Encyclopedia of World Poverty

Mehmet Odekon, Ph.D., Skidmore College, General Editor
Produced by Golson Books, Ltd., Croton-on-Hudson, NY
Published by Sage Publications, Thousand Oaks, CA

SEN-SHORROCKS-THON INDEX

By Victor Aguirregabiria (Boston University)

A poverty index proposed by Shorrocks (1995) based on the pioneering work of Sen (1976). It has also received the name of *modified Sen index* in Shorrocks (1995) and Sen (1997). As noted by Zheng (1997), this index is identical to the limit of Thon's modified Sen index (Thon, 1979 and 1983).

In 1976, Sen proposed an axiomatic approach to poverty measures. He argued that poverty indices should satisfy certain ethically defensible criteria or axioms and that the desirability of a poverty measure should be evaluated in terms of these axioms. Therefore, if we evaluate anti-poverty policies according to their ability to reduce this type of poverty index, our evaluation will be consistent with the ethical criteria that inspire the poverty measure.

Definition. Let $y = (y_1, y_2, \ldots, y_n)$ be the income vector of a population of n individuals with incomes sorted in increasing order of magnitude. Let z be the poverty line and let q be the number of poor persons (i.e., the number of individuals with incomes below the poverty line). The Sen-Shorrocks-Thon (SST) index is defined as:

$$SST = \sum_{i=1}^{q} \left(\frac{2(n-i)+1}{n^2} \right) \left(\frac{z - y_i}{z} \right)$$

The index is a weighted sum of the poverty gap ratios $(z - y_i)/z$ of the poor. The weights decrease with the rank order in the income distribution such that more weight is given to the poverty gap of the poorer individuals. The index is normalized to take values between zero and one: it is equal to zero when all the incomes are above the poverty line and so there are not poor people; it reaches a unit value in the extreme case where all the individuals are poor and they have zero income. Of course, these are extreme hypothetical cases. In a comparative study of 23 OECD countries, Osberg and Xu (2000) report a range of indexes between 0.014 in Austria and 0.125 in US during the 90s.

Properties. This index has some very attractive properties. (1) *Homogeneous of degree zero in y and z:* The index is invariant to changes in the scale of the income distribution and the poverty line. (2) *Focus axiom:* The index does not depend on the income levels of the non-poor. (3) *Impartiality axiom:* It depends only on the vector of ordered incomes and not on the identity of the individuals. (4) *Replication invariant:* The poverty index does not change if it is computed based on an income distribution that is the k-fold replication of the original income distribution. (5) *Monotonicity axiom:* A reduction in a poor person's income, holding other incomes constant, increases the poverty index. (6)

Continuity axiom: The index is a continuous function of individual incomes. (7) *Transfer axiom:* The index increases whenever a pure transfer is made from a poor person to someone with more income.

In order to understand the relevance of these properties, it is useful to compare this index with other commonly used poverty measures. The *poverty rate* or *headcount ratio* is a commonly used poverty measure. It is defined as the proportion of people whose incomes are under the poverty line: $H = \frac{q}{n}$. This measure meets properties (1) to (4), but it violates the monotonicity, the continuity and the transfer axioms because it does not depend on how far and how unevenly the individual incomes of the poor fall below the poverty line. Using this index to design and to evaluate anti-poverty policies can lead to undesirable results. For instance, the easiest way to reduce the poverty rate is to subsidize the richest of the poor with just barely enough additional income to lift them out of poverty. This seems a very controversial policy action. Another commonly used poverty measure is the *average poverty gap ratio of the poor:* $APGR = \sum_{i=1}^{q} \frac{1}{q}\left(\frac{z - y_i}{z}\right)$. This index violates the transfer axiom because it is insensitive to the distribution of income among the poor. An income transfer from one poor person to another poor person without lifting any of the two out of poverty will not change the average poverty gap ratio. Finally, Sen index is defined as: $S = H \left(APGR + (1 - APGR)\, G_P\right)$, where G_P is the Gini coefficient of the income distribution of the poor. The Sen index satisfies properties (1) to (5), but it violates the continuity and the transfer axioms. These limitations of the Sen index lead Shorrocks to propose the modified Sen index or SST index.

Decomposition. The SST index has a multiplicative decomposition in terms of the poverty rate, the average poverty gap ratio and one plus the Gini coefficient of the censored gap ratios:

$$SST = H * APGR * (1 + G_X)$$

where G_X is the Gini coefficient of the censored gap ratios $x_i \equiv \max\left\{\frac{z - y_i}{z} \,;\, 0\right\}$. See Osberg and Xu (2001) for a proof of this property. This decomposition gives a much more straightforward interpretation of poverty intensity. Furthermore, the formula allows us to compute the index when we do not have access to the original individual level data but we have information on the components H, $APGR$ and G_X.

Geometric interpretation. A very attractive feature of the SST index is its ability to be interpreted geometrically. Let $(x_1, x_2, \ldots, x_n)$ be the vector of censored gap ratios as defined above. For $p \in [0,1]$, let $L(x,p)$ be the Lorenz curve of the vector x evaluated at the value p. Define the *poverty gap profile* as the following function:

$$D(x,p) \equiv \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)(1 - L(x,p))$$

Then, it is possible to show (see Shorrocks, 1995) that the SST index is equal to twice the area below the poverty gap profile: i.e., $SST = 2\int_0^1 D(x,p)dp$.

Standard errors. Poverty indexes are typically calculated using a random sample of individuals from the population and not the whole population. Therefore, these values are estimates of the true population value of the index and they are subject to sampling error. Bishop et al. (1997) show that estimates of the index and its components have a jointly asymptotically normal distribution and the variance-covariance structure can be consistently estimated. When the sample size is relatively small, this asymptotic approximation is not precise enough. In these cases, the *bootstrap method* typically provides a better approximation to the standard error than the asymptotic approximation. The bootstrap method is computationally intensive but conceptually very simple. We take M random samples of size n , with replacement, from our original sample. The larger the value of M the better the approximation. Values of M between 100 and 200 are commonly used. Each of the M samples is called a bootstrap sample. We calculate the SST index for every bootstrap sample. Let SST_m be the value of the index for the m-th bootstrap sample. Then, the bootstrap standard error of SST is just the standard deviation of the bootstrap indexes. That is,

$$se(SST) = \sqrt{\frac{1}{M}\sum_{m=1}^{M}(SST_m - SST)^2}$$

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VICTOR AGUIRREGABIRIA
BOSTON UNIVERSITY