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Identification and counterfactuals in dynamic models of market entry and exit

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Abstract This paper addresses a fundamental identification problem in the structural estimation of dynamic oligopoly models of market entry and exit. Using the standard datasets in existing empirical applications, three components of a firm's profit function are not separately identified: the fixed cost of an incumbent firm, the entry cost of a new entrant, and the scrap value of an exiting firm. We study the implications of this result on the power of this class of models to identify the effects of different comparative static exercises and counterfactual public policies. First, we derive a closed-form relationship between the three unknown structural functions and the two functions that are identified from the data. We use this relationship to provide the correct interpretation of the estimated objects that are obtained under the 'normalization assumptions' considered in most applications. Second, we characterize a class of counterfactual experiments that are identified using the estimated model, despite the non-separate identification of the three primitives. Third, we show that there is a general class of counterfactual experiments of economic relevance that are not identified. We present a numerical example that illustrates how ignoring the non-identification of these counterfactuals (i.e., making a 'normalization assumption' on some of the three primitives) generates sizable biases that can modify even the sign of the estimated effects. Finally, we discuss possible solutions to address these identification problems.

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1 Introduction

Dynamic models of market entry and exit are useful tools in the study of different issues and questions on firm competition for which it is important to consider the endogeneity of the market structure and its evolution over time. During recent years, the structural estimation of this class of models has experienced substantial developments, both methodological and empirical, and there are a growing number of empirical applications.[1] In all of these applications, the answer to the empirical questions of interest is based on the implementation of counterfactual experiments using the estimated model. Sometimes, the purpose of a counterfactual experiment is to evaluate the effects of a hypothetical public policy, such as a new tax or subsidy. In other instances, the main purpose of the experiment is to measure the effects of a parameter change. For instance, we may want to obtain the change in market structure, prices, firm profits, and consumer welfare if we reduce the value of a parameter that captures entry costs by 25 percent.

To estimate dynamic structural models of market entry/exit, we distinguish two main components in a firm's profit function: *variable profit* and *fixed cost*. Parameters in the *variable profit* function (i.e., demand and variable cost parameters) can be identified using data on firms' quantities and prices combined with a demand system and a model of competition in prices or quantities.[2] The *fixed cost* is the part of the profit that derives from buying, selling, or renting inputs that are fixed during the whole active life of the firm.[3] The fixed cost is constant with respect to the amount of output that the firm produces and sells in the market, however, this cost depends on the amount and prices of fixed inputs, such as land or fixed capital, and on the firm's current and past incumbent status. The parameters in the fixed cost are estimated using data on firms' choices of whether to be active in the market, combined with a dynamic model of market entry/exit. The identification of this fixed cost function is based on the principle of revealed preference. If a firm chooses to be active

[1] Examples of recent applications are: Ryan (2012) on environmental regulation of an oligopoly industry; Suzuki (2013) on land use regulation and competition in retail industries; Kryukov (2010) on the relationship between market structure and innovation; Sweeting (2013) on competition in the radio industry and the effects of copyright fees; Collard-Wexler (2013) on demand uncertainty and industry dynamics; Snider (2009) on predatory pricing in the airline industry; or Aguirregabiria and Ho (2012) on airlines network structure and entry deterrence.

[2] See Berry and Haile (2010) and Berry et al. (2013) for recent identification results in the estimation of demand and supply models of differentiated products.

[3] There is some abuse of language in using the term "cost" to refer to this component of a firm's profit. This fixed component of the profit may include the positive income/profits associated with sales of owned inputs, such as land and buildings. Therefore, in this paper, we sometimes use "fixed profit" instead of "fixed cost" to denote this component.

in the market, it does so because the firm's expected value of being in the market is greater than its expected value of not being in the market. Therefore, a firm's choice reveals information about structural parameters affecting the firm's profit and value.

This paper addresses a fundamental identification problem in the structural estimation of dynamic models of market entry and exit. Using the standard datasets in existing empirical applications, three key components of a firm's fixed cost function are not separately identified: the fixed cost of an incumbent firm, the entry cost of a new entrant, and the exit value, or scrap value, of an exiting firm.[4] This non-identification result can be considered as an application to dynamic models of market entry and exit of Proposition 2 in Magnac and Thesmar (2002) on the under-identification of a general dynamic discrete choice model. In the existing applications of dynamic models of market entry and exit, the approach to address this identification problem is to normalize one of three functions to zero. This is often referred to as a 'normalization' assumption. The most common 'normalization' is making the scrap value equal to zero. This approach is used in applications such as those presented by Snider (2009), Aguirregabiria and Mira (2007), Ellickson et al. (2012), Lin (2012), Collard-Wexler (2013), Dunne et al. (2013), Igami (2013), Suzuki (2013), or Varela (2013), among others. In other papers, such as Pakes et al. (2007), Ryan (2012), Santos (2013) or Sweeting (2013), the normalization involves making the fixed cost equal to zero.[5]

Using this non-identification result as a starting point, the purpose of this paper is to study the implications of the 'normalization' approach on the interpretation of the estimated structural functions and, most importantly, on the identification of the effects of comparative static exercises or counterfactual experiments using the estimated model. This issue is important because many empirical questions on market competition, as well as on the evaluation of the effects of public policies in oligopoly industries, involve examining counterfactual changes in some of these structural functions, e.g., Ryan (2012), Dunne et al. (2013), Lin (2012), or Varela (2013), among others. We find that a 'normalization' is not always innocuous for some empirical questions. For those cases, we propose alternative approaches to address this identification problem.

First, we derive a closed-form relationship between the three unknown structural functions and the two functions that are identified from the data. We use this relationship to provide the correct interpretation of the estimated objects that are obtained under the 'normalization assumptions' considered in applications. Second, we study the identification of counterfactual experiments. We characterize a class of counterfactuals that are identified using the estimated model, despite the non-separate identification of the three primitives. This class of identified counterfactuals

[4]This identification problem is *fundamental* in that it does not depend on other econometric issues that appear in this class of models, such as the stochastic structure of unobservables, the non-independence between observable and unobservable state variables, or the existence of multiple equilibria in the data. These issues may generate additional identification problems. However, addressing or solving these other identification problems does not help separately identify the three components in the fixed cost function.

[5]Although making the entry cost equal to zero is another possible normalization, this approach has not been common in empirical applications.

consists of an additive change in the structural function(s) where the change is known to the researcher. We also show that there is a general class of counterfactual experiments of economic relevance that are not identified. For instance, the effects of a change in the stochastic process of the price of a fixed input that is an argument in the entry cost, fixed cost, and exit value functions (e.g., land price) is not identified. We present numerical examples that illustrate how ignoring the non-identification of these counterfactuals (i.e., making a 'normalization assumption' on some of the three primitives) generates sizable biases that can modify even the sign of the estimated effects. Finally, we discuss possible solutions to address these identification problems. We show that a particular type of exclusion restrictions provides identification. Furthermore, in industries where the trade of firms is frequent and where the researcher observes transaction prices (Kalouptsidi (Forthcoming)), this information can be used to solve this identification problem. In the absence of this type of data, the researcher can apply a bounds approach in the spirit of Manski (1995). We derive expressions for the bounds of the three functions using this approach.

The rest of the paper proceeds as follows. Section 2 presents the model of market entry and exit. Section 3 describes the identification problem and the relationship between structural functions and identified objects. Section 4 addresses the identification of counterfactual experiments and presents numerical examples. In Section 5, we discuss different approaches to address this identification problem. We summarize and conclude in Section 6.

2 Dynamic model of market entry and exit

2.1 Basic model

We start with a single-firm version of the model or dynamic model of monopolistic competition. Later in this section, we extend our framework to dynamic games of oligopoly competition. Time is discrete and indexed by t. Every period t the firm decides to be active in the market or not. A firm is defined as active if it owns or rents some fixed inputs that are necessary to operate in this market, e.g., land, equipment.[6] Let $a_t \in \{0, 1\}$ be the binary indicator of the firm's decision at period t, such that $a_t = 1$ if the firm decides to be active in the market at period t, and $a_t = 0$ otherwise. The firm takes this action to maximize its expected and discounted flow of profits, $\mathbb{E}_t \left(\sum_{r=0}^{\infty} \beta^r \Pi_{t+r}\right)$, where $\beta \in (0, 1)$ is the discount factor, and Π_t is the firm's profit at period t. We distinguish two main components in the firm's profit at time t: variable profits, VP_t, and fixed profits (or fixed costs), FP_t, with $\Pi_t = VP_t + FP_t$. The variable profit is equal to the difference between revenue and variable costs. It varies continuously with the firm's output and it is equal to zero when output is zero. If active in the market, the firm observes its demand curve and variable cost

[6]In principle, a firm may be active in the market but producing zero output. However, whether we allow for that possibility or not is irrelevant for the (non) identification results in this paper.

function and chooses its price to maximize variable profits at period t. This static price decision determines an indirect variable profit function that relates this component of profit with state variables:

$$VP_t = a_t \ \ vp(\mathbf{z}_t^v) \tag{1}$$

where $vp(.)$ is a real-valued function, and $\mathbf{z}_t^v$ is a vector of exogenous state variables affecting demand and variable costs, e.g., market size, consumers' socioeconomic characteristics, and prices of variable inputs such as wages, price of materials, energy, etc.[7]

The fixed profit is the part of the profit that derives from buying, selling, or renting inputs that are fixed during the active life of the firm and that are necessary for the firm to operate in the industry. These fixed inputs may include land, buildings, some type of equipment, or even managerial skills. We distinguish three components in the fixed profit: the fixed cost of an active firm, FC_t, the entry cost of a new entrant, EC_t, and the scrap value of an exiting firm, SV_t. Each of these three components may depend on a vector of exogenous state variables $\mathbf{z}_t^c$ that includes prices of fixed inputs (e.g., prices of land and fixed capital inputs). This vector may have elements in common with the vector $\mathbf{z}_t^v$ (e.g., the market size may affect both variable profit and fixed costs):

$$\begin{aligned} FP_t &= -FC_t - EC_t + SV_t \\ &= -a_t \, fc\left(\mathbf{z}_t^c\right) - a_t \, (1-k_t) \, ec\left(\mathbf{z}_t^c\right) + (1-a_t) \, k_t sv\left(\mathbf{z}_t^c\right) \end{aligned} \tag{2}$$

where $fc\,(.)$, $ec\,(.)$, and $sv\,(.)$ are real-valued functions and $k_t \equiv a_{t-1}$ is the indicator of the event "the firm was active at period $t-1$", or, equivalently, the firm had the fixed input at period $t-1$. The fixed cost is paid in every period during which the firm is active (i.e., when $a_t = 1$). The fixed cost includes the cost of renting some fixed inputs and taxes that should be paid every period and that depend on the amount of some owned fixed inputs, e.g., property taxes. The entry cost is paid if the firm is active in the current period but was not active in the previous period (i.e., if $a_t = 1$ and $k_t = 0$), and the entry cost includes the cost of purchasing fixed inputs and transaction costs related to the startup of the firm. The firm receives a scrap or exit value if it was active in the previous period but decides to exit in the current period (i.e., if $a_t = 0$ and $k_t = 1$). This scrap value includes earnings from selling owned fixed inputs minus transaction costs related to closing the firm such as compensations to workers and to lessors of capital due to breaking long-term contracts.

[7]The variable profit function is $VP_t = p_t \ D(p_t, \mathbf{z}_t^v) - VC(q_t, \mathbf{z}_t^v)$, where p_t is the firm's price, $D(p_t, \mathbf{z}_t^v)$ is the demand function, and VC is the variable cost function that depends on output q_t. Maximization of variable profit implies the well-known condition of marginal revenue equal to marginal cost, $D(p_t, \mathbf{z}_t^v)+$ $p_t \ [\partial D(p_t, \mathbf{z}_t^v)/\partial p_t]-\ MC(D(p_t, \mathbf{z}_t^v), \mathbf{z}_t^v) \ [\partial D(p_t, \mathbf{z}_t^v)/\partial p_t] = 0$, where MC represents the marginal cost function. Using this condition we can get the optimal pricing function $p_t = p^*(\mathbf{z}_t^v)$, and plugging-in this optimal price into the variable profit function, we get the indirect variable profit function $vp(\mathbf{z}_t^v) \equiv p^*(\mathbf{z}_t^v)$ $D(p^*(\mathbf{z}_t^v), \mathbf{z}_t^v)-\ VC(D(p^*(\mathbf{z}_t^v), \mathbf{z}_t^v), \mathbf{z}_t^v)$.

Example 1 Consider the decision of a hotel chain about whether to operate a hotel in a local market or small town. To start its operation (entry in the market), the firm should purchase or lease some fixed inputs, such as land, a building, furniture, elevators, a restaurant, a kitchen, and other equipment. If this equipment is purchased at the time of entry, the cost of purchasing these inputs is part of the entry cost. Other components of the entry cost are the cost of a building permit or, in the case of franchises, franchise fees. Some fixed inputs are leased. Therefore, a hotel's fixed cost includes the rental cost of leased fixed inputs. It also includes property taxes (that depend on land prices), royalties to the franchisor, and the maintenance costs of owned fixed inputs. At the time of its closure, the hotel operator may recover some money by selling owned fixed inputs such as land, buildings, furniture, and other equipment. These amounts correspond to the scrap value.

The one-period profit function can be described as:[8]

$$\Pi_t = \begin{cases} k_t sv\left(\mathbf{z}_t^c\right) & \text{if } a_t = 0 \\ \\ vp(\mathbf{z}_t^v) - fc\left(\mathbf{z}_t^c\right) - (1-k_t)\, ec\left(\mathbf{z}_t^c\right) & \text{if } a_t = 1 \end{cases} \tag{3}$$

The vector of state variables of this dynamic model is $\{\mathbf{z}_t, k_t\}$, where $\mathbf{z}_t \equiv \{\mathbf{z}_t^v, \mathbf{z}_t^c\}$. The vector of state variables $\mathbf{z}_t$ follows a Markov process with transition probability function $f_z(\mathbf{z}_{t+1}|\mathbf{z}_t)$. The indicator of incumbent status follows the trivial endogenous transition rule, $k_{t+1} = a_t$.

In the econometric or empirical version of the model, we distinguish between two different types of state variables: the variables that are observable to the researcher and those that are unobservable. Here, we consider a general additive specification of the unobservables:

$$\begin{aligned} FC_t &= a_t \left[fc(\mathbf{z}_t^c) + \varepsilon_t^{fc}\right] \\ EC_t &= a_t\,(1-k_t)\left[ec(\mathbf{z}_t^c) + \varepsilon_t^{ec}\right] \\ SV_t &= (1-a_t)\,k_t\left[sv(\mathbf{z}_t^c) + \varepsilon_t^{sv}\right] \end{aligned} \tag{4}$$

where $\varepsilon_t \equiv \{\varepsilon_t^{fc}, \varepsilon_t^{ec}, \varepsilon_t^{sv}\}$ is the vector of state variables that are observable to the firm at period t but unobservable to the researcher. Let $\mathbf{z}_t$ be the vector with all the observable exogenous state variables, i.e., $\mathbf{z}_t = \left(\mathbf{z}_t^v, \mathbf{z}_t^c\right)$. We assume that the unobserved state variables in ε_t are i.i.d. over time and independent of $(\mathbf{z}_t, k_t)$ (Rust 1994). Without loss of generality these unobserved variables have zero mean. Allowing for serially correlated unobservables does not have any

[8] In this version of the model, there is no "time-to-build" or "time-to-exit" such that the decision of being active or not in the market is taken at period t and it is effective at the same period, without any lag. At the end of this section we discuss variations of the model that involve "time-to-build" or/and "time-to-exit". These variations do not have any incidence in our (non) identification results, though they imply some minor changes in the interpretation of the identified objects.

substantive influence on the positive or negative identification results in this paper. Serial correlation in the unobservables creates the so called "initial conditions problem", that is an identification problem of different nature to the one we study in this paper. Whatever the way the researcher deals with the "initial conditions problem", he still faces the problem of separate identification of the three components in the fixed profit.

The model does not include an additive error in the variable profit. This deserves an explanation. First, note that an additive and homoskedastic error term in the variable profit (i.e., $VP_t = a_t \left[vp(\mathbf{z}_t^{vp}) + \varepsilon_t^{vp}\right]$) is redundant with respect to the additive term in the fixed cost because it simply replaces an error $-\varepsilon_t^{fc}$ with an error $\varepsilon_t^{vp} - \varepsilon_t^{fc}$, and the two error structures have the same empirical implications. Second, unobservables in demand and marginal costs do not imply, in general, an error term in the variable profit that is additive and homoskedastic. Therefore, that type of error does not seem plausible. Third, in applications where the researcher has data on prices and quantities, one can estimate demand and marginal cost functions in a first stage, before the estimation of fixed costs in the dynamic structural model. Once these functions are estimated, the unobservables in demand and marginal costs can be recovered as residuals and be treated as observables in the estimation of the fixed costs, i.e., they become part of the observable vector $\mathbf{z}_t^v$ and not of the unobservable ε_t^{vp}. For instance, that is the case of the unobservables ξ and ω in the standard Berry-Levinsohn-Pakes (BLP) model of demand and price competition in a differentiated product industry (Berry et al. 1995). When data on prices and quantities are not available, the typical application assumes that the variable profit is proportional to observable market size and it does not include an error term in the variable profit.

