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DYNAMIC SPATIAL COMPETITION BETWEEN MULTI-STORE RETAILERS*

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We propose a dynamic model of an oligopoly industry characterized by spatial competition between multi-store retailers. Firms compete in prices and decide where to open or close stores depending on demand and cost conditions, the number of competitors at different locations, and on location-specific private-information shocks. The model distinguishes multiple forces in the spatial configuration of store networks, such as cannibalization of revenue between stores of the same retail chain, economies of density, competition, consumer transportation costs, or positive demand spillovers from other stores. We develop an algorithm to approximate a Markov Perfect Equilibrium in our model, and propose a procedure for the estimation of the parameters of the model using panel data on number of stores, prices, and quantities at multiple geographic locations within a city. We also present a numerical example to illustrate the model and algorithm.

I. INTRODUCTION

Retail chains account for more than 60% of sales in U.S. retailing (see Hollander and Omura [1989], and Jarmin, Klimek and Miranda [2009]). Geographic location is in many cases the most important source of product differentiation for these firms. It is also a forward looking decision with significant non-recoverable entry costs, mainly due to capital investments which are both firm and location specific. Thus, the sunk cost of setting up a new store, and the dynamic strategic behavior associated with them, is a potentially important force behind the configuration of the spatial market structure that we observe in retail markets.

Despite its relevance, there have been very few studies analyzing spatial competition as a dynamic oligopoly game. Existent models of industry

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dynamics often lack an explicit account of spatial competition. Although useful applications have emerged from the seminal work by Pakes and McGuire [1994] and Ericson and Pakes [1995], none have explicitly incorporated the spatial and multi-store features which are prevalent in many retailing industries.[1] The literature on spatial competition often restricts the treatment of time. Models based on the seminal work of Hotelling [1929] describe a two or three-period framework where firms choose locations and then compete in the product market.[2] Eaton and Lipsey [1975], Schmalensee [1978], and Bonanno [1987] study the multi-store monopolist under the threat of entry. They find that an incumbent monopolist will strategically locate its stores to successfully preempt the entry of competitors. As noted by Judd [1985], the limited account of time and dynamics in this literature has very important implications on some predictions of these models. Judd notes that the aforementioned models assume that entry and location decisions are completely irreversible, with no possibility of exit or relocation. He shows that *allowing for exit* may result in non-successful spatial preemption by the incumbent. Judd's paper emphasizes that models of spatial competition between multi-store firms need to incorporate dynamics to its full extent, allowing for endogenous entry, exit, and forward-looking strategies. That is the intention of this paper.

In this context, the contribution of this paper is threefold. First, we propose a dynamic model of an oligopoly industry characterized by spatial competition between multi-store firms. In this model, firms compete in prices and decide where to open or close stores depending on the location profile of competitors, demand and cost conditions, and location-specific private information shocks. The model distinguishes multiple forces in the spatial configuration of store networks, such as cannibalization of revenue between stores of the same retail chain, economies of density, competition, consumer transportation costs, or positive demand spillovers from other stores. We define and characterize a Markov Perfect Equilibrium (MPE) in this model. The consideration of multi-store retail firms is a particularly relevant aspect of our model. Important topics in the analysis of competition in retail industries, such as cannibalization, spatial pre-emption, economies of density, network effects, or the value of mergers between store networks,

[1] Ellickson, Beresteanu and Misra [2010] endogenize supermarkets' 'store density,' i.e., the number of stores per capita a firm owns in a market. Holmes [2011] studies the role of economies of density in explaining the spatial evolution of Wal-Mart stores since the 1950s. However, spatial competition is not accounted for in these applications.

[2] Anderson, De Palma and Thisse [1992] present a compilation of static spatial competition models. There is also a large and growing literature on estimation of static structural models of spatial competition and store location. The work by West [1981a] and [1981b] was seminal in this literature. Some recent important contributions to this empirical literature are Pinkse, Slade and Brett [2002], Seim [2006], Zhu and Singh [2009], Ellickson, Houghton and Timmins [2013], Datta and Sudhir [2011], and Vitorino [2012]. Slade [2005] and Pinkse and Slade [2010] provide excellent surveys.

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cannot be studied without a model that recognizes the multi-store nature of retailers. Until recently, empirical games of market entry and store location did not account for the explicit multi-store nature of retailers, e.g., Seim [2006] and Zhu and Singh [2009]. However, recent studies by Jia [2008], Ellickson, Houghton and Timmins [2013], and Nishida [2014] propose and estimate entry games between retail chains. Our paper extends the models in these three previous papers in several important dimensions. First, while these previous models are static, we consider a dynamic game. Incorporating dynamics and firms' forward looking behavior is necessary to study topics such as pre-emption, first-mover advantage, or the implications of firms' uncertainty about future demand or land price. Second, the models in these previous papers do not incorporate spatial differentiation and competition within a market (i.e., within a U.S. county in the papers by Jia [2008] and Ellickson, Houghton and Timmins [2013], and within a 1 km square region in the paper by Nishida [2014]). In contrast, our model incorporates spatial differentiation and the explicit distance between retailers' stores within a city such that we can measure the degree of spatial substitution between stores. Third, Jia [2008] and Ellickson, Houghton and Timmins [2013] impose the restriction that there are only positive (or only negative) spillover effects between stores of the same retail chain. Our model allows for both cannibalization effects on the demand side and economies of scope, scale and density on the cost side.[3] Finally, the specification of the profit function in these previous models is semi-structural in the sense that it does not distinguish between demand and cost parameters. Our framework includes an explicit model of consumer demand with spatial differentiation, and a model of price competition between multi-store retailers. This allows us to distinguish between demand, variable cost, fixed cost and entry cost parameters, and to study welfare implications of alternative policies or firms' strategies.

A second contribution of this paper is to provide a method to compute an equilibrium of the model. The number of possible geographic configurations of a store network, that determines the size of the action space and of the state space in this dynamic game, increases exponentially with the number of geographic locations and with the number of firms. Solving exactly for an equilibrium of the model is an intractable problem even when the number of locations is not too large.[4] In static games of network competition, recent papers have proposed models and methods to deal with the high dimensionality of the action space. Jia [2008] and Nishida [2014] show that under certain restrictions on the profit function the static game is supermodular and this

[3] The model in Nishida [2014] allows for negative spillover effects within a market and positive spillover effects across markets. As in Jia [2008], these restrictions are imposed to satisfy a supermodularity condition in the profit function of a retail chain.

[4] If I is the number of firms, and L is the number of locations in the model, then the number of cells in the state space is 2^{IL}. For instance, with four firms and ten locations the number of cells is greater than one trillion (10^{12}).

