

# Identification of Average Marginal Effects in Fixed Effects Dynamic Discrete Choice Models

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## INTRODUCTION (1/2)

- Consider the **Fixed Effects (FE) Dynamic Binary Logit model** as described by the transition probability:

$$P(y_{it} = 1 | y_{i,t-1}, \alpha_i) = \frac{\exp\{\alpha_i + \beta y_{i,t-1}\}}{1 + \exp\{\alpha_i + \beta y_{i,t-1}\}}$$

where  $p_1(y_{i1} | \alpha_i)$  and  $f_\alpha(\alpha_i)$  are unrestricted, i.e., FE model.

- Given panel data  $(y_{i1}, y_{i2}, \dots, y_{iT})$  with  $T \geq 4$ , **parameter  $\beta$  is identified** (Chamberlain (1985), Honoré & Kyriazidou (2000)):

$$\beta = \log \mathbb{P}(0, 0, 1, 1) - \log \mathbb{P}(0, 1, 0, 1)$$

where  $\mathbb{P}(y_1, y_2, y_3, y_4)$  is the probability of history  $(y_1, y_2, y_3, y_4)$ .

## INTRODUCTION (2/2)

- In this paper, we are interested in the **identification & estimation of Average Marginal Effects (AMEs)**.
- For instance, for the Binary Choice AR(1) model:

$$\begin{aligned}
 AME &= \mathbb{E}_{\alpha} (\mathbb{E} [y_{it} | \alpha_i, y_{i,t-1} = 1] - \mathbb{E} [y_{it} | \alpha_i, y_{i,t-1} = 0]) \\
 &= \int \left( \frac{\exp\{\alpha_i + \beta\}}{1 + \exp\{\alpha_i + \beta\}} - \frac{\exp\{\alpha_i\}}{1 + \exp\{\alpha_i\}} \right) f_{\alpha}(\alpha_i) d\alpha_i
 \end{aligned}$$

- **Common wisdom:** these AMEs are not identified in FE models.
  - They depend on the whole distribution  $f_{\alpha}(\alpha_i)$ , and this distribution is not identified in FE models.

## CONTRIBUTIONS OF THIS PAPER

1. Everything started with the derivation (almost by chance) of a **simple closed-form expression for the AME in BC-AR(1)** in terms of  $\beta$  and probabilities of choice histories.
2. Inspired by this result, we develop a **general method to obtain AMEs** in a broad class of dynamic logit models: binary, multinomial, ordered, with exogenous regressors, with duration dependence.
  - This method is based on the solution of a finite (low-dimension) system of linear equations.
3. Empirical application: dynamic demand of differentiated products with brand switching costs.

## RELATED LITERATURE

- Related papers studying the **identification of AMEs in FE discrete choice models** are:
  - Chernozhukov, Fernandez-Val, Hahn, & Newey (ECMA, 2013)
  - Davezies, D'Haultfoeuille, & Laage (WP, 2021)
  - Pakel & Weidner (WP, 2021)
- Some differences between our paper and these papers:
  - (1) Point identification (us) versus set identification (them).
  - (2) Logit (us) versus more general discrete choice (them).
  - (3) Simple closed-form expressions (us) more computationally intensive methods (them).

## OUTLINE

1. Identification result for AME in BC-AR(1)
2. General identification method
  - Application of the general identification method
  - Multinomial, Exogenous X, Duration, Ordered Logit.
3. Empirical application

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# 1. IDENTIFICATION OF AME IN BC-AR(1) MODEL

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## Identification of AME in BC-AR(1) Model (1/3)

- Define the **individual-level transition probabilities**:

$$\begin{aligned}\pi_{01}(\alpha_i) &\equiv P(y_{it} = 1 | \alpha_i, y_{i,t-1} = 0) = \Lambda(\alpha_i) \\ \pi_{11}(\alpha_i) &\equiv P(y_{it} = 1 | \alpha_i, y_{i,t-1} = 1) = \Lambda(\alpha_i + \beta)\end{aligned}$$

- And the corresponding **average transition probabilities**:

$$\begin{aligned}\Pi_{01} &\equiv \int \pi_{01}(\alpha_i) f_\alpha(\alpha_i) d\alpha_i \\ \Pi_{11} &\equiv \int \pi_{11}(\alpha_i) f_\alpha(\alpha_i) d\alpha_i\end{aligned}$$