The specification of the one-period profit function including unobservable state variables is:

$$\Pi_t = \pi\left(a_t, k_t, z_t\right) + \varepsilon_t\left(a_t\right) = \begin{cases} k_t sv\left(\mathbf{z}_t^c\right) + \varepsilon_t(0) & \text{if } a_t = 0 \\ vp(\mathbf{z}_t^v) - fc\left(\mathbf{z}_t^c\right) - (1-k_t)\, ec\left(\mathbf{z}_t^c\right) + \varepsilon_t(1) & \text{if } a_t = 1 \end{cases} \tag{5}$$

where $\pi(.)$ is the component of the one-period profit that does not depend on unobservables, $\varepsilon_t(0) \equiv k_t\ \varepsilon_t^{sv}$, and $\varepsilon_t(1) \equiv -\varepsilon_t^{fc} - (1-k_t)\,\varepsilon_t^{ec}$.

The value function of the firm, $V\left(k_t, \mathbf{z}_t, \varepsilon_t\right)$, is the unique solution to the Bellman equation:

$$V\left(k_t, \mathbf{z}_t, \varepsilon_t\right) = \max_{a_t \in \{0,1\}} \left\{v\left(a_t, k_t, \mathbf{z}_t\right) + \varepsilon_t\left(a_t\right)\right\} \tag{6}$$

where $v\left(a_t, k_t, z_t\right)$ is the *conditional choice value function*

$$v\left(a_t, k_t, z_t\right) \equiv \pi\left(a_t, k_t, \mathbf{z}_t\right) + \beta \sum_{\mathbf{z}_{t+1} \in \mathbf{Z}} f_z(\mathbf{z}_{t+1}|\mathbf{z}_t) \int V\left(a_t, \mathbf{z}_{t+1}, \varepsilon_{t+1}\right)\, dG\left(\varepsilon_{t+1}\right). \tag{7}$$

where $\beta \in (0, 1)$ is the discount factor, and $G(.)$ is the CDF of ε_t. Similarly, the optimal decision rule of this dynamic programming problem is a function $\alpha(k_t, \mathbf{z}_t, \varepsilon_t)$ from the space of state variables into the action space $\{0, 1\}$ such that:

$$\alpha(k_t, \mathbf{z}_t, \varepsilon_t) = \arg \max_{a_t \in \{0,1\}} \{v(a_t, k_t, \mathbf{z}_t) + \varepsilon_t(a_t)\} \tag{8}$$

By the additivity and the conditional independence of the unobservable ε's, the optimal decision rule has the following threshold structure:

$$\alpha(k_t, \mathbf{z}_t, \varepsilon_t) = 1\{\widetilde{\varepsilon}_t \leq \tilde{v}(k_t, \mathbf{z}_t)\} \tag{9}$$

where $\tilde{v}(k_t, \mathbf{z}_t) = v(1, k_t, \mathbf{z}_t) - v(0, k_t, \mathbf{z}_t)$ and $\widetilde{\varepsilon}_t \equiv \varepsilon_t(0) - \varepsilon_t(1) \equiv (\varepsilon_t^{fc} + \varepsilon_t^{ec}) + k_t (\varepsilon_t^{sv} - \varepsilon_t^{ec})$.

For our analysis, it is helpful to define also the *Conditional Choice Probability* (CCP) function $P(k_t, \mathbf{z}_t)$ that is the optimal decision rule integrated over the unobservables:

$$\begin{aligned} P(k, \mathbf{z}) &\equiv \Pr(\alpha(k_t, \mathbf{z}_t, \varepsilon_t) = 1 | k_t = k, \mathbf{z}_t = \mathbf{z}) \\ &= \Pr(\widetilde{\varepsilon}_t \leq \tilde{v}(k, \mathbf{z})) \\ &= F_{\widetilde{\varepsilon}|k}(\tilde{v}(k, \mathbf{z})) \end{aligned} \tag{10}$$

where $F_{\widetilde{\varepsilon}|k}$ is the CDF of $\widetilde{\varepsilon}_t$ conditional on $k_t = k$.[9] Note that $P(0, \mathbf{z}_t)$ is the probability of market entry, and $[1 - P(1, \mathbf{z}_t)]$ is the probability of market exit.

Appendix A presents three extensions, or variations, of this basic model: (a) model with no re-entry after market exit; (b) model with time-to-build and time-to-exit; and (c) model with capital investment. In the same appendix, we also show that our (non) identification results extend to these models. The next subsection deals with an extension that we consider particularly interesting: a dynamic oligopoly game of market entry and exit.

2.2 Dynamic oligopoly game of entry and exit

We follow the standard structure of dynamic oligopoly models in Ericson and Pakes (1995) but including firms' private information as in Doraszelski and Satterthwaite (2010).[10] There are N firms that may operate in the market. Firms are indexed by $j \in \{1, 2, ..., N\}$. Every period t, the N firms decide simultaneously but independently whether to be active or not in the market. Let a_{jt} be the binary indicator for the event "firm j is active in the market at period t". Variable profits at period t are determined

[9] The distribution of $\widetilde{\varepsilon}_t$ depends on k_t if the entry cost and the scrap value contain unobservable components and these unobservables are different, i.e., $\varepsilon_t^{sv} - \varepsilon_t^{ec} \neq 0$.

[10] In Ericson and Pakes (1995), there is time-to-build in the timing of firms' decisions. Here we consider a version of the dynamic game without time-to-build. As described below, all our results on (non-)identification apply similarly to models with or without time-to-build.

in a static Cournot or Bertrand model played between those firms who choose to be active. This static competition determines the indirect variable profit functions of the N firms: $VP_{jt} = a_{jt}\ vp_j(\boldsymbol{a}_{-jt}, \mathbf{z}_t^v)$, where VP_{jt} is the variable profit of firm j; $\boldsymbol{a}_{-jt}$ is the $N-1$ dimensional vector with the binary indicators for the activity of all the firms except firm j. Note that the variable profit functions, vp_j, can vary across firms due to permanent, common knowledge differences between the firms in variable costs or in the quality of their products.

The three components of the fixed profit function have specifications similar to the case of monopolistic competition, with the only differences that the functions can vary across firms, and the unobservable $\varepsilon's$ are private information shocks of each firm: $FC_{jt} = a_{jt}\ [fc_j(\mathbf{z}_t^c) + \varepsilon_{jt}^{fc}]$; $EC_{jt} = a_{jt}\ \left(1-k_{jt}\right)[ec_j(\mathbf{z}_t^c) + \varepsilon_{jt}^{ec}]$; and $SV_{jt} = \left(1-a_{jt}\right) k_{jt}\ [sv_j(\mathbf{z}_t^c) + \varepsilon_{jt}^{sv}]$. Now, $\varepsilon_{jt} \equiv \{\varepsilon_{jt}^{fc}, \varepsilon_{jt}^{ec}, \varepsilon_{jt}^{sv}\}$ is a vector of state variables that are private information for firm j, they are unobservable to the researcher, and i.i.d. across firms and over time with distribution function G.

Following the literature on dynamic games of oligopoly competition, we assume that the outcome of this game is a Markov Perfect Equilibrium (MPE). In a MPE, a firm's strategy is a function from the space of payoff relevant state variables (known to the firm) into the space of possible actions, that in this model is {0, 1}. Let $\alpha_j\left(\mathbf{k}_t, \mathbf{z}_t, \varepsilon_{jt}\right)$ be a strategy function for firm j , where $\mathbf{k}_t$ is the vector $\{k_{jt} : j = 1, 2, ..., N\}$ with firms' indicators of previous incumbent status, $k_{jt} \equiv a_{jt-1}$. A MPE is a N-tuple of strategy functions, $\alpha \equiv \{\alpha_j : j = 1, 2, ...N\}$ such that every firm maximizes its expected value given the strategies of the other firms.

As we did in the model for a monopolistic firm, we can represent firms' strategies using CCP functions: $P_j(\mathbf{k}, \mathbf{z}) \equiv \Pr\left(\alpha_j\left(\mathbf{k}_t, \mathbf{z}_t, \varepsilon_{jt}\right) = 1 | \mathbf{k}_t = \mathbf{k}, \mathbf{z}_t = \mathbf{z}\right)$. Suppose that firm j believes that the other firms will behave now and in the future according to their respective strategies in the N-tuple of CCP functions $\mathbf{P} \equiv \{P_i : i = 1, 2, ...N\}$. Given these beliefs, the expected profit of firm j is:

$$\pi_j^{\mathbf{P}}\left(a_{jt}, \mathbf{k}_t, \mathbf{z}_t\right) \equiv (1-a_{jt})k_{jt}sv_j\left(\mathbf{z}_t^c\right) - a_{jt}fc_j\left(\mathbf{z}_t^c\right) - a_{jt}\left(1-k_{jt}\right)ec_j\left(\mathbf{z}_t^c\right)$$
$$+\ a_{jt}\sum\nolimits_{\boldsymbol{a}_{-jt}\in\{0,1\}^{N-1}} \Pr\left(\boldsymbol{a}_{-jt} | \mathbf{k}_t, \mathbf{z}_t; \mathbf{P}\right) vp_j(\boldsymbol{a}_{-jt}, \mathbf{z}_t^v) \tag{11}$$

where $\Pr\left(\boldsymbol{a}_{-jt} | \mathbf{k}_t, \mathbf{z}_t; \mathbf{P}\right) \equiv \prod_{i\neq j} P_i(\mathbf{k}_t, \mathbf{z}_t)^{a_{it}}[1 - P_i(\mathbf{k}_t, \mathbf{z}_t)]^{1-a_{it}}$. Similarly, from the point of view of firm j, the expected transition probability of the state variables $(\mathbf{k}_t, \mathbf{z}_t)$ is:

$$f_j^{\mathbf{P}}(\mathbf{k}_{t+1}, \mathbf{z}_{t+1} | a_{jt}, \mathbf{k}_t, \mathbf{z}_t) \equiv 1\left\{k_{jt+1} = a_{jt}\right\} \Pr\left(\mathbf{k}_{-jt+1} | \mathbf{k}_t, \mathbf{z}_t; \mathbf{P}\right) f_z(\mathbf{z}_{t+1} | \mathbf{z}_t) \tag{12}$$

where 1{.} is the indicator function. Therefore, given firm j's beliefs, we can define its value function $V_j^{\mathbf{P}}(\mathbf{k}_t, \mathbf{z}_t, \varepsilon_{jt})$ as the solution of the Bellman equation:

$$V_j^{\mathbf{P}}(\mathbf{k}_t, \mathbf{z}_t, \varepsilon_{jt}) = \max_{a_{jt}\in\{0,1\}} \left\{v_j^{\mathbf{P}}\left(a_{jt}, \mathbf{k}_t, \mathbf{z}_t\right) + \varepsilon_{jt}\left(a_{jt}\right)\right\} \tag{13}$$

with

$$v_j^{\mathbf{P}}\left(a_{jt}, \mathbf{k}_t, \mathbf{z}_t\right) \equiv \pi_j^{\mathbf{P}}\left(a_{jt}, \mathbf{k}_t, \mathbf{z}_t\right) + \beta \sum_{\mathbf{z}', \mathbf{k}'} f_j^{\mathbf{P}}(\mathbf{z}', \mathbf{k}' | a_{jt}, \mathbf{k}_t, \mathbf{z}_t) \int V_j^{\mathbf{P}}\left(\mathbf{z}', \mathbf{k}', \varepsilon_j'\right) dG\left(\varepsilon_j'\right) \tag{14}$$

A firm's best response function is the optimal decision rule associated to this Bellman equation. A MPE is a N-tuple of strategy functions such that the strategy of every firm is the best response to the strategies of the other firms, i.e., every firm maximizes his intertemporal value given the strategies of the other firms. We can represent a MPE as a fixed point of a mapping in the space of players' CCPs. For every player j and any state $(\mathbf{k}, \mathbf{z})$, the equilibrium condition is:

$$P_j(\mathbf{k}, \mathbf{z}) = F_{\widetilde{\varepsilon}|k}\left(\tilde{v}_j^{\mathbf{P}}(\mathbf{k}, \mathbf{z})\right) \tag{15}$$

where $\tilde{v}_j^{\mathbf{P}}(\mathbf{k}, \mathbf{z})$ is the differential value function $v_j^{\mathbf{P}}(1, \mathbf{k}, \mathbf{z}) - v_j^{\mathbf{P}}(0, \mathbf{k}, \mathbf{z})$.

3 Identification of structural functions

3.1 Conditions on data generating process

Suppose that the researcher has panel data with realizations of firms' decisions over multiple markets/locations and time periods. We use the letter m to index markets. The researcher observes a random sample of M markets with information on $\{a_{jmt}, \mathbf{z}_{mt}, k_{jmt} : j = 1, 2, ..., N, t = 1, 2, ..., T\}$, where N and T are small (they can be as small as $N = T = 1$) and M is large. For the identification results in this section, we assume that M is infinite and $T = 1$. For most of the rest of the paper, we assume that the variable profit functions $vp_j(.)$ are known to the researcher or, more precisely, that they have already been identified using data on firms' prices, quantities, and exogenous demand and variable cost characteristics. However, we also discuss the case in which the researcher does not have data on prices, quantities, or revenue to identify the variable profit function in a first step.

We want to use this sample to estimate the structural 'parameters' or functions of the model: the three functions in the fixed profit, $fc_j\left(\mathbf{z}_t^c\right)$, $ec_j\left(\mathbf{z}_t^c\right)$, and $sv_j(\mathbf{z}_t^c)$; the transition probability of the state variables, f_z; and the distribution of the unobservables $F_{\widetilde{\varepsilon}|k}$. Following the standard approach in dynamic decision models, we assume that the discount factor β is known to the researcher. The transition probability function $\{f_z\}$ is nonparametrically identified.[11] Therefore, we assume that $\{vp_j(.), f_z, \beta\}$ are known, and we concentrate on the identification of the functions $fc_j(.)$, $ec_j(.)$, $sv_j(.)$, and $F_{\widetilde{\varepsilon}|k}$.

[11]Note that $f_z(\mathbf{z}'|\mathbf{z}) = \Pr(\mathbf{z}_{mt+1} = \mathbf{z}' \mid \mathbf{z}_{mt} = \mathbf{z})$. Under mild regularity conditions, we can consistently estimate these conditional probabilities using a nonparametric method such as a kernel or sieve method.

All of our identification results apply very similarly to the model of monopolistic competition and to the dynamic game of oligopoly competition. Given the identification of the variable profit function $vp_j(.)$ and given players' CCP functions, it is clear that the expected variable profit of firm j in the dynamic game is also identified.[12] From an identification point of view, a relevant difference between the monopolistic and oligopoly models is that in the oligopoly case, the CCP function of a firm depends not only on its own incumbent status k_{jt} but also on the incumbent status of all firms, as represented by vector k_t. Because a firm's fixed profit does not depend on the incumbent status of the other firms, one may believe that this exclusion restriction might help separately identify the three components of a firm's fixed profit. However, this conjecture is not correct. The oligopoly model provides additional over-identifying restrictions that can be tested; however, these over-identifying restrictions do not help in the separate identification of the three components of the fixed cost. Therefore, for notational simplicity, we omit the firm subindex for the rest of the paper and use the notation of the monopolistic case. When necessary, we comment on some differences to the dynamic oligopoly game and on why the additional restrictions implied by the dynamic game do not help in our identification problem.

3.2 Identification of the distribution of the unobservables

Suppose that the variable profit function is estimated in a first step using data on prices and quantities. Let $vp_t \equiv vp(\mathbf{z}_t^v)$ be the estimated variable profit after this first step such that the vp_t is known to the researcher at every observation in the sample. Proposition 1 establishes conditions for the identification of the distributions $F_{\widetilde{\varepsilon}|0}$ and $F_{\widetilde{\varepsilon}|1}$.

Proposition 1 *Suppose that the following conditions hold: (a) the firm lasts for finite periods;*[13] *(b) the vector of unobservables ε_t is independent of $(k_t, \mathbf{z}_t)$; (c) functions $F_{\widetilde{\varepsilon}|0}$ and $F_{\widetilde{\varepsilon}|1}$ are strictly increasing over the real line; and (d) vp_t has continuous support, and for any value of $(k, \mathbf{z}^c)$, the CCP function $P(vp, k, \mathbf{z}^c) \equiv \Pr(a_t = 1|vp_t = vp, k_t = k, \mathbf{z}_t^c = \mathbf{z}^c)$ is strictly increasing in vp and it can reach values arbitrarily close to 0 and 1. Under these conditions, the distributions $F_{\widetilde{\varepsilon}|0}$ and $F_{\widetilde{\varepsilon}|1}$ are nonparametrically identified. Furthermore, the nonparametric estimation of these distributions can be implemented separately from the estimation of the structural functions in the fixed profit.*

Now, suppose that the dataset does not include information on prices and quantities such that the variable profit cannot be estimated in a first step. In this context, the specification of the (indirect) variable profit function typically follows the approach

[12]Note that the expected variable profit $vp_j^{\mathbf{P}}(\mathbf{k}, \mathbf{z})$ is equal to $\sum_{a_{-j}} [\prod_{i \neq j} P_i(\mathbf{k}, \mathbf{z})^{a_i} (1 - P_i(\mathbf{k}, \mathbf{z}))^{1-a_i}$ $] \, vp_j(a_{-j}, \mathbf{z}^v)$. Therefore, given vp_j and CCPs $\{P_i : i \neq j\}$, the expected variable profit function $vp_j^{\mathbf{P}}$ is known.

