Identification and Counterfactuals in Dynamic Models of Market Entry and Exit

Victor Aguirregabiria and Junichi Suzuki

Virginia, Nov. 15, 2012

Introduction: Context

- This paper deals with a fundamental **identification problem** in the estimation of **models of industry dynamics / market entry-exit**.
- These models are useful to study empirical questions that require to take into account **endogeneity and dynamics of market structure**.
- Examples of recent applications:

  - *Ryan (2012): environmental regulation in the cement industry;*

  - *Kryukov (2010): relationship market structure & innovation;*

  - *Sweeting (2011): radio industry & copyright fees;*

  - *Suzuki (2012): hotel industry & land use regulation;*

Introduction: Structural models of market entry-exit

- Two components in profit function: **variable profit** & **fixed profit**.
- **Parameters in variable-profit** (i.e., demand and variable cost parameters) are estimated using data on firms' quantities & prices, a demand system, and a static model of Cournot/Bertrand competition.
- The **fixed profit** is the part that comes from **buying, selling, or renting inputs** that are **fixed** during the whole active life of the firm.
- **Parameters in fixed-profit** are estimated using data on firms' decisions to be active in the market combined with the dynamic model of market entry-exit. **Revealed preference**.

Introduction: The identification problem in this paper

- The fixed-profit function has three components:

  - **Entry cost**: *Paid by new entrants. Includes transaction costs related to market entry, & cost of purchasing owned fixed inputs;*

  - **Scrap value**: *Received by exiting firms. Includes value of selling owned fixed inputs, minus transaction costs related to exit;*

  - **Fixed cost of an incumbent**: *Paid every period by firms that continue in the market. Includes cost of leasing fixed inputs, and taxes related to fixed inputs (e.g., property taxes).*

- These three functions are **not separately identified**.
- Standard approach: **"normalization" of one function to zero**.

Introduction: Contributions of this project

1. Derive **closed-form relationship** between three unknown structural functions & two functions identified from the data.

   This provides the **correct interpretation** of the estimated objects that are obtained under the **'normalization assumptions'**.

2. Characterize a class of counterfactual exp. that is **identified**;

   Characterize a class of counterfactual exp. that is **not identified**.

3. For non-identified counterfactuals, show that the "normalization" assumption can lead to serious biases.

4. Discuss solutions to this identification problem.

Outline

1. Model
2. Identification of Structural functions
3. Identification of Counterfactuals
4. Numerical Example
5. Solutions

1. Model

Model Variable profit

- Time is discrete and indexed by t.
- Every period t firm decide to be active in the market or not.
- $a_t \in \{0, 1\}$ indicator "active in the market at t"
- Current profit has two components: $\Pi_t = VP_t + FP_t$
- Variable profit is:

$$VP_t \;=\; a_t \; \left[vp(\mathbf{z}_t^{vp}) + \varepsilon_t^{vp} \right]$$

  $\mathbf{z}_t^{vp}$ and ε_t^{vp} are exog. state vars. in demand and var. costs.

- $\mathbf{z}_t^{vp}$ is observable to the researcher, and ε_t^{vp} is unobservable.

Model Fixed Profit

- Fixed profit has three components:

$$\begin{aligned} FP_t = & - a_t \left[fc(\mathbf{z}_t^{fc}) + \varepsilon_t^{fc} \right] && \Longrightarrow \text{Fixed Cost} \\ & - a_t \; (1 - i_t) \; \left[ec(\mathbf{z}_t^{ec}) + \varepsilon_t^{ec} \right] && \Longrightarrow \text{Entry Cost} \\ & + (1 - a_t) \; i_t \; \left[sv(\mathbf{z}_t^{sv}) + \varepsilon_t^{sv} \right] && \Longrightarrow \text{Scrap value} \end{aligned}$$

where $i_t = a_{t-1}$;

- $fc\left(\mathbf{z}_t^{fc}\right)$ = fixed cost function;
- $ec\left(\mathbf{z}_t^{ec}\right)$ = entry cost function;
- $sv\left(\mathbf{z}_t^{sv}\right)$ = scrap value function

Model Profit function

- Then, the profit function is:

$$\Pi_t = \begin{cases} vp(\mathbf{z}_t) - fc(\mathbf{z}_t) - (1 - i_t)\ ec(\mathbf{z}_t) + \varepsilon_t(1) & \text{if } a_t = 1 \\ \\ i_t\ sv(\mathbf{z}_t) + \varepsilon_t(0) & \text{if } a_t = 0 \end{cases}$$

