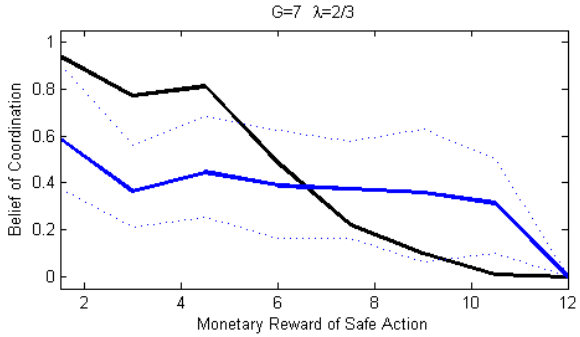
# Estimation Result: Belief

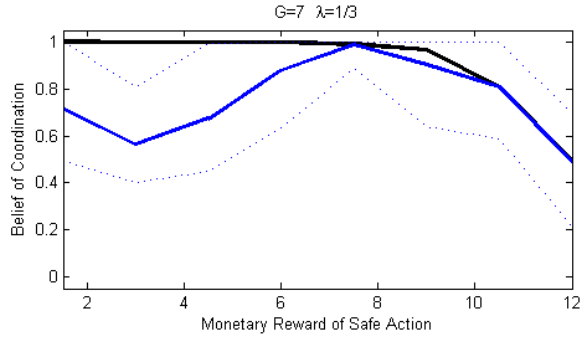


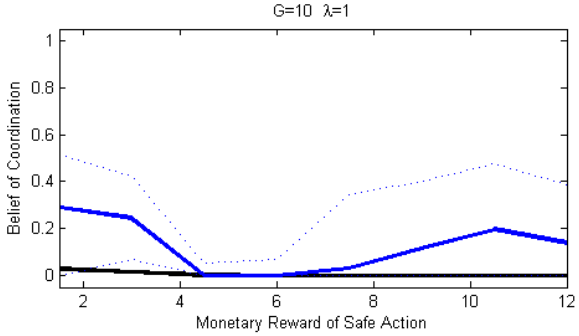



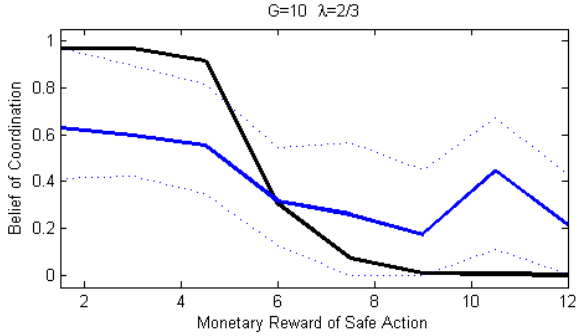



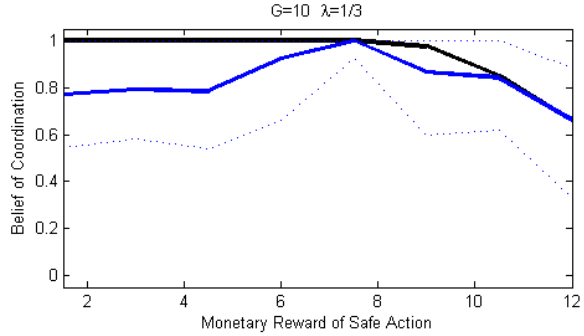



# Conclusion

**Methodology**

- We propose a test of equilibrium belief that is robust to any self-regarding rational preferences
- Player's beliefs and preferences can be separately identified under weak assumption

**Empirical Application**

- Subjects do have biased beliefs in some games
- Utility function is S shaped in monetary payoff