[13]To the best of our knowledge, there is not yet a proof of identification of these distributions in an infinite horizon model.

in the seminal work by Bresnahan and Reiss (1990, 1991a, 1991b, and 1994). In the monopolistic case, this specification is $VP_t = y_t\ \alpha(\mathbf{z}_t^v)$, where y_t represents market size and $\alpha(.)$ may be a constant or a function that depends on variables other than market size. That is, variable profit is proportional to observable market size, and market size does not enter in the fixed profit. Given this semiparametric specification of the variable profit, we have that Proposition 1 applies to this case with only two differences: (1) condition (1d) on the variable vp_t is replaced by a similar condition on market size y_t, i.e., market size y_t has continuous support, and for any value of $(k_t, \mathbf{z}_t^c, \mathbf{z}_t^v)$, the CCP function $P(y, k, \mathbf{z}^c, \mathbf{z}^v) \equiv \Pr(a_t = 1 | y_t = y, k_t = k, \mathbf{z}_t^c = \mathbf{z}^c, \mathbf{z}_t^v = \mathbf{z}^v)$ is strictly increasing in y and it can reach values arbitrarily close to 0 and 1; and (2) the distributions $F_{\widetilde{\varepsilon}|0}$ and $F_{\widetilde{\varepsilon}|1}$ are identified up to a scale parameter. This also implies that the $\alpha(.)$ function in the variable profit, and the functions in the fixed profit are identified up to a scale parameter.

For the rest of the paper, we assume that the distributions $F_{\widetilde{\varepsilon}|0}$ and $F_{\widetilde{\varepsilon}|1}$ are identified. All our results below apply also to the case where these distributions are identified up to scale with the only difference that in this case the results apply to the fixed cost, entry cost, and scrap value functions divided by an unknown scale parameter.

3.3 Identification of functions in the fixed profit

By definition, the CCP function $P(k, \mathbf{z})$ is equal to the conditional expectation $\mathbb{E}(a_{mt} \mid k_{mt} = k, \mathbf{z}_{mt} = \mathbf{z})$ and therefore, it is nonparametrically identified using data on $\{a_{mt}, k_{mt}, \mathbf{z}_{mt}\}$. Given the CCP function $P(k, \mathbf{z})$ and the inverse distribution $F_{\widetilde{\varepsilon}|k}^{-1}$, we have that the differential value function $\tilde{v}\,(k, \mathbf{z})$ is identified from the expression:

$$\tilde{v}\,(k, \mathbf{z}) = F_{\widetilde{\varepsilon}|k}^{-1}\,(P(k, \mathbf{z}))\,. \tag{16}$$

Given the identification of $F_{\widetilde{\varepsilon}|0}$ and $F_{\widetilde{\varepsilon}|1}$, function $\tilde{v}\,(k, \mathbf{z})$ is nonparametrically identified everywhere, such that we can treat $\tilde{v}\,(k, \mathbf{z})$ as a known/identified function. $\tilde{v}\,(k, \mathbf{z})$ is the value of being active in the market minus the value of not being active for a firm with incumbent status "k" at previous period. This differential value is equal to the inverse function of $F_{\widetilde{\varepsilon}|k}$ evaluated at $P(k, \mathbf{z})$, that is the probability of being active in the market for a firm with incumbent status "k" at previous period.

Functions $\tilde{v}\,(k, \mathbf{z})$, $vp(\mathbf{z}^v)$, and $F_{\widetilde{\varepsilon}|k}$ summarize all the information in the data that is relevant for the identification of the three functions in the fixed profit. We now derive a closed-form relationship between these identified functions and the unknown structural functions fc, ec, and sv. Given that by construction $\tilde{v}\,(k, \mathbf{z})$ is equal to $v\,(1, k, z) - v\,(0, k, z)$, and given the definition of conditional choice value function in Equation (7), we have the following system of equations: for any value of $(k, \mathbf{z})$,

$$\tilde{v}(k, \mathbf{z}) = vp(\mathbf{z}^v) - \left[fc\left(\mathbf{z}^c\right) + ec\left(\mathbf{z}^c\right)\right] + k\left[ec\left(\mathbf{z}^c\right) - sv\left(\mathbf{z}^c\right)\right] + \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \left[\bar{V}\left(1, \mathbf{z}'\right) - \bar{V}\left(0, \mathbf{z}'\right)\right] \tag{17}$$

where $\bar{V}(k, \mathbf{z})$ is the integrated value function $\int V(k, \mathbf{z}, \varepsilon)\, dG(\varepsilon)$, i.e., the value function integrated over the distribution of the unobservables in ε. This system summarizes all the restrictions that the model and data impose on the structural functions.

Using the definition of the integrated value function $\bar{V}(k, \mathbf{z})$, we can express it as follows:

$$\begin{aligned} \bar{V}(k, \mathbf{z}) &= \int \max_{a \in \{0,1\}} \{v(a, k, \mathbf{z}) + \varepsilon(a)\}\, dG(\varepsilon) \\ &= v(0, k, \mathbf{z}) + \int \max\{0; \widetilde{v}(k, \mathbf{z}) - \widetilde{\varepsilon}\}\, dF_{\widetilde{\varepsilon}|k}(\widetilde{\varepsilon}) \\ &= v(0, k, \mathbf{z}) + S(\tilde{v}(k, \mathbf{z}), F_{\widetilde{\varepsilon}|k}), \end{aligned} \tag{18}$$

where $S(\tilde{v}(k, \mathbf{z}), F_{\widetilde{\varepsilon}|k})$ represents the function $\int_{-\infty}^{\tilde{v}(k,\mathbf{z})} [\tilde{v}(k, \mathbf{z}) - \widetilde{\varepsilon}]\, dF_{\widetilde{\varepsilon}|k}(\widetilde{\varepsilon})$. Note that the arguments of function S, i.e., functions $\tilde{v}$ and $F_{\widetilde{\varepsilon}|k}$, are identified. Therefore, function S is also a known or identified function. Plugging expression $\bar{V}(k, \mathbf{z}) = v(0, k, \mathbf{z}) + S(\tilde{v}(k, \mathbf{z}), F_{\widetilde{\varepsilon}|k})$ into equation (17), and taking into account that $v(0, 1, \mathbf{z}) - v(0, 0, \mathbf{z}) = sv(\mathbf{z}^c)$, we have the following system of equations that summarizes all the restrictions that the data and model impose on the unknown structural functions fc, ec , and sv.

$$\begin{aligned} \tilde{v}(k, \mathbf{z}) &= vp(\mathbf{z}^v) - \left[fc(\mathbf{z}^c) + ec(\mathbf{z}^c)\right] + k\left[ec(\mathbf{z}^c) - sv(\mathbf{z}^c)\right] + \beta \sum_{\mathbf{z}^{c\prime} \in \mathbf{Z}^c} f_z(\mathbf{z}^{c\prime}|\mathbf{z}) sv(\mathbf{z}^{c\prime}) \\ &\quad + \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \left[S(\tilde{v}(1, \mathbf{z}'), F_{\widetilde{\varepsilon}|1}) - S(\tilde{v}(0, \mathbf{z}'), F_{\widetilde{\varepsilon}|0})\right] \end{aligned} \tag{19}$$

To study the identification of functions fc, ec, and sv, it is convenient to sum up all the identified functions in equation (19) into a single term. Define the function $Q(k, \mathbf{z}) \equiv \tilde{v}(k, \mathbf{z}) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\, [S(\tilde{v}(1, \mathbf{z}'), F_{\widetilde{\varepsilon}|1}) - S(\tilde{v}(0, \mathbf{z}'), F_{\widetilde{\varepsilon}|0})]$. It is clear that function $Q(k, \mathbf{z})$ is identified. This function has also an intuitive interpretation. It represents the difference between the firm's value under two different 'ad-hoc' strategies: the strategy of being in the market today, exiting next period, and remaining out of the market forever in the future, and the strategy of exiting from the market today and remaining out of the market forever in the future. Using this definition for function $Q(k, \mathbf{z})$, we can rewrite the system of equations (19) as follows:

$$\begin{aligned} Q(k, \mathbf{z}) = vp(\mathbf{z}^v) - \left[fc(\mathbf{z}^c) + ec(\mathbf{z}^c)\right] + k\left[ec(\mathbf{z}^c) - sv(\mathbf{z}^c)\right] \\ + \beta \sum_{\mathbf{z}^{c\prime} \in \mathbf{Z}^c} f_z(\mathbf{z}^{c\prime}|\mathbf{z}) sv(\mathbf{z}^{c\prime}) \end{aligned} \tag{20}$$

This system of equations provides a closed form expression for the relationship between the unknown structural functions and the identified function $Q(k, \mathbf{z})$.

Proposition 2 *The structural functions $fc(\mathbf{z}^c)$, $ec(\mathbf{z}^c)$, and $sv(\mathbf{z}^c)$ are not separately identified. However, we can identify two combinations of these structural functions* *which have a clear economic interpretation: (a) the sunk part of the entry cost when entry and exit occur at the same state* $\mathbf{z}$*, i.e.,* $ec(\mathbf{z}^c) - sv(\mathbf{z}^c)$*; and (b) the sum of fixed cost and entry cost minus the discounted expected scrap value in the next period, i.e.,* $fc(\mathbf{z}^c) + ec(\mathbf{z}^c) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\, sv\left(\mathbf{z}^{c\prime}\right)$.

$$\begin{aligned} ec(\mathbf{z}^c) - sv(\mathbf{z}^c) &= Q(1,\mathbf{z}) - Q(0,\mathbf{z}), \\ fc(\mathbf{z}^c) + ec(\mathbf{z}^c) - \beta \sum_{\mathbf{z}^{c\prime} \in \mathbf{Z}^c} f_z(\mathbf{z}^{c\prime}|\mathbf{z}) sv(\mathbf{z}^{c\prime}) &= -Q(0,\mathbf{z}) + vp(\mathbf{z}^v). \end{aligned} \tag{21}$$

3.4 'Normalizations' and interpretation of estimated functions

In empirical applications, the common approach to address this identification problem is to restrict one of the three structural functions to be zero at any value of $\mathbf{z}^c$. This is often referred to as a 'normalization' assumption. Although most papers in the literature admit that setting the fixed cost or the scrap value to zero is not really an assumption but a 'normalization', these papers do not derive the implications of this 'normalization' on the estimated parameters and on the counterfactual experiments using the estimated model. Based on our derivation of the relationship between identified objects and unknown structural functions in the system of equations (20), or equivalently in (21), we can obtain the correct interpretation of the estimated functions under any possible normalization. Ignoring this relationship can lead to misinterpretations of the empirical results.

Table 1 reports the relationship between the estimated structural functions and the true structural functions under different normalizations. Functions $\widehat{fc}$, $\widehat{sv}$, and $\widehat{ec}$ represent the estimated vectors under a given normalization, and they should be distinguished from the true structural functions fc, sv, and ec. The expressions in Table 1 are derived as follows. First, the estimated functions $\widehat{fc}$, $\widehat{sv}$, and $\widehat{ec}$ satisfy the identifying restrictions in (20) and (21). In particular, $\widehat{ec}(\mathbf{z}^c) - \widehat{sv}(\mathbf{z}^c) = Q(1,\mathbf{z}) - Q(0,\mathbf{z})$ and $\widehat{fc}(\mathbf{z}^c) + \widehat{ec}(\mathbf{z}^c) - \beta \sum_{\mathbf{z}^{c\prime} \in \mathbf{Z}^c} f_z(\mathbf{z}^{c\prime}|\mathbf{z})\, \widehat{sv}\left(\mathbf{z}^{c\prime}\right)$. Of course, these conditions are also satisfied by the true values of these functions. Therefore, it should be true that for any normalization we have the following:

$$\begin{aligned} &\widehat{ec}(\mathbf{z}^c) - \widehat{sv}(\mathbf{z}^c) = ec(\mathbf{z}^c) - sv(\mathbf{z}^c), \\ &\widehat{fc}(\mathbf{z}^c) + \widehat{ec}(\mathbf{z}^c) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \widehat{sv}\left(\mathbf{z}^{c\prime}\right) = fc(\mathbf{z}^c) + ec(\mathbf{z}^c) - \beta \sum_{\mathbf{z}^{c\prime} \in \mathbf{Z}^c} f_z(\mathbf{z}^{c\prime}|\mathbf{z}) sv(\mathbf{z}^{c\prime}). \end{aligned} \tag{22}$$

These expressions and the corresponding normalization assumption provide a system of equations that we can solve for the estimated functions and thus obtain the expression of these estimated functions in terms of the true functions. These expressions provide the correct interpretation of the estimated functions.

Suppose that the normalization is $\widehat{sv}(\mathbf{z}^c) = 0$. Including this restriction into the system (22) and solving for $\widehat{ec}(\mathbf{z}^c)$ and $\widehat{fc}(\mathbf{z}^c)$, we get that $\widehat{ec}(\mathbf{z}^c) = ec(\mathbf{z}^c) - sv(\mathbf{z}^c)$, and $\widehat{fc}(\mathbf{z}^c) = fc(\mathbf{z}^c) + sv(\mathbf{z}^c) - \beta \sum_{\mathbf{z}^{c\prime}} f_z(\mathbf{z}^{c\prime}|\mathbf{z})\, sv\left(\mathbf{z}^{c\prime}\right)$. The estimated entry cost

Table 1 Interpretation of estimated structural functions under various "normalizations"

Normalization	Estimated Functions		
	$\widehat{sv}(\mathbf{z}^c)$	$\widehat{fc}(\mathbf{z}^c)$	$\widehat{ec}(\mathbf{z}^c)$
$\widehat{sv}(\mathbf{z}^c) = 0$	0	$fc(z^c) + sv(z^c)$ $-\beta E[sv(z^c_{t+1})\vert z_t = z]$	$ec(z^c) - sv(z^c)$
$\widehat{fc}(\mathbf{z}^c) = 0$	$sv(z^c)$ $+\sum_{r=0}^{\infty} \beta^r E[fc(z^c_{t+r})\vert z_t = z]$	0	$ec(z^c)$ $+\sum_{r=0}^{\infty} \beta^r E[fc(z^c_{t+r})\vert z_t = z]$
$\widehat{ec}(\mathbf{z}^c) = 0$	$sv(z^c) - ec(z^c)$	$fc(z^c) + ec(z^c)$ $-\beta E[ec(z^c_{t+1})\vert z_t = z]$	0

is in fact the entry cost minus the scrap value at the same state, i.e., the 'ex-ante' sunk entry cost.[14] And the estimated fixed cost is the actual fixed cost plus the difference between the current scrap value and the expected, discounted next period scrap value. When the normalization is $\widehat{ec}(\mathbf{z}^c) = 0$, we can perform a similar operation to obtain that $\widehat{sv}(\mathbf{z}^c) = sv(\mathbf{z}^c) - ec(\mathbf{z}^c)$, and $\widehat{fc}(\mathbf{z}^c) = fc(\mathbf{z}^c) + ec(\mathbf{z}^c) - \beta \sum_{\mathbf{z}^{c\prime} \in \mathbf{Z}^c} f_z(\mathbf{z}^{c\prime}|\mathbf{z})\, ec\left(\mathbf{z}^{c\prime}\right)$. That is, the estimated scrap value is equal to the 'ex-ante' sunk entry cost but with the opposite sign, and the estimated fixed cost is equal to the actual fixed cost plus the difference between current entry cost and expected discounted next period entry cost. When the normalization is applied to the fixed cost, such that $\widehat{fc}(\mathbf{z}^c) = 0$, obtaining the solution of the estimated functions in terms of the true functions is a bit more convoluted because the solution of the system of equations is not point-wise or separate for each value of $\mathbf{z}$, but instead we need to solve recursively a system of equations that involves every possible value of $\mathbf{z}$. We have the recursive systems $[\widehat{ec}(\mathbf{z}^c) - ec(\mathbf{z}^c)] = fc(\mathbf{z}^c) + \beta \sum_{\mathbf{z}^{c\prime}} f_z(\mathbf{z}^{c\prime}|\mathbf{z})\, [\widehat{ec}\left(\mathbf{z}^{c\prime}\right) - ec\left(\mathbf{z}^{c\prime}\right)]$ and $[\widehat{sv}(\mathbf{z}^c) - sv(\mathbf{z}^c)] = fc(\mathbf{z}^c) + \beta \sum_{\mathbf{z}^{c\prime}} f_z(\mathbf{z}^{c\prime}|\mathbf{z})\, [\widehat{sv}\left(\mathbf{z}^{c\prime}\right) - sv\left(\mathbf{z}^{c\prime}\right)]$. Solving recursively these functional equations, we get that $\widehat{ec}(\mathbf{z}^c) = ec(\mathbf{z}^c) + \sum_{r=0}^{\infty} \beta^r\, \mathbb{E}[fc(\mathbf{z}^c_{t+r}) \mid \mathbf{z}_t = z]$, and $\widehat{sv}(\mathbf{z}^c) = sv(\mathbf{z}^c) + \sum_{r=0}^{\infty} \beta^r\, \mathbb{E}[fc(\mathbf{z}^c_{t+r}) \mid \mathbf{z}_t = z]$. That is, the estimated entry cost is equal to the actual entry cost plus the discounted and expected sum of the current and future fixed costs of the firm if it would be active forever in the future. A similar interpretation applies to the estimated scrap value.