- $\mathbf{z}_t \equiv (\mathbf{z}_t^{vp}, \mathbf{z}_t^{fc}, \mathbf{z}_t^{ec}, \mathbf{z}_t^{sv})$

- $\varepsilon_t(1) = \varepsilon_t^{vp} - \varepsilon_t^{fc} - (1 - i_t)\, \varepsilon_;^{ec}; \quad \varepsilon_t(0) = -i_t\ \varepsilon_t^{sv}$

- For simplicity, $\varepsilon_t' s$ are assumed independent of $\mathbf{z}_t$ and i.i.d. over time.

Model: Bellman equation

$$V\left(i_t, \mathbf{z}_t, \varepsilon_t\right) = \max\left\{\; v\left(0, i_t, \mathbf{z}_t\right) + \varepsilon_t(0) \quad ; \quad v\left(1, i_t, \mathbf{z}_t\right) + \varepsilon_t(1) \;\right\}$$

with

$$v\left(a_t, i_t, \mathbf{z}_t\right) \equiv \pi\left(a_t, i_t, \mathbf{z}_t\right) + \beta \int V\left(a_t, \mathbf{z}_{t+1}, \varepsilon_{t+1}\right)\; f_z\left(\mathbf{z}_{t+1} \middle| \mathbf{z}_t\right)\; dG\left(\varepsilon_{t+1}\right)$$

- And the optimal decision rule is:

$$\alpha\left(i_t, \mathbf{z}_t, \varepsilon_t\right) = \begin{cases} 1 & \text{if} \quad v\left(1, i_t, \mathbf{z}_t\right) + \varepsilon_t(1) \geq v\left(0, i_t, \mathbf{z}_t\right) + \varepsilon_t(0) \\ \\ 0 & \text{otherwise} \end{cases}$$

Model: Empirical predictions

- Define the Choice Probability:

$$P\left(i_t, \mathbf{z}_t\right) \equiv \Pr(a_t = 1 | i_t, \mathbf{z}_t)$$

- Then,

$$P\left(i_t, \mathbf{z}_t\right) = F_{\widetilde{\varepsilon}}\left(\ v\left(1, i_t, \mathbf{z}_t\right) - v\left(0, i_t, \mathbf{z}_t\right)\ \right)$$

- where $F_{\widetilde{\varepsilon}}$ is the CDF of $\widetilde{\varepsilon}_t \equiv \varepsilon_t(0) - \varepsilon_t(1)$

2. Identification: Structural functions

Identification VP and F

- Data: $\{a_{mt}, \mathbf{z}_{mt}, i_{mt}, p_{mt}, q_{mt} : t = 1, 2, ..., T\}$.
- We assume that the variable profit function $vp(\mathbf{z}_t)$ is identified from $\{\mathbf{z}_{mt}, p_{mt}, q_{mt}\}$.
- Given independence of $\widetilde{\varepsilon}_t$ and $\mathbf{z}_t$ and large support variation of $vp(\mathbf{z}_t)$, then $F_{\widetilde{\varepsilon}}$ is nonparametrically identified.

(Non) identification fc, ec, & sv

- Let $Q(i_t, \mathbf{z}_t)$ be the function that represents the value difference:
 - value of being in the market today, exiting next period, and remaining out of the market forever after that;
 - minus the value of exiting from the market today and remaining out of the market forever after that.

- $Q(i_t, \mathbf{z}_t)$ is identified.

- The following system of equations summarizes all the restrictions on functions *fc*, *ec*, and *sv*:

$$
\begin{aligned}
Q(i, \mathbf{z}) &= vp(\mathbf{z}) - [fc(\mathbf{z}) + ec(\mathbf{z})] + i \ [ec(\mathbf{z}) - sv(\mathbf{z})] \\
&+ \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \ sv(\mathbf{z}')
\end{aligned}
$$

(Non) identification fc, ec, & sv

PROPOSITION 2.