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property can be exploited to define an algorithm for the solution of a Nash Equilibrium that is computationally efficient and practical. Unfortunately, the supermodularity of the static game does not extend to the dynamic version of the game, even under stronger conditions on firms' profits.[5] Ellickson, Houghton and Timmins [2013] relax the supermodularity restriction and propose a 'Profit Inequality' method for the estimation of a static game of competition between retail networks. The method does not require solving for an equilibrium but only evaluating profits at a (not very large) number of retail networks. However, this method cannot solve the dimensionality problem in a dynamic game because this problem appears not only in the dimension of the action space but most importantly in the dimension of the state space. Furthermore, while estimation of structural parameters does not require solving for an equilibrium, the implementation of counterfactual experiments typically involves the computation of an equilibrium, or at least an approximation. In this paper, we propose a method to obtain an approximation of an equilibrium of this dynamic game. The method combines three main ideas: (a) a restriction on the number of stores that a firm can open/close per period; (b) smoothing and interpolation over the geographic space of a city; and (c) a method of simulation and interpolation in the spirit of Rust [1997].[6]

(a) An advantage of the dynamic game is that we can deal, quite easily, with the problem of dimensionality of the action space that appears in the static game. In most real world situations, we find that a retail chain opens/closes only a small number of stores per quarter, or month, or week. We impose this as a restriction. Note that the researcher can observe store openings and closings almost in continuous time, and our model can accommodate any time frequency for firms' store location decisions.[7] (b) We exploit the geographic nature of the model to summarize the information in the state vector (i.e., a high-dimension vector of discrete variables that represent the number of stores of each retail chain at each location) using smooth surfaces in the two dimension geographic space that can be represented using a small number of parameters. (c) Finally, we apply Rust's random grid method and extend it to a multi-agent problem. While Rust proved that his method 'breaks the curse of dimensionality' in the solution of single-agent discrete-choice dynamic programming models with continuous state

[5] In order to apply this type of algorithm to a dynamic version of the game, we need supermodularity not of the one-period profit function but of the intertemporal profit function, i.e., the one-period profit plus the continuation value. This condition requires not only restrictions on the profit function but very unrealistic and *ad hoc* restrictions on the evolution of the endogenous state variables. See Aguirregabiria [2008] for an example in a simple dynamic model of market entry-exit.

[6] In a companion paper (Aguirregabiria and Vicentini [2012]), we provide a manual that describes in detail our programs and procedures. This manual and the software, in GAUSS language, can be downloaded from authors' web pages.

[7] We illustrate this point in section 5 in the context of a longitudinal dataset for the supermarket industry in the city of Greensboro, NC, that has been used in Vicentini [2013].

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variables, in this paper we do not provide any proof that this property extends to our multi-agent spatial competition model. For our purposes, Rust's method provides us with a practical way to reduce significantly the curse of dimensionality when computing equilibrium in the dynamic game.

A third contribution of the paper is that we discuss data requirements and econometric issues in the structural estimation of the parameters of the model. Section IV(i) provides a detailed description of the type of data required to estimate the model, and provides examples of previous applications in empirical IO that have used this type of data. Section IV(ii) describes our specification of the primitive functions and our restrictions on the unobservables of the model. In Section IV(iii), we describe the implementation of a two-step pseudo maximum likelihood method for the estimation of the dynamic game. In the first step of this method, the reduced-form (semiparametric) estimation of player's conditional choice probabilities is particularly challenging in our spatial dynamic model. For the estimation of these probabilities, we propose a flexible but practical reduced-form model that uses the variable profit function from the static Bertrand equilibrium of our model.

The rest of the paper is organized as follows. Section II presents the model and characterizes a Bertrand equilibrium of the static pricing game and a Markov Perfect Equilibrium of the dynamic game of store location by retail chains. In Section III, we describe our algorithms to solve for an equilibrium and to deal with multiple equilibria in the implementation of counterfactual experiments. Section IV deals with structural estimation, data requirements, and estimation methods. We have included in Section V a simple example to illustrate our model and methods. We summarize and conclude in Section VI.

II. MODEL

II(i). *The Market*

Consider a local market of a differentiated retail product (e.g., retail banking, supermarkets). From a geographic point of view the market is a compact set $\mathbb{C}$ in the Euclidean space $\mathbb{R}^2$. The distance between two points in the market, say a and b, is the Euclidean distance denoted by $||a-b||$. There is a finite set of L pre-specified business locations where it is feasible for firms to operate stores. Let $\{z_1, z_2, \ldots, z_L\}$ be the set of geographical coordinates of these feasible locations, where $z_\ell \in \mathbb{C}$. Figure 1 presents an example.[8]

[8] The assumption of a finite number of feasible locations is made for computational convenience. In an empirical application of the model, this assumption implies that the researcher has to discretize the set of business locations. However, there are situations where this assumption can be realistic. For instance, in Canada and the U.S. leasing contracts at shopping centers typically include a 'radius restriction' clause that prohibits tenants in a shopping center from opening another store within a certain radius (see Eckert and West [2006]). Also, some countries and states have zoning laws that apply to 'big box' retailers. These restrictions can be sufficiently strict such that some retailers have only a few feasible locations.

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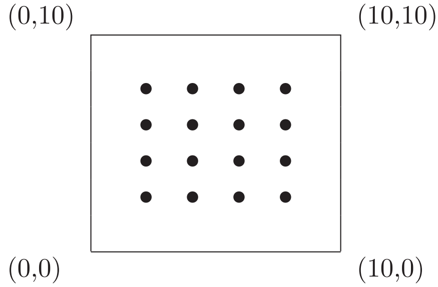

Figure 1
Market and Feasible Business Locations (represented with ●)

Time is discrete. At time t the market is populated by a continuum of consumers. Each consumer is characterized by a geographical location or *home address* $z \in \mathbb{C}$. The geographical distribution of consumers at period t is given by the absolute measure $\phi_t(z)$ such that $\int_{\mathbb{C}} \phi_t(dz) = M_t$, where M_t is the size of the market. This measure ϕ_t evolves over time according to a discrete Markov process. Let Ω be the discrete set of possible functions ϕ_t.[9]

There are I multi-store firms that can potentially operate in the market. We index firms by i and use $\Upsilon = \{1, 2, \ldots, I\}$ to represent the set of firms. At the beginning of period t a firm's network of stores is represented by the vector $n_{it} = (n_{i1t}, n_{i2t}, \ldots, n_{iLt})$, where $n_{i\ell t}$ is the number of stores that firm i operates in location ℓ at period t. For simplicity, we assume that a firm can have at most one store in a location, such that $n_{i\ell t} \in \{0, 1\}$. The model can be easily generalized to the case with more than one store per location and firm.[10] Overlapping of stores from different firms at the same location is allowed. In Section II(vii), we present an extension of the model where business space is scarce in some locations such that there is a maximum number of stores that can operate in each of these locations. The spatial market structure at period t is represented by the vector $\mathbf{n}_t = (n_{1t}, n_{2t}, \ldots, n_{It}) \in \{0, 1\}^{IL}$. A store in this market is identified by a pair (i, ℓ) where i represents the firm, and ℓ identifies the location.