- Define the **individual-level marginal effect**:

$$\Delta(\alpha_i) \equiv \pi_{11}(\alpha_i) - \pi_{01}(\alpha_i)$$

- And the corresponding **Average Marginal Effect (AME)**:

$$AME \equiv \int \Delta(\alpha_i) f_\alpha(\alpha_i) d\alpha_i = \Pi_{11} - \Pi_{01}$$



## Identification of AME in BC-AR(1) Model (2/3)

- We show the following: **identification results**:

$$\left\{ \begin{array}{l} \Pi_{01} = [1 - \exp\{\beta\}] \mathbb{P}_{1,0,1} + \mathbb{P}_{1,1} + \mathbb{P}_{0,1} \\ \Pi_{11} = \exp\{\beta\} \mathbb{P}_{0,1,0} + \mathbb{P}_{0,1,1} + \mathbb{P}_{1,1} \\ AME = [\exp\{\beta\} - 1] [\mathbb{P}_{0,1,0} + \mathbb{P}_{1,0,1}] \end{array} \right.$$

where:

$\mathbb{P}_{y_1, y_2, y_3}$  = empirical probability of  $(y_{i1}, y_{i2}, y_{i3}) = (y_1, y_2, y_3)$

$\mathbb{P}_{y_1, y_2}$  = empirical probability of  $(y_{i1}, y_{i2}) = (y_1, y_2)$

## Proof of Identification of AME (3/3)

- Key in this proof: following **property** of Logit model. For any  $\alpha_i$ :

$$\Delta(\alpha_i) = [\exp \{\beta\} - 1] \pi_{01}(\alpha_i) \pi_{10}(\alpha_i) \quad (1)$$

- For any sequence  $(y_1, y_2, y_3)$ :

$$\mathbb{P}_{y_1, y_2, y_3} = \int p^*(y_1 | \alpha_i) \pi_{y_1, y_2}(\alpha_i) \pi_{y_2, y_3}(\alpha_i) f_\alpha(\alpha_i) d\alpha_i$$

- Applying equation (1) to  $\mathbb{P}_{0,1,0}$  and  $\mathbb{P}_{1,0,1}$ , we have that:

$$\begin{cases} [\exp \{\beta\} - 1] \mathbb{P}_{0,1,0} = \int p^*(0 | \alpha_i) \Delta(\alpha_i) f_\alpha(\alpha_i) d\alpha_i \\ [\exp \{\beta\} - 1] \mathbb{P}_{1,0,1} = \int p^*(1 | \alpha_i) \Delta(\alpha_i) f_\alpha(\alpha_i) d\alpha_i \end{cases}$$

- Adding up these two equations:

$$[\exp \{\beta\} - 1] [\mathbb{P}_{0,1,0} + \mathbb{P}_{1,0,1}] = AME$$

## Identification of n-periods forward AME

- Using a similar approach, we show the identification of the n-periods forward AME, for any  $n \geq 1$ :

$$AME^{(n)} \equiv \mathbb{E}_\alpha (\mathbb{E} [y_{i,t+n} | \alpha_i, y_{it} = 1] - \mathbb{E} [y_{i,t+n} | \alpha_i, y_{it} = 0])$$

- We show that, for  $T \geq 2n + 1$ :

$$AME^{(n)} = [\exp\{\beta\} - 1]^n \left[ \mathbb{P}_{0, \widetilde{\mathbf{1}}^n} + \mathbb{P}_{\widetilde{\mathbf{1}}^n, 1} \right]$$

where  $\widetilde{\mathbf{1}}^n$  represents the repetition n times of of sequence 1, 0.

## Identification of Average Transition Probability in Multinomial Logit

- A similar procedure shows identification of average transition probability  $\Pi_{jj}$  in a dynamic multinomial logit, for  $j = 1, 2, \dots, J$ :

$$\Pi_{jj} \equiv \int \pi_{jj}(\alpha_i) f_{\alpha}(\alpha_i) d\alpha_i$$

with

$$\pi_{jj}(\alpha_i) \equiv P(y_{it} = j | \alpha_i, y_{i,t-1} = j)$$

- Logit model implies that for any triple of choice alternatives  $j, k, \ell$ :

$$\exp \{ \beta_{k\ell} - \beta_{kj} + \beta_{jj} - \beta_{j\ell} \} = \frac{\pi_{k\ell}(\alpha_i) \pi_{jj}(\alpha_i)}{\pi_{kj}(\alpha_i) \pi_{j\ell}(\alpha_i)}$$