Example 2 Suppose an industry where firms need to use a particular capital equipment to operate in the market. For some reason (e.g., informational asymmetries) there is not a rental market for this equipment, or it is always more profitable to purchase the equipment than to rent it. Let z^c be a state variable that represents the current purchasing price of the equipment. Suppose that the entry cost is

[14] The 'ex-ante' sunk entry cost is not necessarily equal to the 'ex-post' or realized sunk cost because the value of the state variables affecting the scrap value may be different at the entry and exit periods.

$ec(z^c) = ec_0 + z^c$, where $ec_0 > 0$ is a parameter that represents costs of entry other than those related to the purchase of capital. The fixed cost depends also on the price of capital through property taxes that firms should pay every period they are active: $fc(z^c) = fc_0 + \tau\, z^c$, where $\tau \in (0, 1)$ is a parameter that captures how the property tax depends on the price of the owned capital. The scrap value function is $sv(z^c) = \lambda\, z^c$, where $\lambda \in (0, 1)$ is a parameter that captures the idea that there is some capital depreciation, or a firm-specific component in the capital equipment, such that there is a wedge between the cost of purchasing capital and the revenue from selling it. Now, consider the identification of these functions. For simplicity, suppose that the real price of capital z^c is constant over time, though it varies across markets in our data such that we can estimate the effect of this state variable. When the normalization is $\widehat{sv}\,(z^c) = 0$, we have that $\widehat{ec}\,(z^c) = ec_0 + (1 - \lambda)\, z^c$ such that $\widehat{ec}\,(z^c) - ec\,(z^c) = -\lambda\, z^c$. An interpretation of $\widehat{ec}\,(z^c)$ as the true entry cost, instead of the sunk entry cost, implies to underestimate the effect of the price of capital on the entry cost. The estimated fixed cost is $\widehat{fc}\,(z^c) = fc_0 + (\tau + (1 - \beta)\lambda)\, z^c$, such that $\widehat{fc}\,(z^c) - fc(z^c) = (1 - \beta)\lambda\, z^c$ and ignoring the effects of the normalization and treating $\widehat{fc}\,(z^c)$ as the true fixed cost leads to an over-estimation of the effect of the price of capital on the fixed cost. That is, we over-estimate the impact of the property tax on the fixed cost. Similar arguments can be applied when the normalization is $\widehat{ec}\,(z^c) = 0$. In particular, $\widehat{fc}\,(z^c) = fc_0 + (\tau + (1 - \beta))\, z^c$, such that the over-estimation of the incidence of the property tax on the fixed cost is $(1 - \beta)\, z^c$ that is even stronger than before. When we normalize the fixed cost to zero, both the scrap value and the entry cost are overestimated by $(fc_0 + \tau\, z^c)/\,(1 - \beta)$. The estimated effect of the price of capital on the cost of entry includes not only the purchasing cost but also the discounted value of the infinite stream of property taxes.

3.5 Dynamic oligopoly game of entry and exit

Appendix A shows that a very similar identification problem arises in three extensions of the basic model. We discuss here the case of the dynamic oligopoly game. Proposition 2 can be extended to the dynamic oligopoly game. In particular, the additional structure in the dynamic game does not help the identification of the components in the fixed profit. Equation (20) also applies to the dynamic game, only with the following modifications: (i) all the functions are firm-specific and should have the firm subindex j; (ii) function Q_j includes as an argument also the past incumbent statuses of the other firms, k_{-j}, such that we have $Q_j([k_j,k_{-j}], \mathbf{z})$; and (iii) the expected variable profit function depends on firms' CCPs and it includes as an argument the incumbent statuses of all the firms, i.e., $vp_j^{\mathbf{P}}(k, \mathbf{z}^v)$. Given this modified version of equation (20), it is straightforward to extend the two parts of Proposition 2 to the dynamic game model. For the same reason, the relationship reported in Table 1 is still applicable to this extension. Note that the dynamic game provides over-identifying restrictions. For instance, we have that for every value of k_{-j} the following equation should hold: $ec_j\,(\mathbf{z}^c) - sv_j\,(\mathbf{z}^c) = Q_j(1,k_{-j}, \mathbf{z}) -$

$Q(0,k_{-j},\mathbf{z})$. This implies that the value of $Q_j(1,k_{-j},\mathbf{z}) - Q(0,k_{-j},\mathbf{z})$ should be the same for any value of k_{-j}, which is a testable over-identifying restriction.

4 Counterfactual experiments

4.1 Definition and identification of counterfactual experiments

Suppose that the researcher is interested in using the estimated structural model to obtain an estimate of the effect on firms' entry-exit behavior of a change in some of the structural functions such that the environment is partly different from the one generating the data. Let $\theta^0 = \{vp^0, fc^0, ec^0, sv^0, \beta^0, f_z^0\}$ represent the structural functions that have generated the data. And let $\theta^* = \{vp^*, fc^*, ec^*, sv^*, \beta^*, f_z^*\}$ be the structural functions in the hypothetical or counterfactual scenario. Define $\Delta_\theta \equiv \{\Delta_{vp}, \Delta_{fc}, \Delta_{ec}, \Delta_{sv}, \Delta_\beta, \Delta_{f_z}\} \equiv \theta^* - \theta^0$. We refer to Δ_θ as the *perturbation* in the primitives of the model defined by the counterfactual experiment. The goal of this counterfactual experiment is to obtain how the perturbation Δ_θ affects firms' behavior as measured by the CCP function. In other words, we want to identify the counterfactual choice probabilities $P(k, \mathbf{z}\, ;\, \theta^0 + \Delta_\theta)$.

The parameter perturbations that have been considered in the counterfactual experiments in recent empirical applications include regulations that increase entry costs (Ryan 2012; Suzuki 2013), entry subsidies (Das et al. 2007; Dunne et al. 2013; Lin 2012), subsidies on R&D investment (Igami 2013), tax on revenue (Sweeting 2013), market size (Bollinger Forthcoming; Igami 2013), economies of scale and scope (Aguirregabiria and Ho 2012; Varela 2013), a ban on some products (Bollinger Forthcoming; Lin 2012), demand fluctuation (Collard-Wexler 2013), time to build (Kalouptsidi Forthcoming), or exchange rates (Das et al. 2007).

Before we present our main identification results on counterfactual experiments (i.e., Propositions 3 and 4), we first describe here the main ideas on the derivation of these results. Propositions 3 and 4 are based on two main building blocks: equations (21) from Proposition 2, and Lemma 1 that we present below. First, Lemma 1 establishes that there is a one-to-one relationship between the vector of choice probabilities $\widetilde{P} \equiv \{P(k, \mathbf{z}) : \textit{for all}\ (k, \mathbf{z})\}$ and the vector of values $\widetilde{Q} \equiv \{Q(k, \mathbf{z}) : \textit{for all}\ (k, \mathbf{z})\}$. By this lemma, the counterfactual choice probabilities $P(.;\, \theta^0 + \Delta_\theta)$ are identified if and only if the counterfactual Q-values $Q(.;\, \theta^0 + \Delta_\theta)$ are identified. Therefore, we can study the identification of counterfactual experiments by looking at the identification of counterfactual Q-values. This is very convenient because to study the identification of $Q(k, \mathbf{z}\, ;\, \theta^0 + \Delta_\theta)$ we can use equations (21) to get a closed-form relationship between Q-values and the counterfactual structural parameters $\theta^0 + \Delta_\theta$, i.e., equation (24) below. Finally, given this closed-form relationship, we can determine what type of counterfactual experiments is identified (Proposition 3) and what type is not (Proposition 4).

We now present Lemma 1 that establishes the one-to-one relationship between CCPs and Qs.[15]

Lemma 1 *Define* $\widetilde{\mathbf{Q}} \equiv \{Q(k, \mathbf{z}) : \textit{for all } (k, \mathbf{z})\}$ *and* $\widetilde{\mathbf{P}} \equiv \{P(k, \mathbf{z}) : \textit{for all } (k, \mathbf{z})\}$. *For given* (β, f_z), *there is a one-to-one (invertible) mapping* $\widetilde{q}(\widetilde{\mathbf{P}}; \beta, f_z)$ *from the space of vectors* $\widetilde{\mathbf{P}}$ *into the space of vectors* $\widetilde{\mathbf{Q}}$ *such that* $\widetilde{\mathbf{Q}} = \widetilde{q}(\widetilde{\mathbf{P}}; \beta, f_z)$ *and* $\widetilde{\mathbf{P}} = \widetilde{q}^{-1}(\widetilde{\mathbf{Q}}; \beta, f_z)$. *The form of this mapping is,* $\widetilde{q}(\widetilde{\mathbf{P}}; \beta, f_z) \equiv \{q(k, \mathbf{z}, \widetilde{\mathbf{P}}; \beta, f_z) : \textit{for all } (k, \mathbf{z})\}$ *with:*

$$\begin{aligned} q(k, \mathbf{z}, \widetilde{\mathbf{P}}; \beta, f_z) \equiv\; & F^{-1}_{\widetilde{\varepsilon}|k}(P(k, \mathbf{z})) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \\ & \left(\int_{-\infty}^{F^{-1}_{\widetilde{\varepsilon}|1}(P(1,\mathbf{z}'))} [F^{-1}_{\widetilde{\varepsilon}|1}\left(P(1, \mathbf{z}')\right) - \widetilde{\varepsilon}]\, dF_{\widetilde{\varepsilon}|1}(\widetilde{\varepsilon}) \right) \\ & + \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \left(\int_{-\infty}^{F^{-1}_{\widetilde{\varepsilon}|0}(P(0,\mathbf{z}'))} [F^{-1}_{\widetilde{\varepsilon}|0}\left(P(0, \mathbf{z}')\right) - \widetilde{\varepsilon}]\, dF_{\widetilde{\varepsilon}|0}(\widetilde{\varepsilon}) \right) \end{aligned} \tag{23}$$

Lemma 1 implies that the vector of counterfactual CCPs $\widetilde{\mathbf{P}}(\theta^0 + \Delta_\theta)$ is identified if and only if the vector of counterfactual Qs, $\widetilde{\mathbf{Q}}(\theta^0 + \Delta_\theta)$, is identified. Therefore, we can study the identification of counterfactual CCPs by looking at the identification of counterfactual Qs. For notational simplicity, we use $P^*(k, \mathbf{z})$ and $Q^*(k, \mathbf{z})$ to represent counterfactuals $P(k, \mathbf{z}; \theta^0 + \Delta_\theta)$ and $Q(k, \mathbf{z}; \theta^0 + \Delta_\theta)$, and $P^0(k, \mathbf{z})$ and $Q^0(k, \mathbf{z})$ to represent factuals $P(k, \mathbf{z}; \theta^0)$ and $Q(k, \mathbf{z}; \theta^0)$, respectively.

For some counterfactual experiments, we need to distinguish two components in the vector of state variables: $\mathbf{z} \equiv (\mathbf{z}^{nosv}, \mathbf{z}^{sv})$, where $\mathbf{z}^{sv}$ is the subvector of the state variables $\mathbf{z}^c$ that affect the scrap value, and $\mathbf{z}^{nosv}$ represents the rest of the state variables. Without loss of generality, we can always represent the transition probability of the observable state variables as $f_z(\mathbf{z}_{t+1}|\mathbf{z}_t) = f_{z,sv}(\mathbf{z}^{sv}_{t+1}|\mathbf{z}_t)\, f_{z,nosv}(\mathbf{z}^{nosv}_{t+1}|\mathbf{z}_t, \mathbf{z}^{sv}_{t+1})$.

Now, taking into account equation (20), we can derive the following equation that relates the counterfactual $Q^*(k, \mathbf{z})$ with the factual $Q^0(k, \mathbf{z})$, and with θ^0 and Δ_θ:

$$\begin{aligned} Q^*(k, \mathbf{z}) =\; & Q^0(k, \mathbf{z}) \\ & + \Delta_{vp}(\mathbf{z}^v) - \left[\Delta_{fc}(\mathbf{z}^c) + \Delta_{ec}(\mathbf{z}^c)\right] + k\left[\Delta_{ec}(\mathbf{z}^c) - \Delta_{sv}(\mathbf{z}^c)\right] \\ & + \beta^0 \sum_{\mathbf{z}^{sv\prime} \in \mathbf{Z}} f^0_{z,sv}(\mathbf{z}^{sv\prime}|\mathbf{z}) \Delta_{sv}(\mathbf{z}^{sv\prime}) \\ & + \beta^0 \sum_{\mathbf{z}^{sv\prime} \in \mathbf{Z}} \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z})[sv^0(\mathbf{z}^{sv\prime}) + \Delta_{sv}(\mathbf{z}^{sv\prime})] \\ & + \Delta_\beta \sum_{\mathbf{z}^{sv\prime} \in \mathbf{Z}} [f^0_{z,sv}(\mathbf{z}^{sv\prime}|\mathbf{z}) + \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z})][sv^0(\mathbf{z}^{sv\prime}) + \Delta_{sv}(\mathbf{z}^{sv\prime})] \end{aligned} \tag{24}$$

[15] Lemma 1 is related but quite different to Proposition 1 in Hotz and Miller (1993). Hotz-Miller Proposition 1 establishes that for every value of the state variables $(k, \mathbf{z})$, there is a one-to-one mapping between CCPs P and differential values $\widetilde{v}$. In our binary choice model, Hotz-Miller Proposition simply establishes that in equation $P(k, \mathbf{z}) = F_{\widetilde{\varepsilon}|k}(\widetilde{v}(k, \mathbf{z}))$ the distribution function $F_{\widetilde{\varepsilon}|k}$ is invertible. In contrast, Lemma 1 establishes the invertibility of the mapping between vector $\widetilde{Q}$ and vector $\widetilde{P}$. This invertibility is not point-wise because every value $Q(k, \mathbf{z})$ depends on the whole vector $\widetilde{P}$.

Remember from Section 3.3 that the factual $Q^0(k, \mathbf{z})$ is identified from the data. By design of the counterfactual experiment, the perturbations in the structural functions, $\Delta_\theta \equiv \{\Delta_{vp}, \Delta_{fc}, \Delta_{ec}, \Delta_{sv}, \Delta_\beta, \Delta_{f_z}\}$, are also known to the researcher. But notice that the right-hand-side of equation (24) includes also the discount factor β^0 and the scrap-value function $sv^0(.)$ that are not identified. Therefore, if a counterfactual experiment is such that the right-hand-side of equation (24) does not depend on β^0 and sv^0, then the experiment is identified. Otherwise, it is not identified. Propositions 3 and 4 characterize the class of experiments that are identified and the class that is not, respectively.

Proposition 3 *Suppose that Δ_θ implies changes only in functions vp, fc, ec, sv, and $f_{z,nosv}$ such that $\beta^* = \beta^0$ and $f^*_{z,sv} = f^0_{z,sv}$. And suppose that the researcher knows the perturbation $\Delta_\theta = \{\Delta_{vp}(\mathbf{z}^v), \Delta_{fc}(\mathbf{z}^c), \Delta_{ec}(\mathbf{z}^c), \Delta_{sv}(\mathbf{z}^c), \Delta_{f_{z,nosv}}\left(\mathbf{z}^{nosv\prime}|\mathbf{z}\right)\}$ (though he does not know neither θ^0 nor θ^*). Then, $Q^*(k, \mathbf{z})$ and $P^*(k, \mathbf{z})$ are identified at every state $(k, \mathbf{z})$.*

Proposition 3 establishes that the non-identification of the three functions in the fixed profit does not imply an identification problem for counterfactuals that can be described as additive changes in these functions. In contrast, the following Proposition 4 shows that there are relevant counterfactual experiments that are not identified.

Proposition 4 *Suppose that Δ_θ implies changes in the discount factor or in the transition probability of the state variables, such that $\Delta_\beta \equiv \beta^* - \beta^0 \neq 0$ or/and $\Delta_{f_{z,sv}} \equiv f^*_{z,sv} - f^0_{z,sv} \neq 0$, where the researcher knows both (β^0, f^0_z) and (β^*, f^*_z). Despite the knowledge of these primitives under the factual and the counterfactual scenarios, the effect of these counterfactuals on firms' behavior, as represented by $Q^*(k, \mathbf{z})$ or $P^*(k, \mathbf{z})$, is NOT identified.*

4.2 Bias induced by normalizations

Suppose that a researcher has estimated the structural parameters of the model under one of the 'normalization' assumptions that we have described in Section 3.4. Furthermore, suppose that given the estimated model, this researcher implements counterfactual experiments by applying the same 'normalization' assumption that has been used in the estimation. For instance, the model has been estimated under the condition that the scrap value is zero, and this condition is also maintained when calculating the counterfactual equilibrium. In this section, we study whether and when this approach introduces a bias in the estimation of the counterfactual effects. We find that this approach does not introduce any bias for the class of (identified) counterfactuals described in Proposition 3. For the class of counterfactuals in Proposition 4, this approach provides a biased estimation. Of course, this bias is not surprising because, as shown in that Proposition, that class of counterfactual experiments is not identified. More interestingly, we show that the magnitude of this bias can be economically very significant.