Every period t, firms observe the spatial market structure $\mathbf{n}_t$, the state of the demand ϕ_t, and some location and firm specific shocks in costs which are private information of each firm. Given this information, incumbent firms compete in prices. Prices can vary over stores within the same firm.

[9] A market where the spatial distribution of consumers is constant over time (i.e., $\phi_t(z) = \phi(z)$ for any $z \in \mathbb{C}$) is a particular case of our model.

[10] In our model, two stores of the same firm and at the same location are perfect substitutes. Therefore, a firm will never find it optimal to have more than one store at the same location. However, it is straightforward to extend our logit demand model to allow for horizontal differentiation between stores with the same firm and location, i.e., the consumer-specific extreme value type 1 variables should vary across stores within a firm location.

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This spatial Bertrand game is static because current prices do not have any effect on future demand or profits. Furthermore, private information shocks affect fixed operating costs and entry costs but not the demand or variable costs. Therefore, these shocks do not have any influence on equilibrium prices. The resulting Bertrand prices determine equilibrium variable profits for each firm i at period t. At the end of period t, firms decide simultaneously their network of stores for the next period. This choice is dynamic because of partial irreversibility in the decision to open a new store, i.e., sunk costs. Firms are allowed to open or close at most one store per period. Exogenous changes in the spatial distribution of demand (i.e., changes in ϕ_t) and firms' location specific shocks generate entry and exit at different locations and changes over time in the spatial market structure. Firms may grow (or decline) over time expanding (contracting) their network of stores, and possibly become a dominant player (or exit from the market). The details of this model are presented in Sections II(ii) to II(vi). In Section II(vii) we present several extensions of our basic framework.

Some assumptions about the basic structure of the model deserve further explanation. First, while our model of entry-exit and store location is dynamic, we assume that the model of price competition is static. This implies that there are no dynamics in demand, such as durable or storable products and consumer switching costs, or in variables costs, such as learning by doing/selling, menu costs, or firm inventories. This structure of the model follows the framework in Ericson and Pakes [1995] and Pakes and McGuire [1994], as well as most of the recent literature on empirical dynamic games of oligopoly competition (see Aguirregabiria and Nevo [2013]). An important reason for using this structure is convenience in the characterization and computation of an equilibrium. However, we also believe that this structure is realistic for many retail industries, especially if the analysis is not particularly focused on the short-run dynamics of prices.[11] Second, our model allows for firms' private information in the dynamic game but assumes complete information in the static pricing game. We admit that allowing for private information in both games would be more realistic. Our assumption of complete information in the pricing game is mainly for convenience and it follows most of the empirical IO literature on models of price competition in differentiated product industries. While it is relatively simple to characterize and compute a Bayesian Nash Equilibrium in a discrete choice game of incomplete information (like a model of market entry-exit), it is substantially more complicated in games where

[11] Sales promotions and price dynamics related to storable products (Hendel and Nevo [2006]), or to firm inventories (Aguirregabiria [1999]) tend to occur at relatively high time frequencies. Our model of price competition can be interpreted in terms of firms' competition in average prices over a quarter or a year.

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players' decisions are continuous variables, and even more if the decision is a vector of prices at each/store location, as in our pricing game.[12]

II(ii). *Consumer Behavior*

The model of consumer demand that we present here builds on De Palma *et al.* [1985]. We extend their model in two dimensions. First, we incorporate vertical differentiation (i.e., they assume that firms have the same qualities ω_i). And second, geographic location in our model has two dimensions while they consider a linear city.

A consumer is fully characterized by a pair (z, v), where z is her location in geographic space or *home address*, and $v \in \mathbb{R}^{IL}$ is a vector representing her idiosyncratic preferences over all possible stores. Consumer behavior is static and demand is unitary. At every period t, consumers know all active stores with their respective locations and prices. A consumer decides whether to buy or not a unit of the good and from which store to buy it. The indirect utility of consumer (z, v) patronizing store (i, ℓ) at time t is:

$$(1) \qquad u_{i\ell t} = \omega_{i\ell} - p_{i\ell t} - \tau g(||z - z_\ell||) + v_{i\ell}$$

$\omega_{i\ell}$ is the quality of the product offered by retail chain i that, in principle, may vary exogenously across locations. All consumers agree on this measure. $p_{i\ell t}$ is the mill price charged by store (i, ℓ) at time t. The term $\tau\, g(||z - z_\ell||)$ represents consumer's transportation costs, where τ is the unit transportation cost and $||z - z_\ell||$ is the Euclidean distance between the consumer's address and the store, and $g(.)$ is a continuous and increasing function.[13] Finally, $v_{i\ell}$ captures consumer idiosyncratic preferences for store (i, ℓ). The utility of the outside alternative (i.e., not purchasing the good) is normalized to zero.

A consumer purchases a unit of the good at store (i, ℓ) if and only if $u_{i\ell t} \geq 0$ and $u_{i\ell t} \geq u_{i'\ell' t}$ for any other store (i', ℓ'). To obtain the aggregate demand at each store we have to integrate individual demands over the distribution of (z, v). We assume that v is independent of z and it has a type 1 extreme value distribution with dispersion parameter μ. The parameter μ measures the importance of horizontal product differentiation, other than spatial differentiation. Integrating over v we obtain the *local demand* for store (i, ℓ) from consumers at location z:

[12] In a discrete choice game of incomplete information, a player's strategy can be represented as a vector of probabilities in the simplex. In contrast, in a continuous decision game of incomplete information, a player's strategy is a multivariate continuous function, which is a substantially more complex object. Most applications of continuous decision games of incomplete information have concentrated in relatively simple one-dimension strategies such as auctions, or price competition in homogeneous product industries.

[13] The $g(.)$ function can be linear, concave, or convex. As in the standard Hotelling model of store location, if this function is convex, then the equilibrium of the model incorporates a consumer transportation cost motive for the agglomeration of stores.