- And using this property, we can show that:

$$\Pi_{jj} = \mathbb{P}_{jj} + \sum_{k \neq j} \left[ \mathbb{P}_{k,j,j} + \sum_{\ell \neq j} \exp \{ \beta_{k\ell} - \beta_{kj} + \beta_{jj} - \beta_{j\ell} \} \mathbb{P}_{k,j,\ell} \right]$$

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## 2. GENERAL METHOD TO SHOW IDENTIFICATION OF AMEs

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## General Dynamic Logit Model

- Consider a dynamic logit model that allows for multinomial  $y$ , exogenous regressors  $(\mathbf{x})$ , and duration  $(d)$  dependence.
- Let  $\mathbf{y}_i \equiv (d_{i1}, y_{i1}, y_{i2}, \dots, y_{iT}) \in \mathcal{D} \times \mathcal{Y}^T$  be individual  $i$ 's choice, and let  $\mathbf{x}_i \equiv (\mathbf{x}_{i1}, \mathbf{x}_{i2}, \dots, \mathbf{x}_{iT}) \in \mathcal{X}^T$
- Let  $\mathbb{P}_{\mathbf{y}|\mathbf{x}}$  represent the probability  $P(\mathbf{y}_i = \mathbf{y} | \mathbf{x}_i = \mathbf{x})$ .
- According to the model, probability  $\mathbb{P}_{\mathbf{y}|\mathbf{x}}$  has the following structure:

$$\mathbb{P}_{\mathbf{y}|\mathbf{x}} = \int G\left(\mathbf{y}^{\{2,T\}} | d_1, y_1, \mathbf{x}, \boldsymbol{\alpha}; \boldsymbol{\theta}\right) p^*(d_1, y_1 | \boldsymbol{\alpha}, \mathbf{x}) f_{\boldsymbol{\alpha}}(\boldsymbol{\alpha} | \mathbf{x}) d\boldsymbol{\alpha},$$

where

$$G\left(\mathbf{y}^{\{2,T\}} | y_1, d_1, \mathbf{x}, \boldsymbol{\alpha}; \boldsymbol{\theta}\right) \equiv \prod_{t=2}^T \Lambda(y_t | y_{t-1}, d_t, \mathbf{x}_t, \boldsymbol{\alpha}; \boldsymbol{\theta})$$

## LEMMA 1

- Consider a FE dynamic discrete choice model characterized by the probability function  $G(\mathbf{y}^{\{2,T\}}|y_1, d_1, \mathbf{x}, \boldsymbol{\alpha}; \boldsymbol{\theta})$ .
- Let  $AME(\mathbf{x}) \equiv \int \Delta(\boldsymbol{\alpha}_i, \mathbf{x}, \boldsymbol{\theta}) f_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}_i|\mathbf{x}) d\boldsymbol{\alpha}_i$  be an average marginal effect of interest.
- This **AME is point identified if and only if** there is a weighting function  $w(\mathbf{y}, \mathbf{x}, \boldsymbol{\theta})$  that satisfies the following equation:

$$\sum_{\mathbf{y}^{\{2,T\}}} w(d_1, y_1, \mathbf{y}^{\{2,T\}}, \mathbf{x}, \boldsymbol{\theta}) G(\mathbf{y}^{\{2,T\}}|y_1, d_1, \mathbf{x}, \boldsymbol{\alpha}; \boldsymbol{\theta}) = \Delta(\boldsymbol{\alpha}, \mathbf{x}, \boldsymbol{\theta}),$$

for every value  $(d_1, y_1) \in \mathcal{D} \times \mathcal{Y}$  and every  $\boldsymbol{\alpha} \in \mathbb{R}^J$ .

- Furthermore, this condition implies that:

$$AME(\mathbf{x}) = \sum_{\mathbf{y}} w(\mathbf{y}, \mathbf{x}, \boldsymbol{\theta}) \mathbb{P}_{\mathbf{y}|\mathbf{x}}$$

## Particular Structure of FE Dynamic Logit

- Lemma 1 does not impose any restriction on the form of function  $G$ .
- In **FE Dynamic Logit model** the probability of a choice history:

$$\log \mathbb{P}(\mathbf{y}_i | \mathbf{x}_i, \boldsymbol{\alpha}_i, \boldsymbol{\theta}) = \mathbf{s}(\mathbf{y}_i, \mathbf{x}_i)' \mathbf{g}(\boldsymbol{\alpha}_i, \mathbf{x}_i, \boldsymbol{\theta}) + \mathbf{c}(\mathbf{y}_i, \mathbf{x}_i)' \boldsymbol{\theta}$$

where  $\mathbf{s}_i \equiv \mathbf{s}(\mathbf{y}_i, \mathbf{x}_i)$  and  $\mathbf{c}_i \equiv \mathbf{c}(\mathbf{y}_i, \mathbf{x}_i)$  are vectors of statistics.