Table 2 Counterfactual experiments in recent empirical studies

Type of Counterfactuals	Normalization Assumption sv = 0	fc = 0	ec = 0	No normalization
Change in β				
Change in transition f_z	Collard-Wexler (2013) Das et al. (2007)			Kalouptsidi (forthcoming)
Change in profit	Aguirregabiria and Ho (2012) Bollinger (forthcoming) Collard-Wexler (2011) Dunne et al. (2013) Igami (2013) Kryukov (2010) Barwick and Pathak (2012) Lin (2012) Suzuki (2013) Varela (2013)	Ryan (2012) Santos (2013) Sweeting (2013)		

Given a normalization used for the estimation of the model (not necessarily $\widehat{sv^0}(\mathbf{z}^c) = 0$), let $\widehat{sv^0}(\mathbf{z}^c)$ be the estimated scrap value function. And let $\widehat{Q^*}(k, \mathbf{z})$ be the estimate of $Q^*(k, \mathbf{z})$ when we use $\widehat{sv^0}$ instead of the true value sv^0. Using the general expression for $Q^*(k, \mathbf{z})$ in equation (24), the bias induced by the normalization is:

$$\begin{aligned} \widehat{Q^*}(k, \mathbf{z}) - Q^*(k, \mathbf{z}) &= \beta^0 \sum_{\mathbf{z}^{sv\prime} \in \mathbf{Z}} \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z}) \left[\widehat{sv^0}(\mathbf{z}^{sv\prime}) - sv^0(\mathbf{z}^{sv\prime})\right] \\ &+ \Delta_\beta \sum_{\mathbf{z}^{sv\prime} \in \mathbf{Z}} [f^0_{z,sv}(\mathbf{z}^{sv\prime}|\mathbf{z}) + \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z})] \left[\widehat{sv^0}(\mathbf{z}^{sv\prime}) - sv^0(\mathbf{z}^{sv\prime})\right] \end{aligned} \quad (25)$$

Proposition 5 *If the counterfactual experiment is such that $\Delta_\beta = 0$ and $\Delta_{f_{z,sv}} = 0$, then the bias $\widehat{Q^*}(k, \mathbf{z}) - Q^*(k, \mathbf{z})$ is zero, and the 'normalization' assumption is innocuous for this class of experiments. Otherwise, the bias $\widehat{Q^*}(k, \mathbf{z}) - Q^*(k, \mathbf{z})$ is not zero and the 'normalization' assumption introduces a bias in the estimated effect of the counterfactual experiment.*

Proposition 5 defines the class of counterfactual experiments for which the 'normalization' assumptions introduce a bias. This class consists of those experiments involving a change in the transition probability of a state variable that enters into the scrap value function or a change in the discount factor. While most of the counterfactual experiments in recent applications examine the impacts of the change in structural functions other than transition functions, several studies have examined the change in firms' behavior under different transition functions. Table 2 lists recent empirical studies using the framework in this paper and implementing counterfactual experiments. We have classified these papers according to two criteria: (a) the type of counterfactual experiment, i.e., change in the profit function, in the discount factor, or in the transition probabilities; and (b) normalization assumption used in the estimation of the model.[16] In terms of normalization, most papers normalize the scrap value to zero, except Ryan (2012), Santos (2013) and Sweeting (2013) that normalize the fixed cost, and Kalouptsidi (Forthcoming) that does not rely on any normalization. We have not found papers that normalize the entry cost to zero. The most common type of counterfactual experiments consists of changes in the parameters of the profit function. Proposition 3 establishes that this type of experiment is identified. We have found two papers that present counterfactual experiments that change the transition probability functions f_z. As shown by Proposition 4, these experiments are not identified when the state variables in the perturbed transition probability affect the scrap value. Collard-Wexler (2013) examines the effect of demand uncertainty on the dynamics of ready-mix concrete markets. His experiments simulate firms' behavior when demand did not fluctuate. According to our results, the normalization used in that study (zero scrap value) is not innocuous if state variables affecting demand have some impact on scrap value. In another example, Das et al. (2007) simulate the change in firms' export decisions when the currencies of these firms' home countries decrease by 20 percent. In this case, the normalization on the scrap value is no longer innocuous if part of the scarp value comes from the sale of their foreign assets (e.g., subsidiaries), and hence depends on the exchange rate. The counterfactual experiments in Kalouptsidi (Forthcoming) are identified despite they imply a change in the transition probability function. The reason is she does not need to impose any normalization assumption in the estimation because her dataset includes information on transaction prices from the acquisition of firms. We discuss this point in more detail in Section 5.3 below.

4.3 Numerical example

In this section, we present a simple example that illustrates how the bias induced by the normalization assumption can be sizeable and economically significant. Consider a retail industry in which market entry requires land ownership. Examples include big-box stores and hotels. Let z_{mt}^c represent the land price in market m at period t. The form of the entry cost, fixed cost, and scrap value functions are as follows: $ec\left(z_{mt}^c\right) = ec_0 + ec_1\ z_{mt}^c$, $fc\left(z_{mt}^c\right) = fc_0 + fc_1\ z_{mt}^c$, and $sv\left(z_{mt}^c\right) = sv_0 + sv_1\ z_{mt}^c$, where ec_0, ec_1, fc_0, fc_1, sv_0, and sv_1 are parameters. Variable profits do not depend on land price: $vp\left(z_{mt}^v\right) = vp_0$, where vp_0 is a parameter. The stochastic process of land price is described by the following AR(1) process: $z_{m,t+1}^c = \alpha_0 + \alpha_1\ z_{mt}^c + \sigma_z\ u_{m,t+1}$, where $u_{m,t+1}$ is an i.i.d. shock with standard normal distribution.

[16] There are several empirical studies that use the framework in this paper but do not conduct counterfactual experiments considered in this paper. Examples include Snider (2009) and Ellickson et al. (2012).

Suppose that there are two groups of markets in the data, types H and L. For instance, each group of markets may represent a metropolitan area or a region. Each group consists of a large number of local markets. Suppose that the only differences between these two groups of markets are the stochastic processes of land price:

$$\textbf{Type}H\ :\ z^c_{m,t+1} = \alpha_0^H + \alpha_1^H z^c_{mt} + \sigma^H u_{m,t+1}$$

$$\textbf{Type}L\ :\ z^c_{m,t+1} = \alpha_0^L + \alpha_1^L z^c_{mt} + \sigma^L u_{m,t+1}$$

where u_{mt} is $iid\ N$ (0, 1). All structural cost functions are the same in all local markets, whether they belong to type H or type L. However, this is not known to the researcher. The researcher observes that the stochastic processes of land prices are different between the two groups; however, he does not know whether this difference is the only structural difference between the two groups.

Given a dataset generated from this model, the researcher observes, or estimates consistently, the CCP functions for each group of markets. For every value of land price z^c, he knows the entry probabilities of potential entrants in market type H and market group L, i.e., $P_H(0, z)$ and $P_L(0, z)$, respectively, and the probabilities that an incumbent stays in the market, i.e., $P_H(1, z)$ and $P_L(1, z)$. The researcher also knows the variable profit vp_0, the discount factor β, and the parameters $\left\{\alpha_0^j, \alpha_1^j, \sigma^j : j = H, L\right\}$ in the stochastic process of land price. Suppose that the main interest of this researcher is to understand the contribution of different structural factors to the differences in the CCP functions in the two groups of markets. More specifically, he is interested in estimating what part of this difference in CCP functions can be attributed purely to the differences in the stochastic processes of land prices, rather than to the differences in cost functions. Unfortunately, as shown in Proposition 5, the researcher cannot obtain an unbiased/consistent estimator of this effect. Performing this experiment requires the knowledge of the scrap value function $sv\,(z^c)$; however, the functions $ec\,(z^c)$, $sv\,(z^c)$, and $fc\,(z^c)$ are not separately identified from the data. Suppose that the researcher makes a normalization assumption on these functions and uses the same normalization to implement the counterfactual experiment. The goal of this numerical exercise is to quantify the extent to which this approach introduces a bias in the estimated effect of the counterfactual experiment.

For our numerical example, we consider the following values for the parameters in the data generating process:

$$\begin{array}{llll} ec_0 = 6.5\ ; & ec_1 = 1\ ; & sv_0 = 0.9\ ; & sv_1 = 0.96 \\ fc_0 = 0.1\ ; & fc_1 = 0.03\ ; & vp_0 = 1.1\ ; & \beta = 0.95 \end{array}$$

$$\begin{array}{lll} \alpha_0^H = 1.0\ ; & \alpha_1^H = 0.9\ ; & \sigma^H = 0.5 \\ \alpha_0^L = 0.9\ ; & \alpha_1^L = 0.9\ ; & \sigma^L = 0.5 \end{array}$$

Under this setting, the average land price of group H is ten percent higher than that of group L (10.0 vs. 9.0), while the standard deviation of land price is the same (1.3). Figure 1 illustrates the distributions of these two groups.

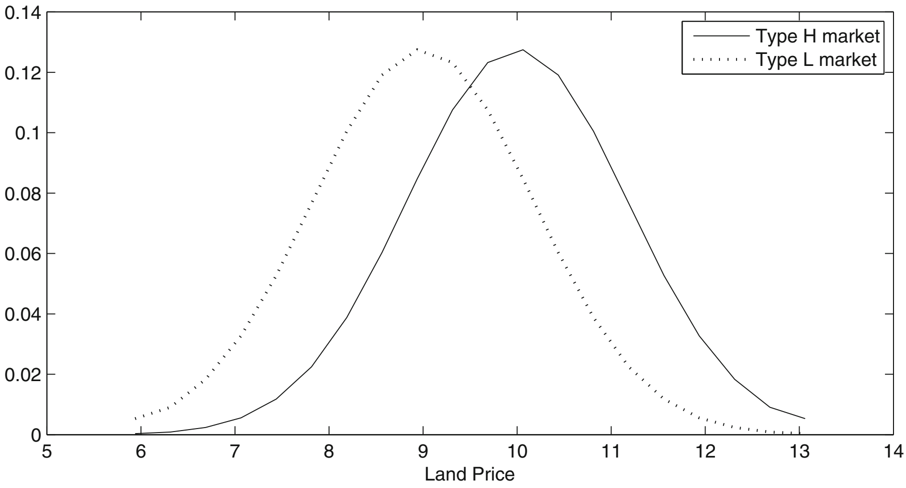

Fig. 1 Stationary Distribution of land price in the two types of markets

Figure 2 shows the CCPs of both a potential entrant and an incumbent in each group of markets. The observed difference in the CCPs of these two groups is entirely due to the difference in the stochastic processes of the land price (though the researcher does not know this fact). For every land price, the probability of entry and the probability of staying in the market is higher for group H than for group L because, at any level of the current land price, a firm's expectation about the future land price is lower in group L than in group H. As a result, a new entrant in group L is more likely to postpone his entry because the future entry cost is (more) likely to be lower. Similarly, an incumbent in group L is more likely to exit today because the scrap value is (more) likely to be lower in the future. For a given value of land price z, there are more entries and fewer exits in type H markets than in type L markets. On average, land prices are lower in group L, which implies that once we average over all possible land prices, there are more entries and fewer exits in type L markets than in type H markets. For a potential entrant, the unconditional probability of being in the market is 7.1 percent in group H and 10.4 percent in group L, and these probabilities are 94.7 percent and 96.5 percent, respectively, for an incumbent.

Having a probability of staying in a market that declines with land price is not by itself evidence of a scrap value that depends on land price. This correlation can also be generated by a model in which the scrap value is constant and the fixed cost of an incumbent firm increases with land price (e.g., property taxes or a leasing price that depends on the price of land). In fact, such a development is the case in our example, where both effects play a role in generating this dependence.

Figure 3 presents the estimated entry cost and fixed cost functions in group H with a zero scrap value normalization. With this normalization, the estimated entry cost function is smaller and less sensitive to a change in land price than the true function, i.e., $\widehat{ec}(z^c) = ec(z^c) - sv(z^c) = [5.6 + 0.4z^c] < [6.5 + z^c] = ec(z^c)$. In contrast, the estimated fixed cost function is larger for most levels of land price and more

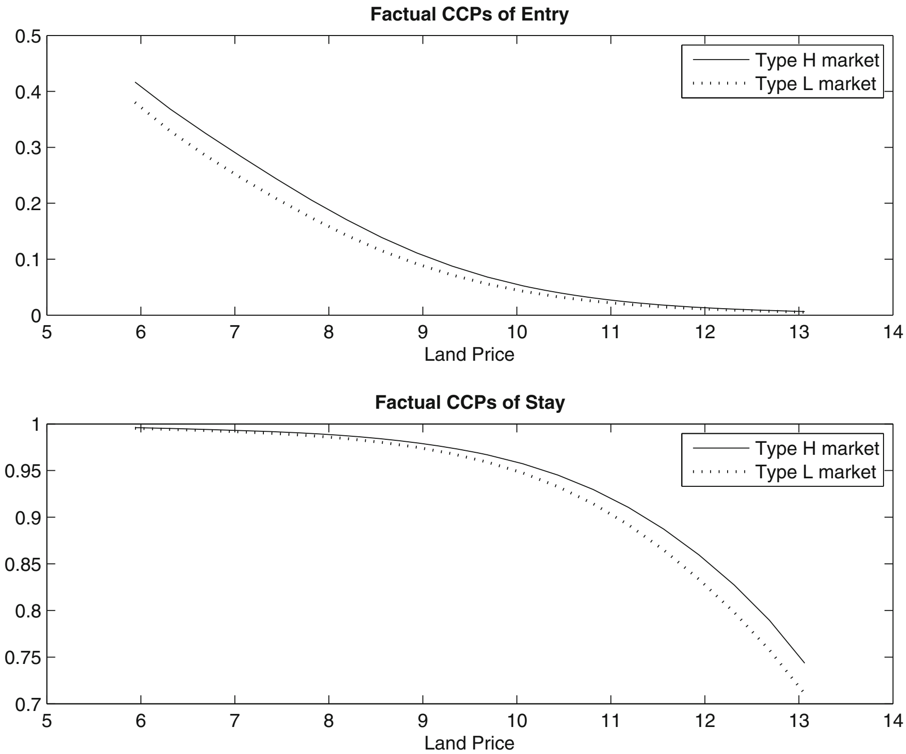

Fig. 2 Estimated probabilities of entry and staying

sensitive to a change in land price than the true function. Under the normalization of the scrap value to zero, the researcher may interpret that the effect of the land price on the probability of exit derives only from the fixed operating cost; however, this effect may also derive from the dependence of the scrap value with respect to the land price, as in this example.

We do not present a figure with the estimated entry cost and fixed cost functions in group L. In this example and under this normalization, the estimated entry cost is the same for the two groups of markets because the stochastic process of land price does not play any role in the estimated entry cost under the zero scrap value normalization. The estimated fixed cost functions are different: $\widehat{fc}_H(z^c) = -0.767 + 0.1692\, z^c$ for group H, and $\widehat{fc}_L(z^c) = -0.675 + 0.1692\, z^c$ for group L. Given these results, the researcher concludes that the sunk entry cost is the same in the two groups of markets. Furthermore, he might even be willing to conjecture that the two groups have the same entry cost function and the same scrap value function. However, this correct conjecture is still not sufficient to identify the effect of a change in the stochastic process of land price. Furthermore, given the difference in the estimated fixed cost function, the researcher cannot identify how much of this difference can be attributed to the difference in the level of land prices and how much can be attributed to possible actual differences in the fixed cost functions of the two groups.