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$$\sigma_{i\ell}(z,\mathbf{n}_t,\mathbf{p}_t)=\frac{n_{i\ell t}\exp\{(\omega_{i\ell}-p_{i\ell t}-\tau g(||z-z_\ell||))/\mu\}}{1+\sum_{i'=1}^{I}\sum_{\ell'=1}^{L} n_{i'\ell' t}\exp\{(\omega_{i'\ell'}-p_{i'\ell' t}-\tau g(||z-z_{\ell'}||))/\mu\}} \tag{2}$$

Integrating these local demands over the spatial distribution of consumers we obtain the *aggregate demand* for store (i,ℓ) at time t:

$$s_{i\ell}(\mathbf{n}_t,\mathbf{p}_t,\phi_t)=\int_{\mathbb{C}}\sigma_{i\ell}(z,\mathbf{n}_t,\mathbf{p}_t)\phi_t(dz) \tag{3}$$

Consumers' substitution patterns depend directly on the distance function $||z-z_\ell||$, so that a store competes more fiercely against closer stores. Stores' market areas are *overlapping* because of the unobserved heterogeneity of consumers, v. Therefore a store serves consumers from all corners of the city $\mathbb{C}$, but more so the nearby patronage. Stores will always face a positive demand and can adjust prices without facing a perfectly elastic demand. Firms face the trade-off between strategic and market share effects. As stores locate closer to each other, the more intense price competition acts as a centrifugal force of dispersion (*strategic* effect). At the same time, firms wish to locate where transportation costs are minimum, which acts as a centripetal force of agglomeration (*market share* effect). An equilibrium spatial market structure would balance these forces, along with the effect of own-firm stores cannibalization.

We note the importance of the parameters μ and τ that capture product differentiation. As $\mu\to 0$ the degree of non-spatial horizontal product differentiation becomes small and every consumer shops at the store with the lowest full price from her location (i.e., quality-adjusted mill price plus transportation costs). At the limit we would observe market areas defined as Voronoi graphs (or Thiessen polygons) with well defined market borders (see Eaton and Lipsey [1975], or Tabuchi [1994], among others). Transportation costs increase the importance of location, serve as a shield for market power and create incentives for firm dispersion.[14]

II(iii). *Price Competition*

For notational simplicity we omit the time subindex in this subsection. Every period firms compete in prices taking as given their network of stores, the state of the demand, and variable costs. Firms may charge different prices at different stores. This price competition is a game of

[14] Besides computing equilbrium prices, our Bertrand algorithm computes demand price elasticities for each location and store at these prices. These elasticities help the researcher better understand what the actual market areas are in geographic space. The detection of the relevant geographical market area has long been debated among antitrust authorities (see Willig [1991] and Baker [1997]).

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complete information. In this game, every firm knows the current networks of stores, the demand system, and the unit costs of all active stores. A firm variable profit function is:

$$R_i(\mathbf{n},\mathbf{p},\phi)=\sum_{\ell=1}^{L}(p_{i\ell}-c_{i\ell})s_{i\ell}(\mathbf{n},\mathbf{p},\phi) \tag{4}$$

$c_{i\ell}$ is the unit variable cost of firm i at location ℓ which is assumed constant over time. Each firm maximizes its variable profit by choosing its best-response vector of prices. The best response of firm i can be characterized by the first-order condition for each price $p_{i\ell}$:

$$s_{i\ell}+(p_{i\ell}-c_{i\ell})\frac{\partial s_{i\ell}}{\partial p_{i\ell}}+\sum_{\ell'\neq\ell}(p_{i\ell'}-c_{i\ell'})\frac{\partial s_{i\ell'}}{\partial p_{i\ell}}=0 \tag{5}$$

The first two terms are the price and quantity effects of $p_{i\ell}$ on the profit at its own store (i,ℓ), while the last term is the quantity effect of $p_{i\ell}$ on all other stores of firm i. This last term is zero for a single-store firm. For a multi-store firm, this term captures how the pricing decision of the firm internalizes the *cannibalization effect* among its own stores. In our demand system, stores of a same firm are *gross substitutes* (i.e., $\partial s_{i\ell'}/\partial p_{i\ell}>0$ for $\ell'\neq\ell$) and therefore the third term is always positive. Given that $\partial s_{i\ell}/\partial p_{i\ell}<0$, we have that, *ceteris paribus*, a multi-store firm will offer higher prices than a single-store firm. The firm knows that by reducing the price in one of its stores there is a business stealing effect on its other stores.[15]

Let $\mathbf{p}$ and $\mathbf{s}(\mathbf{p})$ be vectors with dimension $IL\times 1$ of prices and demands, respectively, for every store. Following Berry [1994] and Berry *et al.* [1995], we define a square matrix $\Lambda(\mathbf{p})$ of dimension $IL\times IL$ with elements:

$$\Lambda_{i\ell}^{j\ell'}=\begin{cases}-\dfrac{\partial s_{j\ell'}}{\partial p_{i\ell}} & \text{if } j=i\\ 0 & \text{otherwise}\end{cases} \tag{6}$$

We can write the entire system of best-response equations in vector form as $\mathbf{s}(\mathbf{p})-\Lambda(\mathbf{p})\cdot(\mathbf{p}-\mathbf{c})=0$, or what is equivalent:

$$\mathbf{p}=\mathbf{c}+\Lambda(\mathbf{p})^{-1}\cdot\mathbf{s}(\mathbf{p}) \tag{7}$$

where $\mathbf{c}$ is the $IL\times 1$ vector of unit costs. A spatial Nash-Bertrand equilibrium is then a vector $\mathbf{p}^{*}$ that solves the fixed-point mapping (7). Given our

[15] Of course, there are cost factors (e.g., economies of scale and density) that can make prices of multi-store firms smaller than prices of single-store firms. The effect that we illustrate in equation (5) is only the cannibalization effect.

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assumptions on the distribution of consumer taste heterogeneity v, the mappings $\mathbf{s}(\mathbf{p})$ and $\Lambda(\mathbf{p})$ are continuously differentiable. Furthermore, it is possible to show that, for every firm i and location ℓ, an equilibrium price $p_{i\ell}^*$ in this game is always greater than or equal to the constant marginal cost $c_{i\ell}$ and smaller than or equal to the equilibrium price at store (i, ℓ) when firm i is a monopolist with stores at every location in the city. Therefore, the vector of prices $\mathbf{p}$ belongs to a compact set: $\mathbf{p}$ belongs to the IL-dimension rectangle (i.e., lattice) $< \mathbf{c}, \mathbf{p}^{Mon} >$, where $\mathbf{p}^{Mon}$ is the vector of monopoly prices for each firm-location. By Brower's fixed-point theorem, a Nash-Bertrand equilibrium exists.