- This equation implies that:

**(1)**  $\mathbf{s}_i$  is a sufficient statistic for  $\boldsymbol{\alpha}_i$ .

**(2)** Given  $\boldsymbol{\theta}$ , the distribution of  $\mathbf{s}_i$  contains all the information in the data about the distribution of  $\boldsymbol{\alpha}_i$ , and therefore, about AMEs.

**(3)** The form of  $\mathbb{P}_{\mathbf{s}|\mathbf{x}}$  is:

$$\mathbb{P}_{\mathbf{s}|\mathbf{x}} = \sum_{\mathbf{y}: \mathbf{s}(\mathbf{y}, \mathbf{x}) = \mathbf{s}} \left[ \int \exp\{\mathbf{s}(\mathbf{y}, \mathbf{x})' \mathbf{g}(\boldsymbol{\alpha}, \mathbf{x}, \boldsymbol{\theta}) + \mathbf{c}(\mathbf{y}, \mathbf{x})' \boldsymbol{\theta}\} f_{\boldsymbol{\alpha}}(\boldsymbol{\alpha} | \mathbf{x}) d\boldsymbol{\alpha} \right]$$



## LEMMA 2

- Consider a FE Dynamic Logit model.
- Let  $AME(\mathbf{x}) \equiv \int \Delta(\boldsymbol{\alpha}_i, \mathbf{x}, \boldsymbol{\theta}) f_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}_i | \mathbf{x}) d\boldsymbol{\alpha}_i$  be an AME of interest.
- This **AME is point identified if and only if** there is a weighting function  $m(\mathbf{s}, \mathbf{x}, \boldsymbol{\theta})$  that satisfies the following equation:

$$\sum_{\tilde{\mathbf{s}} \in \tilde{\mathcal{S}}} m(d_1, y_1, \tilde{\mathbf{s}}, \mathbf{x}, \boldsymbol{\theta}) \exp\{(d_1, y_1, \tilde{\mathbf{s}})' \mathbf{g}(\boldsymbol{\alpha}, \mathbf{x}, \boldsymbol{\theta})\} = \Delta(\boldsymbol{\alpha}, \mathbf{x}, \boldsymbol{\theta}),$$

for every value  $(d_1, y_1)$  and every  $\boldsymbol{\alpha} \in \mathbb{R}^J$ .

- Furthermore, this condition implies that:

$$AME(\mathbf{x}) = \sum_{\mathbf{s} \in \mathcal{S}} \frac{m(\mathbf{s}, \mathbf{x}, \boldsymbol{\theta})}{\sum_{\mathbf{y}: \mathbf{s}(\mathbf{y}, \mathbf{x}) = \mathbf{s}} \exp\{\mathbf{c}(\mathbf{y}, \mathbf{x})' \boldsymbol{\theta}\}} \mathbb{P}_{\mathbf{s} | \mathbf{x}}$$

## System with Infinite Restrictions and Finite Unknowns (1/2)

- The identification condition in Lemma 2 defines an infinite system of equations – as many as values of  $\alpha_j$ .
- The researcher knows functions  $\mathbf{g}(\alpha, \mathbf{x}, \theta)$  and  $\Delta(\alpha, \mathbf{x}, \theta)$ .
- The unknowns are the weights  $m(\mathbf{s}, \mathbf{x}, \theta)$ .
- Without some structure, this system with infinite restrictions and finite unknowns would not have a solution.

## System with Infinite Restrictions and Finite Unknowns (2/2)

- Lemma 3 shows that, in the FE dynamic logit model, the structure of functions  $\mathbf{g}(\boldsymbol{\alpha}, \mathbf{x}, \boldsymbol{\theta})$  and  $\Delta(\boldsymbol{\alpha}, \mathbf{x}, \boldsymbol{\theta})$  is such that the **identification condition can be represented as a finite order polynomial** in the variables  $\exp\{\alpha_i(j)\}$  for  $j = 1, 2, \dots, J$ .
- Since these variables are always strictly positive, there is a solution to the system **if and only if the coefficients multiplying every monomial term in this polynomial are all equal to zero.**
- This property transforms the infinite system of equations into a **finite system with finite unknowns.**
- Furthermore, if a solution exists, this solution implies a closed-form expression for the weights  $m(\mathbf{s}, \mathbf{x}, \boldsymbol{\theta})$ , and therefore, for  $AME$ .