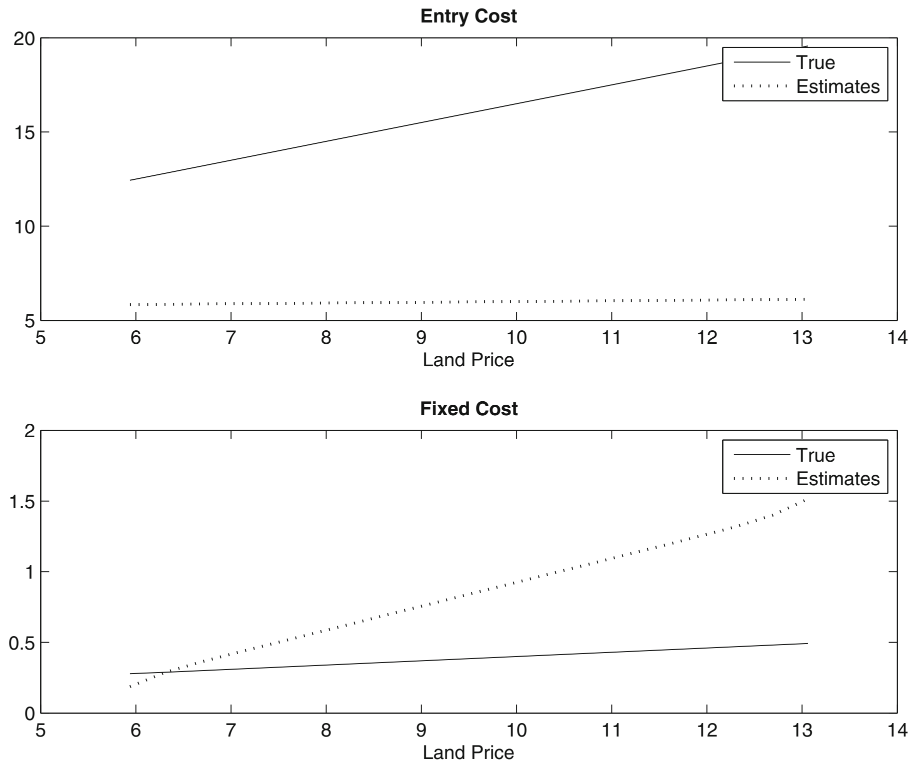

Fig. 3 Estimated (and true) entry cost and fixed cost

Figure 4 presents the main results of a counterfactual experiment that measures to what extent the differences in average land prices can explain the different firm turnover rates between type H and type L markets.[17] This experiment consists of solving for the equilibrium CCPs in a type H market if we keep the structural cost functions at the estimated values for group H but replace the stochastic process of land price with that of type L. The upper panel shows the true and the estimated changes in the entry probability (i.e., the difference between counterfactual and factual probabilities) where the estimated values are based on a zero scrap value normalization. Similarly, the lower panel shows the change in the probability of staying in the market for an incumbent.

The true effect of this counterfactual experiment that consists of reducing the average land price by ten percent is that both the new entrants and the incumbents are less likely to be in the market at every land price, i.e., the schedules that represent the probability of entry and stay, as functions of z^c, move downward. The reduction in the average land price implies that, at any level of the current land price, a firm's

[17]The land price is discretized into 20 equally spaced distinct points between the first and ninety-ninth percentiles.

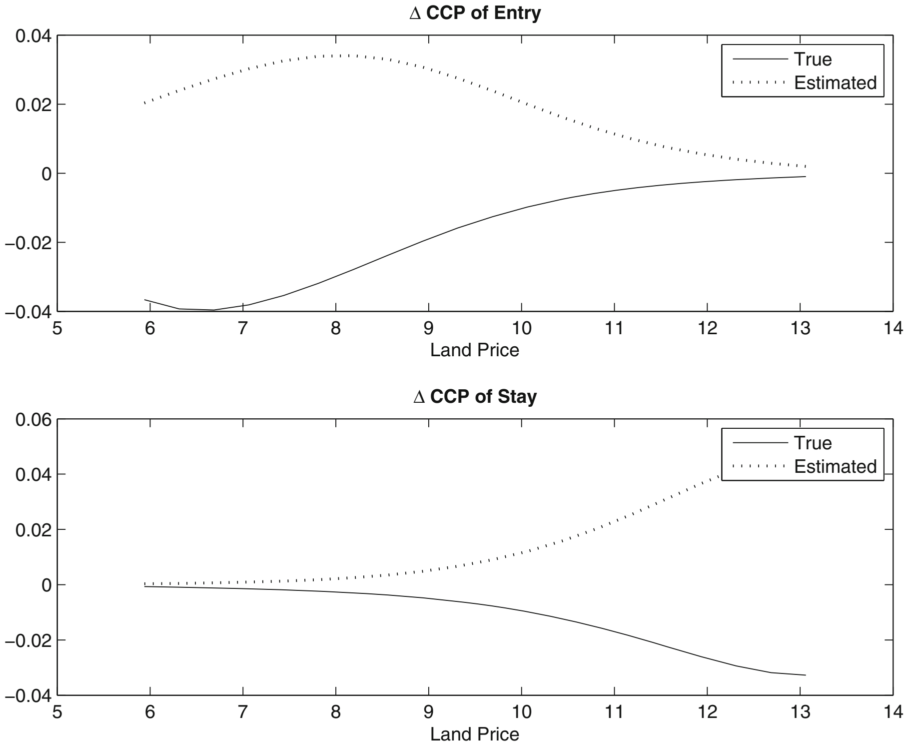

Fig. 4 Estimated (and true) counterfactual change

expectation about the future land price decreases. As a result, a new entrant is more likely to postpone its entry because the future entry cost is likely to be lower and an incumbent is more likely to exit today because the scrap value is likely to decrease in the future. Despite this downward shift in these probability schedules with the lower mean land price, there are on average more entries and fewer exits: the counterfactual change increases the unconditional probability of being in the market from 7.1 percent to 10.4 percent for a potential entrant and from 94.7 percent to 96.5 percent for an incumbent.

The predictions are considerably different if we perform the same counterfactual experiment using the estimated structural cost functions that are 'identified' from data under the zero scrap value restriction. In contrast to the true effect, the estimated effect shows an upward shift in the schedules that represent the probability of entry and the probability of stay for every possible land price. For instance, for the mean land price in the original distribution ($z^c = 10$), the probability of being in the market increases by 2.0 percentage points (from 5.3 to 7.3) for a new entrant and by 1.2 percentage points (from 95.7 to 96.9) for an incumbent. This experiment also overestimates the unconditional probability of being in a market in the next period by 4.7 percentage points for a potential entrant (15.1 vs. 10.4) and by 1.4 percentage points for an incumbent (97.9 vs. 96.5).

This bias is generated by the difference between the estimated and true structural cost functions. As shown in the first row of Table 1, imposing a zero scrap value restriction leads to an overestimation of fixed cost and an underestimation of entry cost. In addition, fixed cost estimates under this restriction depend on the land price, while both entry cost and scrap value do not. Under the original distribution of land price, a firm's equilibrium policy under this (incorrect) cost structure is exactly equal to the policy under the true cost structure. However, if the distribution shifts to the left, a firm is more likely to choose to be in the market for every land price level because a firm expects lower fixed cost in the future. In this environment, firms do not consider the change in entry cost and scrap value because they do not depend on the land price any more.

5 Identifying conditions

In this section we present a discussion of three approaches to deal with the non-identification of the three structural functions in the fixed cost and some of the counterfactual experiments: (a) partial (interval) identification; (b) exclusion restrictions, and (c) using data on firms' value or scrap values.

5.1 Partial identification

There are weak and plausible restrictions on the fixed profits functions that provide bounds of our estimates of these functions. For instance, suppose that the researcher is willing to assume that the three components of the fixed profit are always positive, i.e., $ec(\mathbf{z}^c) \geq 0$, $fc(\mathbf{z}^c) \geq 0$, and $sv(\mathbf{z}^c) \geq 0$. These restrictions, together with equation (21), imply sharper lower-bounds for the entry cost and the scrap value. More specifically, we have that $ec(\mathbf{z}^c) \geq ec_{LOW}(\mathbf{z}) \equiv \max\{0, Q(1, \mathbf{z}) - Q(0, \mathbf{z})\}$, and $sv(\mathbf{z}^c) \geq sv_{LOW}(\mathbf{z}) = -ec_{LOW}(\mathbf{z})$, such that the lower bounds for the entry cost function, $ec_{LOW}(\mathbf{z})$, and for the scrap value function, $sv_{LOW}(\mathbf{z})$, are identified. That is, if the 'ex ante' sunk cost is strictly positive (i.e., $Q(1, \mathbf{z}) - Q(0, \mathbf{z}) > 0$), then the entry cost should be at least as large as the sunk cost, and if the sunk cost is strictly negative (i.e., $Q(1, \mathbf{z}) - Q(0, \mathbf{z}) < 0$), then the scrap value should be at least as large as the negative sunk cost. Other restriction that seems plausible is that the current scrap value should be greater than the discounted and expected value of future scrap value, i.e., $sv(\mathbf{z}^c) \geq \beta\, \mathbb{E}(sv(\mathbf{z}^c_{t+1}) \mid \mathbf{z}_t = \mathbf{z})$. Combining this restriction with equation (21), we have the following upper bound on the fixed cost function: $fc(\mathbf{z}^c) \leq fc_{UP}(\mathbf{z}) \equiv -Q(1, \mathbf{z}) + vp(\mathbf{z}^v)$. The restriction that current entry cost should be greater than the discounted and expected value of future entry cost (i.e., $ec(\mathbf{z}^c) \geq \beta\, \mathbb{E}(ec(\mathbf{z}^c_{t+1}) \mid \mathbf{z}_t = \mathbf{z})$) also provides an upper bound on the fixed cost.

5.2 Exclusion restrictions

In some applications, the researcher may be willing to assume some observables in $\mathbf{z}$ enter only in one of the three structural cost functions. Proposition 6 explains when these exclusion restrictions help identification of structural cost functions, while

Proposition 7 shows that this additional identification result extends the set of identified counterfactual experiments. For simplicity, we assume that the space of the vector of exogenous state variables $\mathcal{Z}$ is finite and discrete. We use $|\mathcal{Z}|$ to denote the number of elements in a finite set $\mathcal{Z}$.

Proposition 6 *Suppose that the vector of exogenous state variables $\mathbf{z}^c$ is such that $\mathbf{z}^c = \left(\mathbf{z}_1^c, \mathbf{z}_2^c\right)$ with $\mathbf{z}_1^c \in \mathcal{Z}_1^c$ and $\mathbf{z}_2^c \in \mathcal{Z}_2^c$. For any value $\mathbf{z}_2^{c0} \in \mathcal{Z}_2^c$, define the transition matrix $\mathbf{F}_{z_1}(\mathbf{z}_2^{c0})$ with elements $\{f_z(\mathbf{z}_1^{c\prime}|\mathbf{z}_1^c, \mathbf{z}_2^{c0})\}$. Suppose that the following conditions hold: (1) $\mathbf{z}_2^c$ does not enter the fixed cost function, i.e., $fc\,(\mathbf{z}^c) = fc\left(\mathbf{z}_1^c\right)$; (2) $\mathbf{z}_2^c$ may enter either entry cost function $ec\,(\mathbf{z}^c)$ or scrap value function $sv\,(\mathbf{z}^c)$, but not both; and (3) there exit two different values in $\mathcal{Z}_2^c$, say $\mathbf{z}_2^{c0}$ and $\mathbf{z}_2^{c1}$, such that the matrix $\mathbf{F}_{z_1^c}(\mathbf{z}_2^{c1}) - \mathbf{F}_{z_1^c}(\mathbf{z}_2^{c0})$ has rank equal to $|\mathcal{Z}_1^c| - 1$. Under these conditions, each of the structural cost functions $fc\,(\mathbf{z}^c)$, $ec\,(\mathbf{z}^c)$ and $sv\,(\mathbf{z}^c)$ is identified up to an additive constant.*

A few remarks clarify the implication of Proposition 6. First, the normalization considered in this proposition is less restrictive than the one considered before. In this proposition, the normalization means to fix, say, scrap value for a particular realization of $\mathbf{z}_1^c$, while the normalization considered before fixes scrap values to be zero for every possible realization of $\mathbf{z}_1^c$. Second, this identification result does not require a large support for $\mathcal{Z}_2^c$. The set $\mathcal{Z}_2^c$ may contain as few points as two. Larger $\left|\mathcal{Z}_2^c\right|$ does not bring full identification as the corresponding matrix never has full column rank. Third, the key source of identification here is the variation in the transition probabilities of $\mathbf{z}_1^c$ caused by the change in $\mathbf{z}_2^c$. The required rank condition means that the impacts of $\mathbf{z}_2^c$ on the transition probabilities must vary across different $\mathbf{z}_1^c$.

Example 3 Suppose that z_1^c represents the property tax rate in the market, and that this variable enters into the three structural functions. Suppose that, at period t, it is announced whether or not the market property tax will increase next period as a result of, say, the passage of a new law. Let $z_{2t}^c \in \{0, 1\}$ be the dummy variable that indicates whether the market belongs to the "experimental" group of markets with an announcement of a future tax increase (i.e., $z_{2t}^c = 1$) or to the "control" group of markets without that announcement (i.e., $z_{2t}^c = 0$). Following Proposition 6, we need the following two identification assumptions. First, entry cost, scrap value, and fixed cost at period t do not depend on the announcement dummy z_{2t}^c once we control for the current property tax z_{1t}^c.[18] The second identification assumption is that the announcement has an effect on the transition probability of the property tax between periods t and $t+1$, i.e., $\Pr(z_{1t+1}^c|z_{1t}^c, z_{2t}^c = 0) \neq \Pr(z_{1t+1}^c|z_{1t}^c, z_{2t}^c = 1)$. At period t, firms in an "experimental" market have different beliefs about the

[18]Note that the "announcement dummy" z_{2t}^c may be correlated with the level of tax at period t. That is, the average property tax at period t in markets in the "experimental" group may be larger (or smaller) than in the "control" group.

distribution of future property tax than firms in a "control" market, even if the current level of property tax is the same in the two markets. Note that this condition on the transition probability of property tax is testable using the data. The restrictions that exclude the announcement dummy from the structural functions are not testable.

Proposition 7 *Let $\widehat{fc}\,(\mathbf{z}^c)$, $\widehat{ec}\,(\mathbf{z}^c)$, and $\widehat{sv}\,(\mathbf{z}^c)$ be the identified functions (up to a constant) under the conditions of Proposition 6. Then, the counterfactuals $\Delta_Q(k, \mathbf{z})$, $\Delta_P(k, \mathbf{z})$, and $P(k, \mathbf{z}\,; \theta^0 + \Delta_\theta)$ are identified even if $\Delta_{f_z} \neq 0$, as long as $\Delta_\beta = 0$.*

Note that the identification of counterfactual experiments that change the discount rate β is still not identified as it requires the complete identification of scrap value function $sv\,(\mathbf{z}^c)$.

5.3 Using data on transaction prices from the acquisition of firms

In some industries, a common form of firm exit (and entry) is that the owner of an incumbent firm sells all the firm's assets to a new entrant. For instance, this is very frequent in the bulk shipping industry, as shown in Kalouptsidi (Forthcoming), and it is also common in some retail industries intensive in land input such as the hotel industry. Sometimes, the researcher has data on firm acquisition prices. Under some assumptions, these additional data can be used to deal with the identification problem that we study in this paper. We now illustrate this approach in a simple framework.

For simplicity, suppose that the industry is such that the only form of entry is by acquiring an incumbent firm, and similarly the only form of exit is by selling your assets to a new entrant. Then, the entry cost has two components: $ec(\mathbf{z}_t^c) = r(\mathbf{z}_t^c) + \tau_{en}(\mathbf{z}_t^c)$, where $r(\mathbf{z}_t^c)$ represents the acquisition price that the new entrant should pay to the exiting incumbent, and $\tau_{en}(\mathbf{z}_t^c)$ represents costs of entry other than the acquisition price. Similarly, the exit value of a firm has also two components: $sv(\mathbf{z}_t^c) = r(\mathbf{z}_t^c) - \tau_{ex}(\mathbf{z}_t^c)$, where $\tau_{ex}(\mathbf{z}_t^c)$ represents costs associated with market exit. We assume that the acquisition price $r(\mathbf{z}_t^c)$ is the solution of a Nash bargaining problem between the seller and the buyer. Taking into account that $\bar{V}\,(1, \mathbf{z}) - \bar{V}\,(0, \mathbf{z})$ is the value of being an incumbent minus the value of being a potential entrant, we have that the surplus of the buyer is $\bar{V}\,(1, \mathbf{z}) - \bar{V}\,(0, \mathbf{z}) - ec(\mathbf{z}^c)$, and the surplus of the seller is $sv(\mathbf{z}^c) - \bar{V}\,(1, \mathbf{z}) + \bar{V}\,(0, \mathbf{z})$. The Nash bargaining solution implies that:

$$r(\mathbf{z}) = \bar{V}\,(1, \mathbf{z}) - \bar{V}\,(0, \mathbf{z}) + \alpha\tau_{ex}(\mathbf{z}^c) - (1 - \alpha)\tau_{en}(\mathbf{z}^c) \tag{26}$$

where $\alpha \in (0, 1)$ is a parameter that represents the seller bargaining power.