The equilibrium is not necessarily unique. Multiplicity of equilibria may be a problem when we use the model for comparative statics or to evaluate the effects of public policies. A possible way of dealing with multiplicity of equilibria is to impose an equilibrium selection mechanism, i.e., a criterion that selects a specific type of equilibrium such that when we do comparative statics the same equilibrium type is always selected. To implement an equilibrium selection mechanism in practice, we need an algorithm that can find the specific type of equilibrium for any possible specification of the primitives of the model. We describe here an algorithm with these features that exploits the supermodularity of this Bertrand game.

The algorithm is based on Topkis [1979], [1998] and Echenique [2007]. Following Vives [1999] (page 32), our Bertrand game is smooth supermodular if it satisfies the following conditions: (i) the space of prices is a lattice; (ii) the profit function $R_i(.)$ is twice continuously differentiable in prices; (iii) $\partial^2 R_i / \partial p_{i\ell} \partial p_{jm} \geq 0$ for any $i \neq j$ and any pair of locations ℓ and m; and (iv) $\partial^2 R_i / \partial p_{i\ell} \partial p_{im} \geq 0$ for any $\ell \neq m$. As mentioned above, conditions (i) and (ii) are satisfied in this pricing game because $\mathbf{p}$ belongs to the lattice $< \mathbf{c}, \mathbf{p}^{Mon} >$ and the revenue function is twice continuously differentiable within that set. A sufficient (but not necessary) condition for (iii) and (iv) to hold is that the local market shares $\sigma_{i\ell}(z, \mathbf{p})$ are never larger than a half within the set $< \mathbf{c}, \mathbf{p}^{Mon} >$.[16] More generally, this Bertrand game is smooth

[16] For condition (iii) we have that for $i \neq j$:

$$\frac{\partial^2 R_i}{\partial p_{i\ell} \partial p_{jm}} = \left[\frac{\partial s_{i\ell}}{\partial p_{jm}} \right] + \left[(p_{i\ell} - c_{i\ell}) \frac{\partial^2 s_{i\ell}}{\partial p_{i\ell} \partial p_{jm}} \right] + \left[\sum_{\ell' \neq \ell} (p_{i\ell'} - c_{i\ell'}) \frac{\partial^2 s_{i\ell'}}{\partial p_{i\ell} \partial p_{jm}} \right]$$

The first and the third terms in brackets are always positive. For the second term, we have that: $\partial^2 s_{i\ell} / \partial p_{i\ell} \partial p_{jm} = \mu^{-1} \int \partial \sigma_{i\ell}(z) / \partial p_{jm} (1 - 2\sigma_{i\ell}(z)) \phi(dz)$. Since $\partial \sigma_{i\ell}(z) / \partial p_{jm} > 0$, a sufficient condition for this second term to be positive is that, for any location $z \in \mathbb{C}$, the local market shares $\sigma_{i\ell}(z)$ are smaller than 1/2. This condition holds when qualities are not too large and the degree of horizontal product differentiation μ is not too small relative to transportation costs. However, it is clear that this sufficient condition is far from being necessary. Local market shares greater than 1/2 are perfectly compatible with a positive value for $\partial^2 s_{i\ell} / \partial p_{i\ell} \partial p_{jm}$. Furthermore, it is clear that the cross-price second derivative of the profit function can be

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supermodular if quality differences between firms are not too large and the degree of horizontal product differentiation (μ) is not too small relative to transportation costs (τ).

The following Lemma is a straightforward application to our Bertrand game of a general result by Topkis [1979] for supermodular games. The Lemma establishes a simple algorithm to obtain the equilibria with the smallest and with the largest prices, respectively.

Lemma. Define the best response mapping in vector form, $\mathbf{b}(\mathbf{p}) \equiv \mathbf{c} + \Lambda(\mathbf{p})^{-1}\mathbf{s}(\mathbf{p})$. Consider the following algorithm (best response function iteration): start with an initial vector of prices $\mathbf{p}^0$; at iteration $k \geq 1$, $\mathbf{p}^k = \mathbf{b}(\mathbf{p}^{k-1})$; stop when $\mathbf{p}^k = \mathbf{p}^{k-1}$. If the game is supermodular then:

i. if we start with $\mathbf{p}^0 = \mathbf{c}$, then the algorithm stops at the Nash-Bertrand equilibrium with smallest prices, $\mathbf{p}^{low}$;

ii. if we start with $\mathbf{p}^0 = \mathbf{p}^{Mon}$, then the algorithm stops at the Nash-Bertrand equilibrium with largest prices, $\mathbf{p}^{high}$.

Topkis [1979] proved this Lemma for supermodular games, and Topkis [1998] extended the result to a more general class of games with strategic complementarities (GSC). The proof is relatively simple. If the game is supermodular, then the best response mapping $\mathbf{b}(\mathbf{p})$ is monotonically increasing. This implies that $\mathbf{b}(\mathbf{c}) \geq \mathbf{c}$, and $\mathbf{b}^2(\mathbf{c}) \equiv \mathbf{b}(\mathbf{b}(\mathbf{c})) \geq \mathbf{c}, \ldots$, and $\mathbf{b}^k(\mathbf{c}) \geq \mathbf{b}^{k-1}(\mathbf{c})$, such that iterating in $\mathbf{b}(.)$ generates a monotonically increasing sequence in the space of prices $< \mathbf{c}, \mathbf{p}^{Mon} >$. But the space of prices is compact, so there must be an iteration such that $\mathbf{b}^k(\mathbf{c}) = \mathbf{b}^{k-1}(\mathbf{c})$, and this implies that $\mathbf{b}^k(\mathbf{c})$ is a Nash-Bertrand equilibrium. Let $\mathbf{p}^{low}$ be that equilibrium, and let $\mathbf{p}^*$ be another equilibrium of the game. Since $\mathbf{p}^* \geq \mathbf{c}$ and the best response mapping is monotonically increasing, we have that $\mathbf{p}^{low} = \mathbf{b}^k(\mathbf{c}) \leq \mathbf{b}^k(\mathbf{p}^*) = \mathbf{p}^*$ for any equilibrium $\mathbf{p}^*$. Therefore, $\mathbf{p}^{low}$ is the equilibrium with smallest prices. Similarly, if we initialize the algorithm with $\mathbf{p}^0 = \mathbf{p}^{Mon}$, we generate a monotonically increasing sequence, $\mathbf{b}(\mathbf{p}^{Mon}) \leq \mathbf{b}^2(\mathbf{p}^{Mon}) \leq \ldots \leq \mathbf{b}^k(\mathbf{p}^{Mon})$ that should converge in the compact space of

positive when $\partial^2 s_{i\ell}/\partial p_{i\ell}\partial p_{jm}$ is negative just because the other two terms can be larger in absolute value. For condition (iv) we have that for $\ell \neq m$:

$$\frac{\partial^2 R_i}{\partial p_{i\ell}\partial p_{im}} = \left[\frac{\partial s_{i\ell}}{\partial p_{im}} + \frac{\partial s_{im}}{\partial p_{i\ell}}\right] + \left[(p_{i\ell} - c_{i\ell})\frac{\partial^2 s_{i\ell}}{\partial p_{i\ell}\partial p_{im}} + (p_{im} - c_{im})\frac{\partial^2 s_{im}}{\partial p_{i\ell}\partial p_{im}}\right] + \left[\sum_{\ell' \neq \{\ell, m\}} (p_{i\ell'} - c_{i\ell'})\frac{\partial^2 s_{i\ell'}}{\partial p_{i\ell}\partial p_{im}}\right]$$

Again, the first and the third terms in brackets are always positive. The second term is also positive under the same conditions as mentioned above.