## LEMMA 3

- Consider the FE dynamic logit model.
- The identification condition in Lemma 2 can be represented as a finite order polynomial in the variables  $\exp\{\alpha_i(j)\}$  for  $j = 1, 2, \dots, J$ .
- This implies a **finite system of linear equations** with unknowns the finite number of weights  $m(\mathbf{s}, \mathbf{x}, \boldsymbol{\theta})$  for every  $\mathbf{s} \in \mathcal{S}$ .

## EXAMPLE: AME in BC-AR(1) (1/2)

- $\mathbf{s} = (y_1, y_T, n_1)$  with  $n_1 = \sum_{t=2}^T y_t$ ;  $\mathbf{c} = \sum_{t=2}^T y_{t-1}y_t$ , and:

$$\begin{cases} \Delta(\alpha_i) = \frac{e^{\alpha_i}(e^\beta - 1)}{(1 + e^{\alpha_i + \beta})(1 + e^{\alpha_i})} \\ e^{\mathbf{s}' \mathbf{g}(\alpha)} = \left(\frac{1}{1 + e^\alpha}\right)^{T-1} \left(\frac{1 + e^{\alpha + \beta}}{1 + e^\alpha}\right)^{y_T - y_1} \left(\frac{e^\alpha(1 + e^\alpha)}{1 + e^{\alpha + \beta}}\right)^{n_1} \end{cases}$$

- Therefore, the identification condition is:

$$\begin{aligned} \sum_{y_T, n_1} m(y_1, y_T, n_1) \left(\frac{1}{1 + e^\alpha}\right)^{T-1} \left(\frac{1 + e^{\alpha + \beta}}{1 + e^\alpha}\right)^{y_T - y_1} \left(\frac{e^\alpha(1 + e^\alpha)}{1 + e^{\alpha + \beta}}\right)^{n_1} \\ = \frac{e^\alpha(e^\beta - 1)}{(1 + e^{\alpha + \beta})(1 + e^\alpha)} \end{aligned}$$

## EXAMPLE: AME in BC-AR(1) (2/2)

$$\sum_{y_T, n_1} m(y_1, y_T, n_1) \left( \frac{1}{1 + e^\alpha} \right)^{T-1} \left( \frac{1 + e^{\alpha+\beta}}{1 + e^\alpha} \right)^{y_T - y_1} \left( \frac{e^\alpha (1 + e^\alpha)}{1 + e^{\alpha+\beta}} \right)^{n_1} - \frac{e^\alpha (e^\beta - 1)}{(1 + e^{\alpha+\beta})(1 + e^\alpha)} = 0$$

- Multiplying this equation times  $(1 + e^{\alpha+\beta})(1 + e^\alpha)$  to eliminate denominators, we obtain a **polynomial of order  $2T - 2$  in  $e^\alpha$** .
- Since  $e^\alpha > 0$ , this equation holds for every value of  $\alpha$  iff the coefficients multiplying each of the  $2T - 2$  monomials are zero.
- These coefficients are linear in the weights  $m_{y_1, y_T, n_1}$ , and this defines a system of  $2T - 2$  linear equations with  $2T - 2$  unknowns.

## Application of the general identification method

- We apply this general approach to show identification of different AMEs in different versions of the FE dynamic logit model.
1.  $\Pi_{11}$ ,  $\Pi_{00}$ , and  $AME^{(n)}$  in BC-AR(1).
  2. Average transition probability  $\Pi_{jj}$  in multinomial and ordered logit.
  3. AME of change in duration.
  4. All these AMEs in model with exogenous  $\mathbf{x}$ .

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# 3. EMPIRICAL APPLICATION

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## PRELIMINARIES

- Demand of differentiated product / state dependence in consumer brand choice.
- Data (Nielsen scanner panel data) from **Erdem, Imai, and Keane, 2003 (EIK)**, and similar model.
- The main goal is to determine the relative contribution of unobserved heterogeneity and state dependence to explain the observed time persistence of consumer brand choices.
- All previous studies estimate Random Effects models.