Let R_t be the selling price of a firm, and suppose that the researcher observes this price when a transaction actually occurs. For simplicity, to abstract from selection problems, consider that R_t is a deterministic function of the observable state variables $\mathbf{z}_t$ plus a measurement error ξ_t that is not a state variable of the model, has zero mean, and is independent of the observed state variables $\mathbf{z}_t$ (i.e., classical measurement error): $R_t = r(\mathbf{z}_t) + \xi_t$, with $\mathbb{E}(\xi_t|\mathbf{z}_t) = 0$.[19] Under these conditions, the pricing function $r(\mathbf{z})$ is identified from the data. Now, the researcher has three sets of restrictions to identify the three unknown functions $fc(\mathbf{z}^c)$, $\tau_{en}(\mathbf{z}^c)$, and $\tau_{ex}(\mathbf{z}^c)$:

$$\tau_{en}(\mathbf{z}^c) + \tau_{ex}(\mathbf{z}^c) = Q(1, \mathbf{z}) - Q(0, \mathbf{z}),$$

$$fc(\mathbf{z}^c) + r(\mathbf{z}) + \tau_{en}(\mathbf{z}^c) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \left[r\left(\mathbf{z}'\right) - \tau_{ex}\left(\mathbf{z}^{c\prime}\right) \right] = -Q(0, \mathbf{z}) + vp(\mathbf{z}^v).$$

$$\beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \left[r\left(\mathbf{z}'\right) - \alpha \tau_{ex}\left(\mathbf{z}^{c\prime}\right) + (1-\alpha)\tau_{en}\left(\mathbf{z}^{c\prime}\right) \right] = \widetilde{v}(k, \mathbf{z}) - Q(k, \mathbf{z}) \tag{27}$$

where the third set of restrictions comes from combining the price equation in (26) with the definition of $Q(k, \mathbf{z})$ as $Q(k, \mathbf{z}) \equiv \widetilde{v}(k, \mathbf{z}) - \beta \sum_{\mathbf{z}'} f_z(\mathbf{z}'|\mathbf{z})$ $[\bar{V}\left(1, \mathbf{z}'\right) - \bar{V}\left(0, \mathbf{z}'\right)]$.

Proposition 8 *Given a value of the parameter that represents seller bargaining power, α the set of restrictions in (27) identify the functions $\tau_{en}(.)$, $\tau_{ex}(.)$, and $fc(.)$.*

6 Conclusion

The solution and estimation of dynamic structural models of oligopoly competition is a useful tool in industrial organization and other fields of empirical microeconomics such as trade, health economics, or public economics. Empirical applications of these models typically use panel data on firms' entry and exit decisions in multiple local markets, combined with information on exogenous market and firm characteristics, and sometimes information on firms' prices and quantities. In this class of models, the functions that represent fixed operating cost, entry cost, and exit value play an important role in firms' entry and exit behavior. Furthermore, many public and managerial policies can be described in terms of changes in some of these functions. This paper is motivated by a fundamental identification problem: these three functions cannot be separately identified using these data. The conventional approach to address this identification problem has been to 'normalize' some of the structural parameters to zero. We study the implications of this 'normalization' approach. First, we obtained closed-form expressions that provide the correct interpretation of the estimated objects that are obtained under the 'normalization assumptions' that have been considered in applications. Second, we show that there

[19] This assumption is testable: it implies that the residual price $\xi \equiv [R - \mathbb{E}(R|\mathbf{z})]$ should be independent of firms' entry and exit decisions. In general, unless the dataset is rich enough to include in $\mathbf{z}$ all the relevant variables affecting the price of a firm, we should expect that this assumption will be rejected by the data. That is, we expect $R = r(\mathbf{z}, \varepsilon) + \xi$, where ε is the vector of state variables observable to firms but unobservable to the researcher. Allowing for this type of unobservables as determinants of the transaction price implies that we should deal with a potential selection problem. We only observe the transaction price for those firm-market-period observations when a firm is sold, but those firms that are sold can be systematically different in terms of unobserved state variables ε from those firms that are not sold, i.e., $\mathbb{E}(R|\mathbf{z}, \text{firm is sold}) \neq \mathbb{E}(R|\mathbf{z})$.

is a class of counterfactual experiments that are identified and for which the normalization assumptions are innocuous. We also show that there is a class of experiments for which the normalization assumptions introduce a bias. Using a simple numerical experiment, we show that this bias can be very significant both quantitatively and economically. We also discuss alternative approaches to address this identification problem.

Appendix

A Extensions of the basic model

A.1 Model with no re-entry after market exit and no waiting before market entry

Some models and empirical applications of industry dynamics assume that a new entrant has only one opportunity to enter and an incumbent can not reenter after exit from the market (e.g., Ryan 2012). This model is practically the same as the basic model presented in Section 2.1, with the only difference that the value of not entering for a potential entrant is 0 and the value of exiting for an incumbent is the scrap value, i.e., $v\,(0, 0, \mathbf{z}_t) = 0$ and $v\,(0, 1, \mathbf{z}_t) = sv(\mathbf{z}_t^c) + \varepsilon_t^{sv}$.

A.2 Model with time-to-build and time-to-exit

In this version of the model, it takes one period to make entry and exit decisions effective, though the entry cost is paid at the period when the entry decision is made, and similarly the scrap value is received at the period when the exit decision is taken. Now, a_t is the binary indicator of the event "the firm will be active in the market at period $t + 1$", and $k_t = a_{t-1}$ is the binary indicator of the event "the firm is active in the market at period t". For this model, the one-period profit function is:

$$\Pi_t = \begin{cases} k_t[vp(\mathbf{z}_t^v) - fc\left(\mathbf{z}_t^c\right) + sv\left(\mathbf{z}_t^c\right)] + \varepsilon_t(0) & \text{if } a_t = 0 \\ k_t[vp(\mathbf{z}_t^v) - fc\left(\mathbf{z}_t^c\right)] - (1 - k_t)\, ec\left(\mathbf{z}_t^c\right) + \varepsilon_t(1) & \text{if } a_t = 1 \end{cases} \tag{A.1}$$

Given this structure of the profit function, we have that the Bellman equation, optimal decision rule, and CCP function are defined exactly the same as above in equations (6), (9), and (10), respectively.

A.3 Model with investment

The basic model and the previous extensions assume that the only dynamic decision of a firm is to be active or not in the market. However, our (non) identification results extend to more general models where incumbent firms make investments in product quality, capacity, etc. Here we present a relatively simple model with investment. Suppose that there is a quasi-fixed input, say capital, and the firm decides every period the amount of capital to use. Let $a_t \in \{0, 1, \cdots, K\}$ denote the firm's decision at period t where K is the largest possible capital level. When a_t is zero, the firm is inactive in the current period. The firm's variable profit depends on the current amount of capital a_t, e.g., the amount of capital may affect the quality of the product and therefore demand, and also variable costs. The fixed profit depends both on current capital a_t and on the amount of capital installed at previous period, $k_t \equiv a_{t-1}$. The one-period profit function of this firm is:

$$\Pi_t = \begin{cases} -ic\left(0, k_t, \mathbf{z}_t^c\right) + \varepsilon_t(0) & \text{if } a_t = 0 \\ \\ vp(a_t, \mathbf{z}_t^v) - fc\left(a_t, \mathbf{z}_t^c\right) - ic\left(a_t, k_t, \mathbf{z}_t^c\right) + \varepsilon_t(a_t) & \text{if } a_t > 0 \end{cases} \tag{A.2}$$

where $ic\left(a_t, k_t, \mathbf{z}_t^c\right)$ is the investment cost function, which represents the cost the firm incurs to change its capital level from k to a taking $\mathbf{z}_t^c$ as given. We assume $ic\left(a_t, k_t, \mathbf{z}_t^c\right) = 0$ when $a_t = k_t$. In this specification, $ic\left(a, 0, \mathbf{z}_t^c\right)$ represents the entry cost for a firm that decides to enter in the market with an initial level of capital equal to a. Similarly, $-ic\left(0, k, \mathbf{z}_t^c\right)$ represents the scrap value of an incumbent firm with installed capital equal to k. It is clear that our baseline model is a special case of this general model when $K = 1$.

A.4 Identification of the model with no re-entry after market exit and no waiting before market entry

In this extension, $v\,(0, k, \mathbf{z}_t)$ is equal to $k\ sv\,(\mathbf{z}_t)$. The system of equations corresponding to (19) is:

$$\begin{aligned}\tilde{v}(k, \mathbf{z}) \ =\ & vp(\mathbf{z}^v) - \left[fc\,(\mathbf{z}^c) + ec\,(\mathbf{z}^c)\right] + k\left[ec\,(\mathbf{z}^c) - sv\,(\mathbf{z}^c)\right] + \beta \textstyle\sum_{\mathbf{z}^{c\prime} \in \mathbf{Z}^c} f_z(\mathbf{z}^{c\prime}|\mathbf{z}) sv\left(\mathbf{z}^{c\prime}\right) \\ & + \beta \textstyle\sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) S(\tilde{v}\left(1, \mathbf{z}'\right), F_{\tilde{\varepsilon}|1})\end{aligned} \tag{A.3}$$

Define the function $Q(k, \mathbf{z}) \equiv \tilde{v}(k, \mathbf{z}) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\ S(\tilde{v}\left(1, \mathbf{z}'\right), F_{\tilde{\varepsilon}|1})$. With this new definition of $Q(k, \mathbf{z})$, we have exactly the same system of equations as (20). The relationship reported in Table 1 is directly applicable to this extension.

A.5 Identification of the model with time-to-build and time-to-exit

Most of the expressions for the basic model still hold for this extension, except that now the one-period payoff $\pi\,(a, k, \mathbf{z})$ has a different form. In particular, now $\pi\,(1, k, \mathbf{z}) - \pi\,(0, k, \mathbf{z}) = -k\ sv(\mathbf{z}^c) - (1 - k)\ ec\,(\mathbf{z}^c)$, and this implies that the expression for the differential value function $\tilde{v}(k, \mathbf{z})$ is:

$$\tilde{v}(k, \mathbf{z}) = -ksv(\mathbf{z}^c) - (1 - k)ec\left(\mathbf{z}^c\right) + \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \left[\bar{V}\left(1, \mathbf{z}'\right) - \bar{V}\left(0, \mathbf{z}'\right)\right] \tag{A.4}$$

Also, now we have that $v\,(0, 1, \mathbf{z}) - v\,(0, 0, \mathbf{z}) = \pi\,(0, 1, \mathbf{z}) - \pi\,(0, 0, \mathbf{z}) = vp(\mathbf{z}^v) - fc\,(\mathbf{z}^c) + sv\,(\mathbf{z}^c)$. Therefore, the system of identifying restrictions (20) becomes:

$$Q(k, \mathbf{z}) \ =\ -ksv(\mathbf{z}^c) - (1 - k)ec\,(\mathbf{z}^c) + \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})[vp(\mathbf{z}^{v\prime}) - fc\left(\mathbf{z}^{c\prime}\right) + sv\left(\mathbf{z}^{c\prime}\right)] \tag{A.5}$$

Table 3 Interpretation of estimated structural functions in the model with time-to-build

Normalization	Estimated Functions		
	$\widehat{sv}(\mathbf{z}^c)$	$\beta\,\mathbb{E}(\widehat{fc}(\mathbf{z}^c_{t+1})\,\vert\mathbf{z}_t=\mathbf{z})$	$\widehat{ec}(\mathbf{z}^c)$
$\widehat{sv}(\mathbf{z}^c)=0$	0	$sv(z^c)$ $+\beta E[fc(z^c_{t+1})\vert z_t=z]$ $-\beta E[sv(z^c{}_{t+1})\vert z_t=z]$	$ec(z^c)-sv(z^c)$
$\widehat{fc}(\mathbf{z}^c)=0$	$sv(z^c)$ $+\sum_{r=1}^{\infty}\beta^r E[fc(z^c_{t+r})\vert z_t=z]$	0	$ec(z^c)$ $+\sum_{r=1}^{\infty}\beta^r E[fc(z^c_{t+r})\vert z_t=z]$
$\widehat{ec}(\mathbf{z}^c)=0$	$sv(z^c)-ec(z^c)$	$ec(z^c)$ $+\beta E[fc(z^c_{t+1})\vert z_t=z]$ $-\beta E[ec(z^c_{t+1})\vert z_t=z]$	0

where $Q(k,\mathbf{z})$ has exactly the same definition as before in the model without time-to-build. Given this system of equations, Proposition 2 also applies to this model with the only difference that now we have the following relationship between true functions and identified objects:

$$\begin{aligned} ec\,(\mathbf{z}^c)-sv\,(\mathbf{z}^c) &= Q(1,\mathbf{z})-Q(0,\mathbf{z}) \\ ec\,(\mathbf{z}^c)+\beta\sum_{\mathbf{z}^{c\prime}\in\mathbf{Z}^c} f_z(\mathbf{z}^{c\prime}|\mathbf{z})[fc\left(\mathbf{z}^{c\prime}\right)-sv\left(\mathbf{z}^{c\prime}\right)] &= -Q(0,\mathbf{z})+\beta\sum_{\mathbf{z}^{v\prime}\in\mathbf{Z}^v} f_z(\mathbf{z}^{v\prime}|\mathbf{z})vp(\mathbf{z}^{v\prime}) \end{aligned} \tag{A.6}$$

The first equation is exactly the same as in the model without time to build. The second equation is slightly different: instead of current value of variable profit minus the fixed cost, $vp(\mathbf{z}^v)-fc(\mathbf{z}^c)$, now we have the discounted and expected value of this function at the next period, i.e., $\beta\sum_{\mathbf{z}'\in\mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\;[vp\left(\mathbf{z}^{v\prime}\right)-fc\left(\mathbf{z}^{c\prime}\right)]$. Table 3 reports the relationship between estimated functions and unknown structural functions.

A.6 Identification of the model with investment

Under mild regularity conditions the CCPs $P(a|k,\mathbf{z})$ are identified, and given CCPs we can also identify the differential value function $\tilde{v}(a,k,\mathbf{z})\equiv v(a,k,\mathbf{z})-v(0,k,\mathbf{z})$. The model implies the following restrictions:

$$\begin{aligned} \tilde{v}\,(a,k,\mathbf{z}) &= vp(a,\mathbf{z}^v)-fc\,(a,\mathbf{z}^c)-ic\,(a,k,\mathbf{z}^c)+ic\,(0,k,\mathbf{z}^c) \\ &\quad+\beta\sum_{\mathbf{z}'\in\mathbf{Z}} f_z\left(\mathbf{z}'|\mathbf{z}\right)\left[\bar{V}\left(a,\mathbf{z}'\right)-\bar{V}\left(0,\mathbf{z}'\right)\right] \end{aligned} \tag{28}$$

The same logic used to derive (18) implies that $\bar{V}\,(k,\mathbf{z})=v\,(0,k,\mathbf{z})+S(\tilde{v}\,(.,k,\mathbf{z})\,,$ $F_{\widetilde{\varepsilon}|k})$ where $S(\tilde{v}\,(.,k,\mathbf{z})\,,\;F_{\widetilde{\varepsilon}|k})\;\equiv\;v\,(0,k,\mathbf{z})+\int\max_{a>0}\{0\;,\;\tilde{v}\,(a,k,\mathbf{z})\;-\;\widetilde{\varepsilon}(a)\}$

$dF_{\widetilde{\varepsilon}|k}\left(\widetilde{\varepsilon}\right)$. Using the definition of $v\left(a,k,\mathbf{z}\right)$, we have that $v\left(0,k,\mathbf{z}\right)-v\left(0,0,\mathbf{z}\right)=\pi\left(0,k,\mathbf{z}\right)-\pi\left(0,0,\mathbf{z}\right)=-ic\left(0,k,\mathbf{z}^{c}\right)$, that is the scrap value of a firm with k units of capital. Therefore, we have that $\bar{V}\left(a,\mathbf{z}\right)-\bar{V}\left(0,\mathbf{z}\right)=-ic\left(0,k,\mathbf{z}^{c}\right)+S(\tilde{v}\left(.,k,\mathbf{z}\right),F_{\widetilde{\varepsilon}|k})-S(\tilde{v}\left(.,0,\mathbf{z}\right),F_{\widetilde{\varepsilon}|0})$. Define the function $Q\left(a,k,\mathbf{z}\right)\equiv\tilde{v}\left(a,k,\mathbf{z}\right)-\beta\sum_{\mathbf{z}'}f_{z}\left(\mathbf{z}'|\mathbf{z}\right)\left[S(\tilde{v}\left(.,k,\mathbf{z}\right),F_{\widetilde{\varepsilon}|k})-S(\tilde{v}\left(.,k,\mathbf{z}\right),F_{\widetilde{\varepsilon}|k})\right]$, that is identified from the data. Then, the restrictions shown in (28) can be written as:

$$\begin{aligned}Q\left(a,k,\mathbf{z}\right)&=vp(a,\mathbf{z}^{v})-fc\left(a,\mathbf{z}^{c}\right)-ic\left(a,k,\mathbf{z}^{c}\right)+ic\left(0,k,\mathbf{z}^{c}\right)\\&-\beta\sum_{\mathbf{z}^{c\prime}\in\mathbf{Z}^{c}}f_{z}\left(\mathbf{z}^{c\prime}|\mathbf{z}\right)ic\left(0,a,\mathbf{z}^{c\prime}\right)\end{aligned}\tag{A.7}$$