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prices. Let $\mathbf{p}^{high}$ be that equilibrium. Since any equilibrium $\mathbf{p}^*$ is such that $\mathbf{p}^* \leq \mathbf{p}^{Mon}$, we have that $\mathbf{p}^{high} = \mathbf{b}^k(\mathbf{p}^{Mon}) \geq \mathbf{b}^k(\mathbf{p}^*) = \mathbf{p}^*$ for any equilibrium $\mathbf{p}^*$. Therefore, $\mathbf{p}^{high}$ is the equilibrium with largest prices.[17]

Based on this Lemma, we can use Topkis algorithm to select always the same type of Nash-Bertrand equilibrium, e.g., the equilibrium with minimum prices. The two extremal equilibria coincide with the Pareto best and the Pareto worst equilibria from the point of view of firms (see Vives [1999], page 152). In the numerical examples in Section V, we use Topkis algorithm to select the Nash-Bertrand equilibrium with smallest prices: the worst equilibria from the point of view of firms.[18]

Let $\mathbf{p}^*(\mathbf{n}, \phi)$ be the vector of equilibrium prices associated with a value $(\mathbf{n}, \phi)$ of the state variables. Solving this vector into the variable profit function one obtains the equilibrium variable profit function:

$$R_i^*(\mathbf{n}, \phi) \equiv R_i(\mathbf{n}, \mathbf{p}^*(\mathbf{n}, \phi), \phi) \tag{8}$$

II(iv). *Dynamic Game*

At the end of period t firms simultaneously choose their network of stores $\mathbf{n}_{t+1}$ with an understanding that they will affect their variable profits at future periods. We model the choice of store location as a game of *incomplete information*, so that each firm i has to form beliefs about other firms' choices of networks. More specifically, there are components of the entry costs and exit values of a store which are firm-specific and private information. There are two main reasons why we include incomplete information in our model. First, as shown by Doraszelski and Satterthwaite [2010], in the Ericson-Pakes complete-information model of industry dynamics an equilibrium in pure strategies does not necessarily exist. Doraszelski and Satterthwaite also show that the introduction of private information variables with continuous distribution function and large support guarantees the existence of an equilibrium in this class of games of industry dynamics. Second, the recent literature on estimation of dynamic games has also considered games of incomplete information because these variables are convenient sources of unobserved heterogeneity from the point of view of the

[17] See also Echenique [2007], who has developed an efficient algorithm to find all the equilibria in GSC. The implementation of Echenique's algorithm is relatively simple. Once we have obtained the lowest equilibrium, $\mathbf{p}^{low}$, we can define a new game with the same payoff functions as our game but where the set of feasible prices is the lattice $< \mathbf{p}^{low} + \delta, \mathbf{p}^{Mon} >$, and $\delta > 0$ is a small constant. Then, we can apply Topkis algorithm to obtain the lowest equilibrium of this game. It is straightforward to show that this equilibrium is the one with the second lowest prices in our Bertrand game. We can proceed in this way to obtain a sequence of equilibria sorted by the value of prices. The algorithm continues until an iteration K where the K-lowest equilibrium is exactly the highest equilibrium, $\mathbf{p}^{high}$.

[18] We can also use the Lemma to check for multiplicity of equilibria. If the smallest equilibrium coincides with the largest equilibrium, then the equilibrium is unique.

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researcher (see Aguirregabiria and Mira [2007], or Pakes, Ostrovsky and Berry [2007]).

We assume that a firm may open or close *at most one store* per period. Given that we can make the frequency of firms' decisions arbitrarily high, this is a plausible assumption that reduces significantly the cost of computing an equilibrium in this model. Let a_{it} be the decision of firm i at period t such that: $a_{it}=\ell_+$ represents the decision to open a new store at location ℓ; $a_{it}=\ell_-$ means that a store at location ℓ is closed; and $a_{it}=0$ means the firm chooses to do nothing. Therefore, the choice set is $A=\{0,\ell_+,\ell_- : \ell=1,2,\ldots,L\}$. Some of the choice alternatives in A are not feasible for a firm given its current network n_{it}. In particular, a firm can not close a store in a submarket where it has no stores, and it cannot open a new store in a location where it already has a store. The set of feasible choices for firm i at period t is denoted $A(n_{it})$ such that $A(n_{it})= \{0\}\cup \{\ell_+ : n_{i\ell t}=0\}\cup \{\ell_- : n_{i\ell t}=1\}$. Note that this choice set has exactly $L+1$ choice alternatives.

We represent the transition rule of market structure as $\mathbf{n}_{t+1}=\mathbf{n}_t+1[a_t]$, where $1[a_t]$ is a $IL\times 1$ vector such that its (i,ℓ)-element is equal to $+1$ when $a_{it}=\ell_+$, to -1 when $a_{it}=\ell_-$, and to zero otherwise. That is, the (i,ℓ)-element of $1[a_t]$ is equal to $1\{a_{it}=\ell_+\}-1\{a_{it}=\ell_-\}$, where $1\{.\}$ is the indicator function.