## DATA

- Product: Ketchup.
- Same working sample as EIK. 996 households; 123 weeks.
- For our analysis, a time period is a *household purchase occasion*.
- Number observation purchase occasions =  $\sum_{i=1}^N T_i = 9,562$

Table 3

Distribution of number of purchase occasions ( $T_i$ )

<i>Minimum</i>	<i>5%</i>	<i>25%</i>	<i>Median</i>	<i>75%</i>	<i>95%</i>	<i>Maximum</i>
3	4	5	8	12	21	52

## BRAND CHOICE PERSISTENCE

**Table 4**  
**Matrix of Transition Probabilities of Brand Choices**  
**(percentage points)**

<i>Brand choice at t</i>	<i>Brand choice at t + 1</i>				<i>Total</i>
	<i>Heinz</i> <i>(j = 0)</i>	<i>Hunts</i> <i>(j = 1)</i>	<i>Del Monte</i> <i>(j = 2)</i>	<i>Store</i> <i>(j = 3)</i>	
<i>Heinz (j = 0)</i>	<b>78.95</b>	10.67	6.98	3.40	100.00
<i>Hunts (j = 1)</i>	45.16	<b>32.30</b>	15.76	6.78	100.00
<i>Del Monte (j = 2)</i>	41.11	18.98	<b>34.07</b>	5.83	100.00
<i>Store (j = 3)</i>	42.32	17.11	13.38	<b>27.19</b>	100.00
<i>Market share (<math>\mathbb{P}_j</math>)</i>	66.65	15.63	12.19	5.53	100.00
<i>Persistence (<math>\mathbb{P}_{jj} - \mathbb{P}_j</math>)</i>	12.30	16.67	21.88	21.66	

## MODEL

- We consider the following dynamic structural model:

$$y_{it} = \arg \max_{j \in \{0,1,2,3\}} \{ \alpha_i(j) + \varepsilon_{it}(j) + \beta_{jj} 1\{y_{i,t-1} = j\} + v_i(j, y_{i,t-1}) \}$$

- $\beta_{jj}$  represents habits in the purchase/consumption of brand  $j$ : additional utility from buying the same brand as in last purchase.
- To illustrate our method using a short panel, we split the purchasing histories in the original sample into subs-histories of length  $T$ , where  $T$  is small.
- We present results for  $T = 6$  and  $T = 8$ .

## ESTIMATION OF BETA PARAMETERS

**Table 5**  
**Conditional Maximum Likelihood Estimates**  
**of Brand Habit ( $\beta_{jj}$ ) Parameters**

<i>Parameter</i> $\beta_{jj}$	<i>T = 6 sub-histories</i>		<i>T = 8 sub-histories</i>	
	<i>Estimate</i>	<i>(s.e.)<sup>(1)</sup></i>	<i>Estimate</i>	<i>(s.e.)<sup>(1)</sup></i>
<i>Heinz</i>	0.00	(.)	0.00	(.)
<i>Hunts</i>	0.2312	(0.0590)	0.2566	(0.0570)
<i>Del Monte</i>	0.1155	(0.0718)	0.1191	(0.0722)
<i>Store</i>	0.3245	(0.1166)	0.4675	(0.1106)
<i># histories of length T</i>	4,764		3,396	

(1) Standard errors (s.e) are obtained using a bootstrap method. We generate 1,000 resamples (independent, with replacement, and with  $N = 996$ ) from the 996 purchasing histories in the original dataset. Then, we split each history of the bootstrap sample into all the possible sub-histories of length  $T$ .

## STATE DEPENDENCE vs UH, USING AME

**Table 6**  
**FE Estimates of ATPs and AMEs**

	$T = 8$ sub-histories			
	<i>Pers</i> (s.e.)	<i>ATP</i> (s.e.)	<i>ATE</i> (s.e.)	<i>UHet</i> (s.e.)
<i>Heinz</i>	0.1230 (0.0033)	0.6708 (0.0062)	0.0043 (0.0067)	0.1187 (0.0069)
<i>Hunts</i>	0.1667 (0.0077)	0.1788 (0.0072)	0.0225 (0.0106)	0.1442 (0.0109)
<i>Del Monte</i>	0.2188 (0.0090)	0.1345 (0.0062)	0.0126 (0.0110)	0.2062 (0.0113)
<i>Store</i>	0.2166 (0.0062)	0.0805 (0.0072)	0.0252 (0.0094)	0.1914 (0.0099)

## CONCLUSIONS

- AME are useful parameters to represent causal effects.
- In FE nonlinear panel data models with short panels, the distribution of the UH, and this problem has been associated with the common belief that AMEs are not identified.
- In the context of dynamic logit models, we prove the identification of AMEs associated with changes in lagged dependent variables and in duration variables.
- Our proofs provide simple closed-form expressions for the AMEs in terms of frequencies of choice histories.
- We illustrate our identification results using consumer scanner data in dynamic demand model with state dependence.