- $fc(\mathbf{z})$, $ec(\mathbf{z})$, *and* $sv(\mathbf{z})$ *are not separately identified.*
- *But we can identify two combinations of these functions:*

$$
\begin{aligned}
ec(\mathbf{z}) - sv(\mathbf{z}) &= Q(1,\mathbf{z}) - Q(0,\mathbf{z}) \\
fc(\mathbf{z}) + ec(\mathbf{z}) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\ sv(\mathbf{z}') &= -Q(0,\mathbf{z}) + vp(\mathbf{z})
\end{aligned}
$$

Identification: Normalization sv(z)=0

- Under this normalization, the estimated functions $\widehat{fc}(\mathbf{z})$ and $\widehat{ec}(\mathbf{z})$ are:

$$
\begin{aligned}
\widehat{ec}(\mathbf{z}) &= ec(\mathbf{z}) - sv(\mathbf{z}) \\
\widehat{fc}(\mathbf{z}) &= fc(\mathbf{z}) + \left[sv(\mathbf{z}) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \; sv(\mathbf{z}') \right]
\end{aligned}
$$

- One should be careful when interpreting $\widehat{ec}(\mathbf{z})$ and $\widehat{fc}(\mathbf{z})$.

Identification: Normalization fc(z)=0

- Under this normalization, the estimated functions $\widehat{ec}\left(\mathbf{z}\right)$ and $\widehat{sv}\left(\mathbf{z}\right)$ are:

$$\widehat{ec}\left(\mathbf{z}\right) = ec\left(\mathbf{z}\right) + \sum_{r=0}^{\infty} \beta^{r} \; E\left[fc(\mathbf{z}_{t+r}) \mid \mathbf{z}_{t} = \mathbf{z}\right]$$

$$\widehat{sv}\left(\mathbf{z}\right) = sv\left(\mathbf{z}\right) + \sum_{r=0}^{\infty} \beta^{r} \; E\left[fc(\mathbf{z}_{t+r}) \mid \mathbf{z}_{t} = \mathbf{z}\right]$$

3. Identification: Counterfactual experiments

Description of counterfactuals

- Let $\theta^0 = \{vp^0, fc^0, ec^0, sv^0, f_z^0\}$ be the values of these functions in the environment that has generated the data.
- Let $P(i, \mathbf{z} ; \theta^0)$ be CCP associated to θ^0.
- A **counterfactual experiment** is described by $\Delta_\theta = \{\Delta_{vp}(\mathbf{z}), \Delta_{fc}(\mathbf{z}), \Delta_{ec}(\mathbf{z}), \Delta_{sc}(\mathbf{z}), \Delta_{fz}(\mathbf{z})\}$ that represents a set of changes in structural functions.
- Δ_θ **is known to the researcher**, though some components of θ^0 and $\theta^0 + \Delta_\theta$ are unknown.

Description of counterfactuals (2)

- We want to obtain how the perturbation Δ_θ affects firms' behavior as measured by CCP function.

- Identify the $\Delta_P(i, \mathbf{z})$ associated to Δ_θ, where

$$\Delta_P(i, \mathbf{z}) \equiv P(i, \mathbf{z}\ ;\ \theta^0 + \Delta_\theta) - P(i, \mathbf{z}\ ;\ \theta^0)$$

- **LEMMA:** There is a one-to-one relationship between $Q(i, \mathbf{z})$ and $P(i, \mathbf{z})$.

- Therefore, $\Delta_P(i, z)$ **is identified iff** $\Delta_Q(i, \mathbf{z})$ is identified, where;

$$\Delta_Q(i, \mathbf{z}) \equiv Q(i, \mathbf{z}\ ;\ \theta^0 + \Delta_\theta) - Q(i, \mathbf{z}\ ;\ \theta^0)$$

Identified Counterfactuals

- Distinguish two components in the vector of state variables: $\mathbf{z} \equiv (\mathbf{z}^{nosv}, \mathbf{z}^{sv})$, where $\mathbf{z}^{sv}$ is the subvector of the state variables that affect the scrap value:

$$f_z(\mathbf{z}_{t+1}|\mathbf{z}_t) = f_{z,sv}(\mathbf{z}_{t+1}^{sv}|\mathbf{z}_t)\ f_{z,nosv}(\mathbf{z}_{t+1}^{nosv}|\mathbf{z}_t, \mathbf{z}_{t+1}^{sv}).$$

PROPOSITION 3:

- *Suppose that Δ_θ implies changes only in functions vp, fc, ec, sv, and $f_{z,nosv}$ such that $\beta^* = \beta^0$ and $f_{z,sv}^* = f_{z,sv}^0$.*
- *And suppose that the researcher knows the perturbation $\Delta_\theta = \{\Delta_{vp}(\mathbf{z}), \Delta_{f_{z,nosv}}(\mathbf{z}'|\mathbf{z}), \Delta_{ec}(\mathbf{z}), \Delta_{sv}(\mathbf{z})\}$ (though he does not know neither $\boldsymbol{\theta}^0$ nor $\boldsymbol{\theta}^*$).*
- *Then, $\Delta_Q(i, \mathbf{z})$, $\Delta_P(i, \mathbf{z})$, and $P(i, \mathbf{z}\ ; \theta^0 + \Delta_\theta)$ are identified.*

Non-identified Counterfactuals

PROPOSITION 4:

- *Suppose that* Δ_θ is *such that* $\Delta_\beta \equiv \beta^* - \beta^0 \neq 0$ *or/and* $\Delta_{f_{z,sv}} \equiv f^*_{z,sv} - f^0_{z,sv} \neq 0.$
- *Despite the researcher knows both* (β^0, f^0_z) *and* (β^*, f^*_z), *the effect of these counterfactuals on firms' behavior is NOT identified.*

$$
\begin{aligned}
\Delta_Q(i, \mathbf{z}) \;\; &= \;\; \beta^0 \sum_{\mathbf{z}^{sv\prime} \in \mathbf{Z}} \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z}) \; sv^0(\mathbf{z}^{sv\prime}) \\
&+ \;\; \Delta_\beta \sum_{\mathbf{z}^{sv\prime} \in \mathbf{Z}} [f^0_{z,sv}(\mathbf{z}^{sv\prime}|\mathbf{z}) + \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z})] \; sv^0(\mathbf{z}^{sv\prime})
\end{aligned}
$$

4. Numerical Example

Numerical Example

- Consider a retail industry in which market entry requires land ownership, e.g., hotels.
- Let z_t represent land price.
- Structural functions:

$$\begin{aligned} fc(z) &= fc_0 + \lambda \ z \\ ec(z) &= ec_0 + z \\ sv(z) &= sv_0 + z \\ z_{t+1} &= \alpha_0 + \alpha_1 \ z_t + \sigma_z u_{t+1} \text{ where } u \sim iid \ N(0,1) \end{aligned}$$

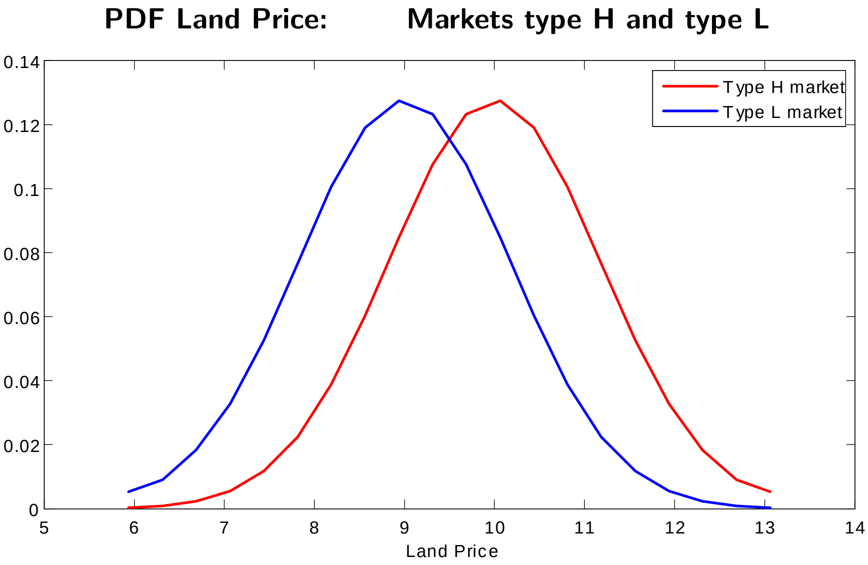
$ec_0 = 6.5$, $sv_0 = 0.5$, $fc_0 = 0.4$, $\lambda = 0.1$, $vp_0 = 1.1$.