Note that $Q\left(k,\mathbf{z}\right)$ in equation (20) is a special case when $K=1$ and $a=1$. Using this expression, we can also obtain a system of equations that correspond or extend the ones in (21). For any $a>0$ and any $(k,\mathbf{z})$:

$$\begin{aligned}&-ic\left(a,k,\mathbf{z}^{c}\right)+ic\left(0,k,\mathbf{z}^{c}\right)+ic\left(a,0,\mathbf{z}^{c}\right)=Q\left(a,k,\mathbf{z}\right)-Q\left(a,0,\mathbf{z}\right)\\&fc\left(a,\mathbf{z}^{c}\right)+ic\left(a,k,\mathbf{z}^{c}\right)-ic\left(0,k,\mathbf{z}^{c}\right)+\beta\sum_{\mathbf{z}^{c\prime}\in\mathbf{Z}^{c}}f_{z}\left(\mathbf{z}^{c\prime}|\mathbf{z}\right)ic\left(0,a,\mathbf{z}^{c\prime}\right)\\&=-Q\left(a,k,\mathbf{z}\right)+vp\left(a,\mathbf{z}^{v}\right)\end{aligned}\tag{A.8}$$

The same logic used to prove Proposition 2 implies no identification of structural cost functions $fc\left(a,\mathbf{z}^{c}\right)$ and $ic\left(a,k,\mathbf{z}^{c}\right)$. However, we can still identify the difference between entry cost $(ic\left(a,0,\mathbf{z}^{c}\right))$ and scrap value $(-ic\left(0,k,\mathbf{z}^{c}\right))$ for the same $\mathbf{z}$ as we have

$$Q\left(k,k,\mathbf{z}\right)-Q\left(k,0,\mathbf{z}\right)=ic\left(0,k,\mathbf{z}^{c}\right)+ic\left(a,0,\mathbf{z}^{c}\right)\tag{A.9}$$

The relationship between estimated functions under some normalization and structural cost functions is similar to that of the baseline model. For example, when we normalize scrap value (i.e., $\widehat{ic}\left(0,k,\mathbf{z}^{c}\right)=0$), our estimates of investment cost function $\widehat{ic}\left(a,k,\mathbf{z}^{c}\right)$ and fixed cost function $fc\left(k,\mathbf{z}^{c}\right)$ are written as

$$\begin{aligned}\widehat{ic}\left(a,k,\mathbf{z}^{c}\right)&=ic\left(a,k,\mathbf{z}^{c}\right)+ic\left(0,a,\mathbf{z}^{c}\right)-ic\left(0,k,\mathbf{z}^{c}\right)\\\widehat{fc}\left(k,\mathbf{z}^{c}\right)&=fc\left(k,\mathbf{z}^{c}\right)-ic\left(0,k,\mathbf{z}^{c}\right)+\beta\mathbb{E}\left[ic(0,k,\mathbf{z}_{t+1}^{c})|\mathbf{z}_{t}=\mathbf{z}\right]\end{aligned}\tag{A.10}$$

B Proofs of Lemmas and Propositions

Proof of Lemma 1. By definition $Q(k,\mathbf{z})$ is equal to $\tilde{v}(k,\mathbf{z})-\beta\sum_{\mathbf{z}'}f_{z}(\mathbf{z}'|\mathbf{z})\left[S(\tilde{v}\left(1,\mathbf{z}\right),F_{\widetilde{\varepsilon}|1})-S(\tilde{v}\left(0,\mathbf{z}\right),F_{\widetilde{\varepsilon}|0})\right]$, where $\tilde{v}\left(k,\mathbf{z}\right)=F_{\widetilde{\varepsilon}|k}^{-1}\left(P(k,\mathbf{z})\right)$ and $S(\tilde{v}\left(k,\mathbf{z}\right),F_{\widetilde{\varepsilon}|k})\equiv\int_{-\infty}^{\tilde{v}(k,\mathbf{z})}[\tilde{v}(k,\mathbf{z})-\widetilde{\varepsilon}]\,dF_{\widetilde{\varepsilon}|k}\left(\widetilde{\varepsilon}\right)$. These expressions imply the mapping in equation (23). We can describe mapping $\widetilde{q}(\widetilde{P};\beta,f_{z})$ as the composition of two mappings: (1) the mapping from CCPs to differential values, i.e., $\tilde{v}\left(k,\mathbf{z}\right)=F_{\widetilde{\varepsilon}|k}^{-1}\left(P(k,\mathbf{z})\right)$; and (2) the mapping from differential values to $Q's$, i.e., $Q(k,\mathbf{z})=\tilde{v}(k,\mathbf{z})-\beta\sum_{\mathbf{z}'}f_{z}(\mathbf{z}'|\mathbf{z})\left[S(\tilde{v}\left(1,\mathbf{z}\right),F_{\widetilde{\varepsilon}|1})-S(\tilde{v}\left(0,\mathbf{z}\right),F_{\widetilde{\varepsilon}|0})\right]$. The first mapping is point-wise for every value of $(k,\mathbf{z})$, and in our binary choice model it is obviously invertible. Proposition 1 in (Hotz and Miller 1993) establishes the invertibility of that mapping for multinomial choice models. Therefore, we should prove the invertibility of the second mapping, between the vector $\widetilde{v} \equiv \{\widetilde{v}(k, \mathbf{z}) : \text{for all } (k, \mathbf{z})\}$ and the vector $\widetilde{Q} \equiv \{Q(k, \mathbf{z}) : \text{for all } (k, \mathbf{z})\}$. Define this mapping as $\widetilde{Q} = \widetilde{g}(\widetilde{v})$ where $\widetilde{g}(\widetilde{v}) \equiv \{g(k, \mathbf{z}, \widetilde{v}) : \text{for all } (k, \mathbf{z})\}$ and:

$$g(k, \mathbf{z}, \widetilde{v}) \equiv \widetilde{v}(k, \mathbf{z}) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \left(\int_{-\infty}^{\widetilde{v}(1,\mathbf{z}')} [\widetilde{v}(1, \mathbf{z}') - \widetilde{\varepsilon}] \, dF_{\widetilde{\varepsilon}|1}(\widetilde{\varepsilon}) \right)$$

$$+ \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \left(\int_{-\infty}^{\widetilde{v}(0,\mathbf{z}')} [\widetilde{v}(0, \mathbf{z}') - \widetilde{\varepsilon}] \, dF_{\widetilde{\varepsilon}|0}(\widetilde{\varepsilon}) \right)$$

In vector form, we can express this mapping as:

$$\widetilde{g}(\widetilde{v}) \equiv \begin{bmatrix} \widetilde{g}(0, \widetilde{v}) \\ \widetilde{g}(1, \widetilde{v}) \end{bmatrix} = \begin{bmatrix} \widetilde{v}(0, .) - \beta \mathbf{F}_z \widetilde{e}(\widetilde{v}) \\ \widetilde{v}(1, .) - \beta \mathbf{F}_z \widetilde{e}(\widetilde{v}) \end{bmatrix}$$

where $\mathbf{F}_z$ is the transition probability matrix with elements $f_z(\mathbf{z}'|\mathbf{z})$, and $\widetilde{e}(\widetilde{v}) \equiv \{\widetilde{e}(\mathbf{z}; \widetilde{v}) : \text{for all } \mathbf{z}\}$ with $\widetilde{e}(\mathbf{z}; \widetilde{v}) = \int_{-\infty}^{\widetilde{v}(1,\mathbf{z})} [\widetilde{v}(1, \mathbf{z}) - \widetilde{\varepsilon}] \, dF_{\widetilde{\varepsilon}|1}(\widetilde{\varepsilon}) - \int_{-\infty}^{\widetilde{v}(0,\mathbf{z})} [\widetilde{v}(0, \mathbf{z}) - \widetilde{\varepsilon}] \, dF_{\widetilde{\varepsilon}|0}(\widetilde{\varepsilon})$. The mapping $\widetilde{g}(\widetilde{v})$ is globally invertible if and only if its Jacobian matrix $\mathbf{J}(\widetilde{v}) \equiv \partial \widetilde{g}(\widetilde{v}) / \partial \widetilde{v}'$ is non-singular for every value of $\widetilde{v}$. It is simple to show that this Jacobian matrix has the following form:

$$\mathbf{J}(\widetilde{v}) = \mathbf{I} - \beta \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} \otimes \left(\mathbf{F}_z \frac{\partial \widetilde{e}(\widetilde{v})}{\partial \widetilde{v}'} \right) \right]$$

where $\mathbf{I}$ is the identity matrix. Given the form of function $\widetilde{e}(\mathbf{z}; \widetilde{v})$, it is straightforward to show that $\partial \widetilde{e}(\mathbf{z}; \widetilde{v}) / \partial \widetilde{v}(0, \mathbf{z}) = F_{\widetilde{\varepsilon}|0}(\widetilde{v}(0, \mathbf{z})) = -P(0, \mathbf{z})$, and $\partial \widetilde{e}(\mathbf{z}; \widetilde{v}) / \partial \widetilde{v}(1, \mathbf{z}) = F_{\widetilde{\varepsilon}|1}(\widetilde{v}(1, \mathbf{z})) = P(1, \mathbf{z})$. Therefore,

$$\frac{\partial \widetilde{e}(\widetilde{v})}{\partial \widetilde{v}'} = [-diag\{P(0, .)\}; diag\{P(1, .)\}]$$

where $diag\{P(k, .)\}$ is diagonal matrix with elements $\{P(k, \mathbf{z})\}$ for every value of $\mathbf{z}$. The Jacobian matrix $\mathbf{J}(\widetilde{v})$ is invertible for every value of $\widetilde{v}$. □

Proof of Proposition 1 The proof of this Proposition 1 is a direct application of Proposition 4 in Aguirregabiria (2010) to our model of market entry and exit. Proposition 4 in Aguirregabiria (2010) applies to a general class of binary choice dynamic structural models with finite horizon, and it builds on previous results by Matzkin (1992, 1994). □

Proof of Proposition 2 (i) No identification. Let ec, fc, and sv be the true values of the functions in the population. Based on these true functions, define the functions: $ec^* (\mathbf{z}^c) = ec(\mathbf{z}^c) + \lambda(\mathbf{z}^c)$; $sv^* (\mathbf{z}^c) = sv(\mathbf{z}^c) + \lambda(\mathbf{z}^c)$, and $fc^* (\mathbf{z}^c) = fc(\mathbf{z}^c) - \lambda(\mathbf{z}^c) + \beta \sum_{\mathbf{z}^{c'} \in \mathbf{Z}^c} f_z(\mathbf{z}^{c'}|\mathbf{z}) \, \lambda\left(\mathbf{z}^{c'}\right)$, where $\lambda(\mathbf{z}^c) \neq 0$ is an arbitrary function. It is clear that ec^*, sv^*, and fc^* also satisfy the system of equations (20). Therefore, ec, fc, and sv cannot be uniquely identified from the restrictions in equations (20). *(ii) Identification of two combinations of the three structural functions.* We can derive equations in equations (21) after simple operations in the system (20). □

Proof of Proposition 3 Under the conditions of Proposition 3, we have that equation (24) becomes:

$$\begin{aligned} & Q(k, \mathbf{z}; \theta^0 + \Delta_\theta) = Q^0(k, \mathbf{z}) + \Delta_{vp}(\mathbf{z}^v) - \\ & \left[\Delta_{fc}\left(\mathbf{z}^c\right) + \Delta_{ec}\left(\mathbf{z}^c\right)\right] + k\left[\Delta_{ec}\left(\mathbf{z}^c\right) - \Delta_{sv}\left(\mathbf{z}^c\right)\right] \\ & + \beta^0 \sum_{\mathbf{z}^{sv\prime} \in \mathbf{Z}} f^0_{z,sv}(\mathbf{z}^{sv\prime}|\mathbf{z}) \Delta_{sv}(\mathbf{z}^{sv\prime}) \end{aligned} \tag{29}$$

The researcher knows all the elements in the right-hand-side of this equation, and therefore $Q(k, \mathbf{z}; \theta^0 + \Delta_\theta)$ is identified. Given $\widetilde{\mathbf{Q}}(\theta^0 + \Delta_\theta)$, we can use the inverse mapping $\widetilde{q}^{-1}$ to get $\widetilde{\mathbf{P}}(\theta^0 + \Delta_\theta) = \widetilde{q}^{-1}(\widetilde{\mathbf{Q}}(\theta^0 + \Delta_\theta); \beta^0, f_z^*)$. □

Proof of Proposition 4 Under the conditions of Proposition 4 (and making $\Delta_{vp} = \Delta_{fc} = \Delta_{ec} = \Delta_{sv} = \Delta_{f_{z,nosv}} = 0$, for simplicity but without loss of generality), equation (24) becomes:

$$\begin{aligned} Q(k, \mathbf{z}; \theta^0 + \Delta_\theta) &= Q^0(k, \mathbf{z}) + \beta^0 \sum_{\mathbf{z}^{sv\prime} \in \mathbf{Z}} \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z}) sv^0(\mathbf{z}^{sv\prime}) \\ &\quad + \Delta_\beta \sum_{\mathbf{z}^{sv\prime} \in \mathbf{Z}} [f^0_{z,sv}(\mathbf{z}^{sv\prime}|\mathbf{z}) + \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z})] sv^0(\mathbf{z}^{sv\prime}) \end{aligned} \tag{30}$$

Since the scrap value sv^0 is not identified, none of the terms that form $Q(k, \mathbf{z}; \theta^0 + \Delta_\theta) - Q^0(k, \mathbf{z})$ are identified. □

Proof of Proposition 5 It follows simply from equation (25). □

Proof of Proposition 6 Suppose $\mathbf{z}_2^c$ enters only in the entry cost $ec\,(\mathbf{z}^c)$. Under the conditions of Proposition 6, the system of equations (21) becomes:

$$fc\left(\mathbf{z}_1^c\right) + sv\left(\mathbf{z}_1^c\right) - \beta \sum_{\mathbf{z}_1^{c\prime} \in \mathbf{Z}^c} f_z(\mathbf{z}_1^{c\prime}|\mathbf{z}^v, \mathbf{z}_1^c, \mathbf{z}_2^c) sv\left(\mathbf{z}_1^{c\prime}\right) = -Q(1, \mathbf{z}) + vp(\mathbf{z}^v). \tag{31}$$

The difference between this equation evaluated at $\mathbf{z}_2^{c1}$ and at $\mathbf{z}_2^{c0}$ is:

$$\begin{aligned} & \sum_{\mathbf{z}_1' \in \mathbf{Z}} [f_z(\mathbf{z}_1^{c\prime}|\mathbf{z}^v, \mathbf{z}_1^c, \mathbf{z}_2^{c1}) - f_z(\mathbf{z}_1^{c\prime}|\mathbf{z}^v, \mathbf{z}_1^c, \mathbf{z}_2^{c0})] sv\left(\mathbf{z}_1^{c\prime}\right) \\ & = \frac{1}{\beta}\left[Q(1, \mathbf{z}^v, \mathbf{z}_1^c, \mathbf{z}_2^{c1}) - Q(1, \mathbf{z}^v, \mathbf{z}_1^c, \mathbf{z}_2^{c0})\right]. \end{aligned}$$

In matrix form, we can express this system of equations as

$$\left[\mathbf{F}_{z_1^c}(\mathbf{z}_2^{c1}) - \mathbf{F}_{z_1^c}(\mathbf{z}_2^{c0})\right] \mathbf{sv} = \frac{1}{\beta}\left[\mathbf{Q}(1, \mathbf{z}_2^{c1}) - \mathbf{Q}(1, \mathbf{z}_2^{c0})\right]. \tag{32}$$

Note that matrix $\mathbf{F}_{z_1^c}(\mathbf{z}_2^{c1}) - \mathbf{F}_{z_1^c}(\mathbf{z}_2^{c0})$ does not have full column rank because any matrix that is a difference of transition matrices is singular. However, the rank of $\mathbf{F}_{z_1^c}(\mathbf{z}_2^{c1}) - \mathbf{F}_{z_1^c}(\mathbf{z}_2^{c0})$ can be $|\mathcal{Z}_1^c| - 1$. If that is the case, we can combine the system of equations (32) with a normalization assumption on one single element of the vector $\mathbf{sv}$ (e.g., $sv(\mathbf{z}_1^{c0}) = 0$ for some value $\mathbf{z}_1^{c0}$) to uniquely identify the vector $\mathbf{sv}$. $\square$

Proof of Proposition 7 The identified function $\widehat{sv}(\mathbf{z}^c)$ is such that $\widehat{sv}(\mathbf{z}^c) = sv^0(\mathbf{z}^c) + \kappa$ where $sv^0(\mathbf{z}^c)$ is the true scrap value function and κ is an unknown constant. Given $\Delta_\beta = 0$, we can write equation (30) as

$$\begin{aligned} Q(k, \mathbf{z}; \theta^0 + \Delta_\theta) &= Q^0(k, \mathbf{z}) + \beta^0 \sum_{\mathbf{z}^{sv\prime} \in \mathbf{Z}} \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z}) \left[\widehat{sv}\left(\mathbf{z}^{sv\prime}\right) - \kappa\right] \\ &= \beta^0 \sum_{\mathbf{z}^{sv\prime} \in \mathbf{Z}} \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z}) \widehat{sv}\left(\mathbf{z}^{sv\prime}\right) \end{aligned}$$

where the last equality holds as we always have $\sum_{\mathbf{z}^{sv\prime} \in \mathbf{Z}} \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z}) = 0$. Therefore, we can calculate $Q(k, \mathbf{z}; \theta^0 + \Delta_\theta)$, and the effect of the counterfactual experiment on CCPs is identified. $\square$

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