II(v). *Specification of the Profit Function*

Firm i's current profit is:

$$\Pi_{it} = R_i^*(\mathbf{n}_t,\phi_t)-FC_{it}-EC_{it}+EV_{it} \tag{9}$$

FC_{it} is the fixed cost of operating all the stores of firm i. EC_{it} is the entry or set-up cost of a new store. And EV_{it} is the exit value of closing a store. Fixed operating costs depend on the number of stores but also on their location.

$$FC_{it} = \sum_{\ell=1}^{L} \theta_{i\ell}^{FC} n_{i\ell t} \tag{10}$$

$\theta_{i\ell}^{FC}$ is the fixed cost of operating a store in submarket ℓ. In Section II(vii), we extend this basic specification to incorporate economies of scale and density in the fixed cost of a retail chain. The specification of entry cost is:

$$EC_{it} = \sum_{\ell=1}^{L} 1\{a_{it}=\ell_+\}\left(\theta_{i\ell}^{EC}+\varepsilon_{i\ell t}^{EC}\right) \tag{11}$$

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$\theta_{i\ell}^{EC}$ is the entry cost at location ℓ. The variable $\varepsilon_{i\ell t}^{EC}$ represents a firm and location specific component of the entry cost. This idiosyncratic shock is private information of firm i. The specification of the exit value is:

$$EV_{it} = \sum_{\ell=1}^{L} 1\{a_{it}=\ell_{-}\}\left(\theta_{i\ell}^{EV}+\varepsilon_{i\ell t}^{EV}\right) \tag{12}$$

$\theta_{i\ell}^{EV}$ is the scrapping or exit value of a store in location ℓ. The variable $\varepsilon_{i\ell t}^{EV}$ is a firm and location specific shock in the exit value of a store.

The vector of private information variables for firm i at period t is $\varepsilon_{it}=\{\varepsilon_{i\ell t}^{EC}, \varepsilon_{i\ell t}^{EV} : \ell=1,2,\dots,L\}$. We make two assumptions on its distribution. First, we assume that ε_{it} is independent of demand conditions ϕ_t, and independently distributed across firms and over time. Independence across firms implies that a firm cannot learn about other firms' ε's by using its own private information. And independence over time means that a firm cannot use other firms' histories of previous decisions to infer their current ε's. These assumptions simplify significantly the definition and the computation of an equilibrium in this dynamic game. Second, we assume that ε_{it} has a cumulative distribution function $G_i(.)$ that is strictly increasing and continuously differentiable with respect to the Lebesgue measure in $\mathbb{R}^{2L}$. These two assumptions allow for a broad range of specifications for the ε_{it}'s, including spatially correlated shocks.

It will be convenient to distinguish two additive components in the current profit function:

$$\Pi_{it}=\pi_i(a_{it},\mathbf{n}_t,\phi_t)+\varepsilon_{it}(a_{it}) \tag{13}$$

where $\pi_i(a_{it},\mathbf{n}_t)$ is the current profit function excluding the private information variables, and $\varepsilon_{it}(a_{it})$ represents the private information shock associated with action a_{it}.

II(vi). *Markov Perfect Equilibrium*

We consider that a firm's strategy depends only on its payoff relevant state variables $(\mathbf{n}_t,\phi_t,\varepsilon_{it})$. For the sake of notational simplicity, hereinafter we omit the state of the demand ϕ_t as an argument of the different functions. Let $\alpha\equiv\{\alpha_i(\mathbf{n}_t,\varepsilon_{it}) : i\in\Upsilon\}$ be a set of strategy functions, one for each firm, such that α_i is a function from $\{0,1\}^{IL}\times\mathbb{R}^{2L}$ into A. Given a set of strategy functions α, we can define a value function $V_i^{\alpha}(\mathbf{n}_t,\varepsilon_{it})$ that represents the value of firm i given that the other firms behave according to their strategy functions in α and firm i responds optimally. The value function V_i^{α} is the unique solution of the following Bellman equation:

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$$V_i^{\alpha}(\mathbf{n}_t, \varepsilon_{it}) = \max_{a_{it} \in A(n_{it})} \left\{ v_i^{\alpha}(a_{it}, \mathbf{n}_t) + \varepsilon_{it}(a_{it}) \right\} \tag{14}$$

where the functions $v_i^{\alpha}(a_{it}, \mathbf{n}_t)$ are *choice specific value functions* which are defined as:

$$\begin{aligned} v_i^{\alpha}(a_{it}, \mathbf{n}_t) \equiv \pi_i(a_{it}, \mathbf{n}_t) + \beta \int V_i^{\alpha}\left(\mathbf{n}_t + 1[a_{it}, \alpha_{-i}(\mathbf{n}_t, \varepsilon_{-it})], \varepsilon_{i,t+1}\right) \\ \times dG_i(\varepsilon_{i,t+1}) \left[\prod_{j \neq i} dG_j(\varepsilon_{jt}) \right] \end{aligned} \tag{15}$$

Similarly, we can define firm i's best response function, $\alpha_i^{BR}(\mathbf{n}_t, \varepsilon_{it}, \alpha_{-i})$, as:

$$\alpha_i^{BR}(\mathbf{n}_t, \varepsilon_{it}, \alpha_{-i}) = \arg\max_{a_{it} \in A(n_{it})} \left\{ v_i^{\alpha}(a_{it}, \mathbf{n}_t) + \varepsilon_{it}(a_{it}) \right\} \tag{16}$$

A Markov perfect equilibrium (MPE) in this game is a set of strategy functions such that each firm's strategy maximizes the value of the firm for each possible $(\mathbf{n}_t, \varepsilon_{it})$ and taking other firms' strategies as given.

Definition. A set of strategy functions $\alpha^* \equiv \{\alpha_i^*(\mathbf{n}_t, \varepsilon_{it}) : i \in \Upsilon\}$ is an MPE if and only if for any firm i and any state $(\mathbf{n}_t, \varepsilon_{it})$ we have that:

$$\alpha_i^*(\mathbf{n}_t, \varepsilon_{it}) = \alpha_i^{BR}(\mathbf{n}_t, \varepsilon_{it}; \alpha_{-i}^*) \tag{17}$$

Next, we follow Aguirregabiria and Mira ([2007], pp. 7–13) to represent an MPE as a fixed point in a space of choice probabilities. The algorithm that we use to compute an equilibrium (in Section III(ii)) uses this representation. We start by defining three objects: conditional choice probabilities; integrated value function; and best response probability function.

Conditional choice probabilities (CCP's). Given any set of strategy functions α, we can define a set of conditional choice probabilities $\mathbf{P}^{\alpha} \equiv \{P_i^{\alpha}(a_{it}|\mathbf{n}_t) : i \in \Upsilon; a_{it} \in A; \mathbf{n}_t \in \{0,1\}^{IL}\}$ such that

$$P_i^{\alpha}(a_{it}|\mathbf{n}_t) \equiv \Pr\left(\alpha_i(\mathbf{n}_t, \varepsilon_{it}) = a_{it}|\mathbf{n}_t\right) = \int 1\{\alpha_i(\mathbf{n}_t, \varepsilon_{it}) = a_{it}\} dG_i(\varepsilon_{it}) \tag{18}$$

The probabilities in $\mathbf{P}^{\alpha}$ represent firms' expected behavior, from the point of view of the competitors, when firms follow their respective strategies in α. Given that a_{it} and $\mathbf{n}_t$ are discrete variables with finite support, $\mathbf{P}^{\alpha}$ is a vector in an Euclidean space of finite dimension.[19] More precisely,

[19] Note that we have assumed that ϕ_t can take only a finite number of values. Therefore, $P_i^{\alpha}(a_{it}|\mathbf{n}_t, \phi_t)$ also belongs to a finite-dimension Euclidean space.