DGP

- There are two groups of markets in the data, e.g., two cities (regions) where each city has a large number of local markets.
- Markets in city H, and markets city L.
- All the structural functions are the same in the two cities. But this is unknwon to the researcher.
- The only difference between the two cities is in the level of land price:

$$\text{City H} \quad : \quad z_{t+1} = 1.0 + 0.9\ z_t + 0.5\ u_{t+1}$$

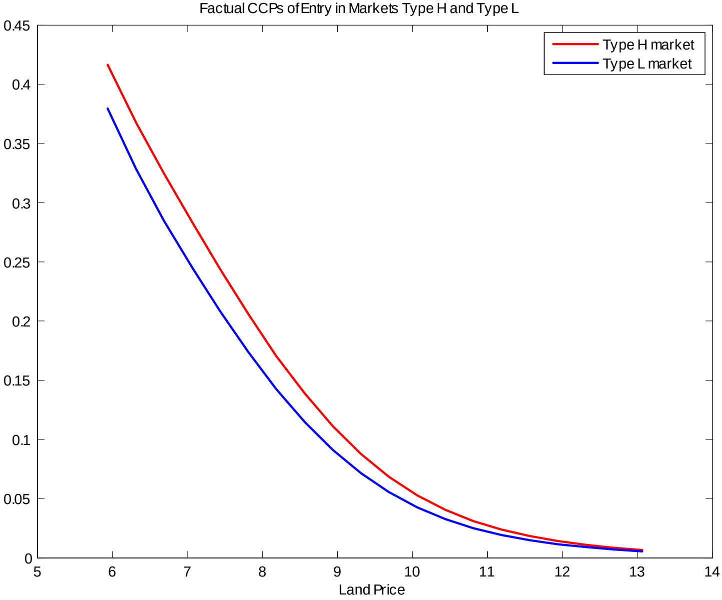
$$\text{City L} \quad : \quad z_{t+1} = 0.9 + 0.9\ z_t + 0.5\ u_{t+1}$$


PDF Land Price: Markets type H and type L
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
5
6
7
8
9
10
11
12
13
14
Land Price
Type H market
Type L market

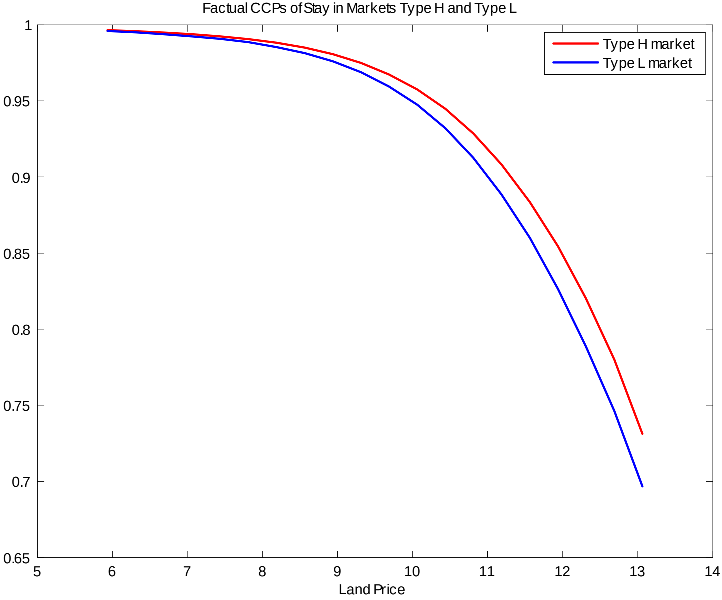
Estimation of CCPs

- The researcher does not can estimate nonparametrically the CCPs separately for the two groups of markets.
- He finds that, for every value of land price, the CCP of entry in City H is above the one in City L.
- Similarly, for every value of land price, the CCP of staying in City H is above the one in City L.

CCP Entry: Markets type H and type L



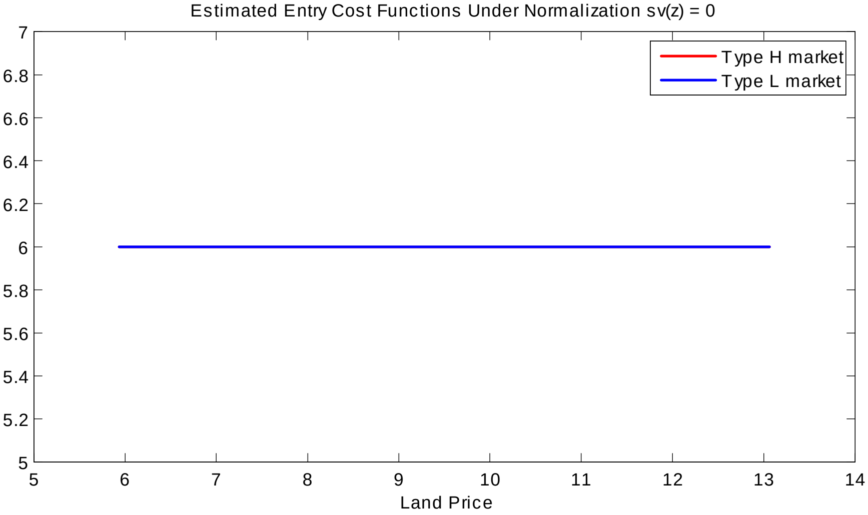
CCP Stay: Markets type H and type L



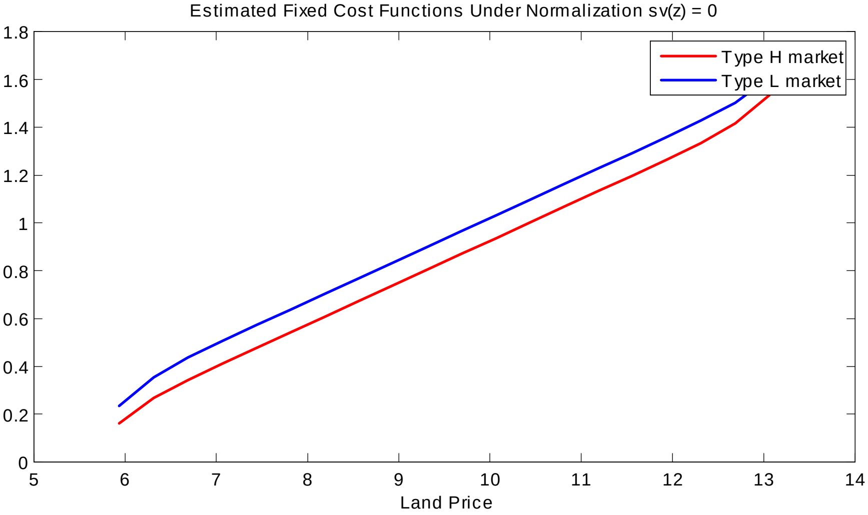
Estimation of structural functions

- The researcher wants to understand what explains this difference in the CCPs of the two cities
- Is it differences in the *fc* function, or *ec* function, or *sv* function?
- Or is it simply the difference in the price of land?
- The truth is that the difference in the average price of land between the two cities explains the whole difference between the CCPs. But the research does not know it.
- Can he identify it from the data?

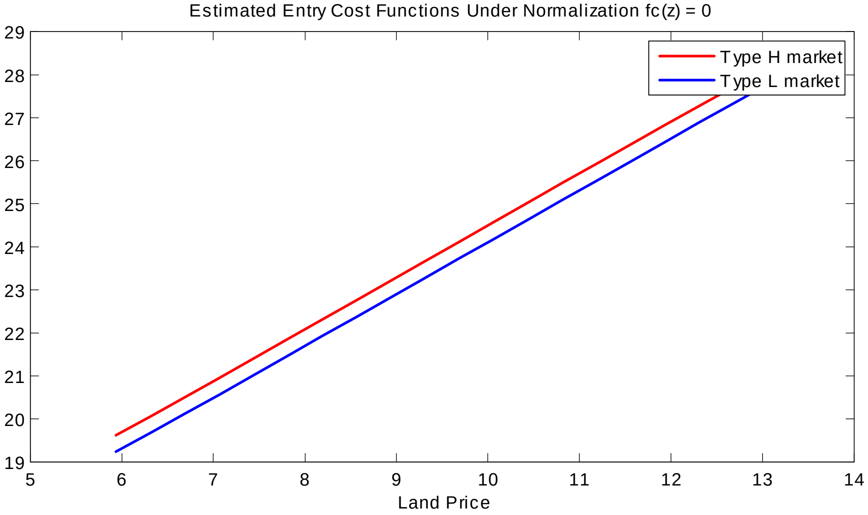
Estimate Entry Costs: Normalization sv(z) = 0



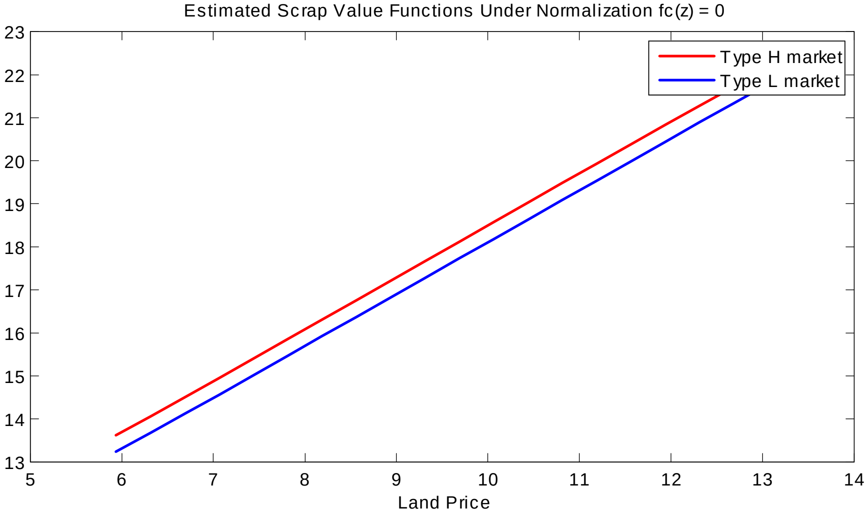
Estimate Fixed Cost: Normalization sv(z) = 0



Estimate Entry Cost: Normalization fc(z) = 0



Estimate Scrap Value: Normalization fc(z) = 0



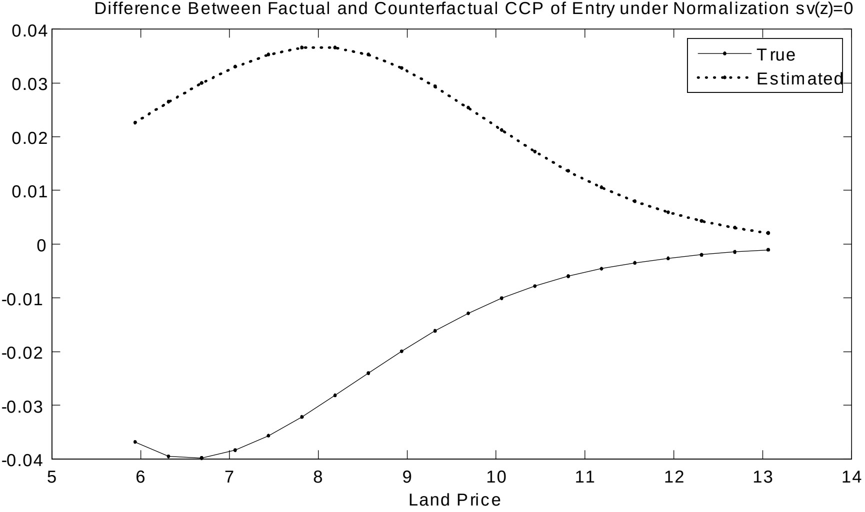
Counterfactual Experiment

- What is the CCPs of entry and stay in the markets of city H if the price of land (stochastic process) were the one in city L?
- 10% permanent reduction in the level of price.

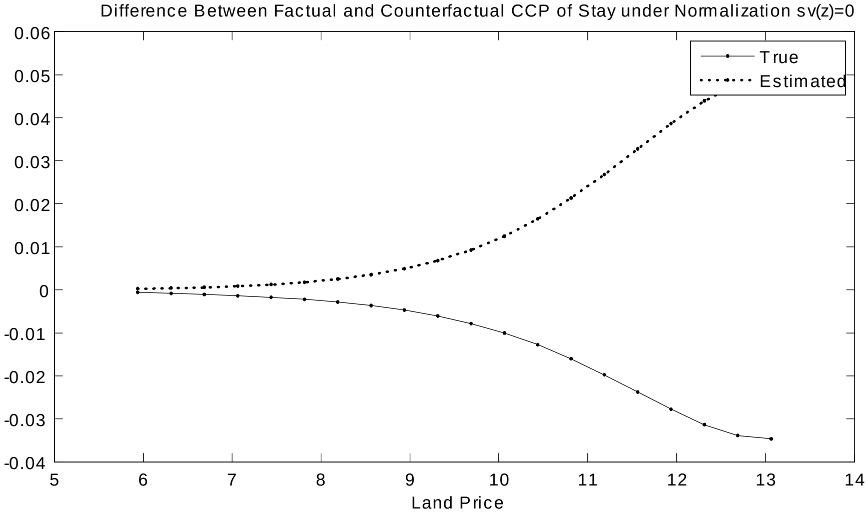
	Factual	Correct CF	Estimated CF $\widehat{sv}(\mathbf{z}) = 0$
Avg. Prob. Entry	0.067	0.087	0.134
			[0.047]
Avg. Prob. Staying	0.950	0.963	0.978
			[0.015]
Avg. Prob. Active	0.572	0.699	0.861
			[0.162]

The numbers in brackets are the biases.

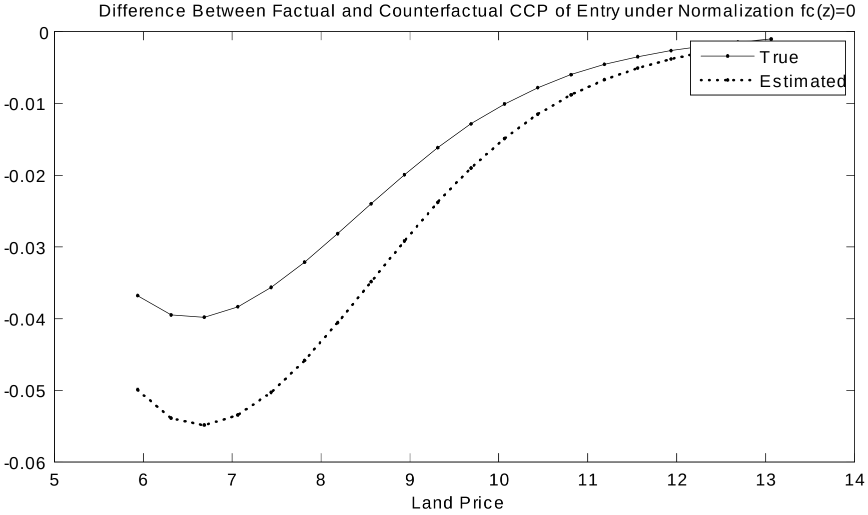
True & Estimated Count. Effect CCP Entry: Norm. sv(z) = 0



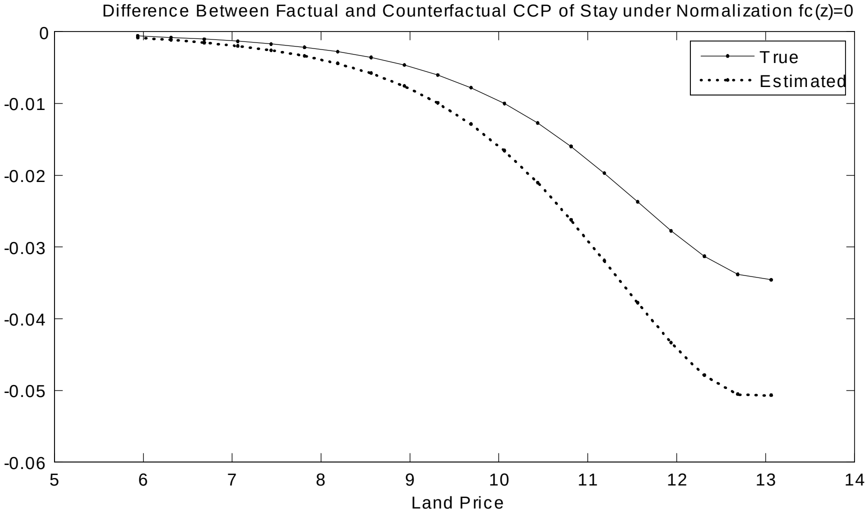
True & Estimated Count. Effect CCP Stay: Norm. sv(z) = 0



True & Estimated Count. Effect CCP Entry: Norm. fc(z) = 0



True & Estimated Count. Effect CCP Stay: Norm. fc(z) = 0



5. Solutions

Solutions

- Manski's bound approach.

  - Not very informative.

- Using data on transaction prices from the acquisition of firms.

- In some industries, a common form of firm exit (and entry) is that the owner of an incumbent firm sells all the firm's assets to a new entrant.

- Under some assumptions, these additional data can be used to deal with the identification problem that we study in this paper.