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$\mathbf{P}^{\alpha} \in [0,1]^{D}$ where $D = I * L * 2^{IL}$ is the number of free probabilities in the vector $\mathbf{P}^{\alpha}$.

Define the *integrated value function* $\bar{V}_i^{\alpha}(\mathbf{n}_t) \equiv \int V_i^{\alpha}(\mathbf{n}_t, \varepsilon_{it}) dG_i(\varepsilon_{it})$. Applying the definitions of CCP's and integrated value function to the Bellman equation in (14)-(15) we get the following *integrated Bellman equation*:

$$\begin{aligned} \bar{V}_i^{\alpha}(\mathbf{n}_t) = \int \max_{a_{it}} \{ & \pi_i(a_{it}, \mathbf{n}_t) + \varepsilon_{it}(a_{it}) \\ & + \beta \sum_{a_{-it}} \bar{V}_i^{\alpha}(\mathbf{n}_t + 1[a_{it}, a_{-it}]) \left[\prod_{j \neq i} P_j^{\alpha}(a_{jt} | \mathbf{n}_t) \right] \Bigg\} dG_i(\varepsilon_{it}) \end{aligned} \tag{19}$$

The integrated value function $\bar{V}_i^{\alpha}$ is the unique fixed point of this Bellman equation. Notice that the fixed point mapping that defines $\bar{V}_i^{\alpha}$ depends on firms' strategies only through the vector of choice probabilities $\mathbf{P}^{\alpha}$. To emphasize this point and to define an MPE in probability space, we change the notation slightly and use the symbol $\bar{V}_i^{\mathbf{P}}$ instead of $\bar{V}_i^{\alpha}$ to denote the integrated value function. For the same reason, we use $v_i^{\mathbf{P}}(a_{it}, \mathbf{n}_t)$ to represent the choice-specific value functions, which can be written as:

$$v_i^{\mathbf{P}}(a_{it}, \mathbf{n}_t) \equiv \pi_i(a_{it}, \mathbf{n}_t) + \beta \sum_{a_{-it}} \bar{V}_i^{\mathbf{P}}(\mathbf{n}_t + 1[a_{it}, a_{-it}]) \left[\prod_{j \neq i} P_j^{\alpha}(a_{jt} | \mathbf{n}_t) \right] \tag{20}$$

Given these value functions, we can re-write the best response function as: $\alpha_i^{BR}(\mathbf{n}_t, \varepsilon_{it}, \mathbf{P}) = \arg\max_{a_{it} \in A(n_{it})} \{ v_i^{\mathbf{P}}(a_{it}, \mathbf{n}_t) + \varepsilon_{it}(a_{it}) \}$. Note that we have replaced α_{-i} by $\mathbf{P}$ as an argument in the best response function. This is because CCP's contain all the information about competitors' strategies that a firm needs to construct its best response.

The *best response probability function*, $\Psi_i(a_{it} | \mathbf{n}_t, \mathbf{P})$, is the probability that action a_{it} is firm i's best response given that the state of the market is $\mathbf{n}_t$ and the other firms behave according to their choice probabilities in $\mathbf{P}$. It is the best response function α_i^{BR} integrated over the distribution of private information variables.

$$\Psi_i(a_{it} | \mathbf{n}_t, \mathbf{P}) \equiv \int 1 \left\{ a_{it} = \arg \max_{a \in A(n_{it})} \{ v_i^{\mathbf{P}}(a, \mathbf{n}_t) + \varepsilon_{it}(a) \} \right\} dG_i(\varepsilon_{it}) \tag{21}$$

This function maps CCP's into CCP's. The best response probability function in vector form is $\Psi(\mathbf{P}) = \{ \Psi_i(a_{it} | \mathbf{n}_t, \mathbf{P}) : (i, a_{it}, \mathbf{n}_t) \in \Upsilon \times A \times \{0,1\}^{IL} \}$.

Let α^* be a set of MPE strategies and let $\mathbf{P}^*$ be the vector of CCP's associated to α^*. Using the previous definitions it is simple to verify that $\mathbf{P}^*$ should be a fixed point of the mapping Ψ. Inversely, let $\mathbf{P}^*$ be a fixed point of the mapping Ψ, and define the set of strategy functions α^* with

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$\alpha_i^*(\mathbf{n}_t, \varepsilon_{it}) = \arg\max_{a_{it}}\{v_i^{\mathbf{P}^*}(a_{it}, \mathbf{n}_t) + \varepsilon_{it}(a_{it})\}$. Then, it is also simple to verify that α^* is an MPE (see Aguirregabiria and Mira [2007], for further details). Therefore, we can represent any MPE in this model as a fixed point of the best response probability mapping. Equilibrium probabilities solve the coupled fixed-point problems defined by equations (19), (20) and (21). Given a vector of probabilities **P**, we obtain value functions $\bar{V}_i^{\mathbf{P}}$ as solutions of the I Bellman equations in (19), and given these value functions, we obtain best response probabilities using (21).

Given this representation of an equilibrium, the proof of existence of an MPE is a straightforward application of Brower's theorem. The distribution $G_i(.)$ has support over the entire $\mathbb{R}^{2L}$ and it is continuous and strictly increasing with respect to every argument. This implies that the fixed-point mapping Ψ is continuous on the compact set $[0,1]^D$. Thus, by Brower's theorem, an equilibrium exits.

Example. The functional forms of the integrated Bellman equation and of the best response probability mapping depend on the distribution of the private information variables. A special case in which these functions have close form expressions is when the private information variables have a type 1 extreme value distribution. Suppose that the private information shocks $\{\varepsilon_{it}(a) : a \in A\}$ are independently and identically distributed over (i, t, a) with type 1 extreme value distribution. Then, the integrated Bellman equation is:

$$\bar{V}_i^{\mathbf{P}}(\mathbf{n}_t) = \log\left(\sum_{a_{it}\in A(n_{it})} \exp\left\{v_i^{\mathbf{P}}(a_{it}, \mathbf{n}_t)\right\}\right) \tag{22}$$

And the best response probability function is:

$$\Psi_i(a_{it}|\mathbf{n}_t, \mathbf{P}) = \frac{\exp\left\{v_i^{\mathbf{P}}(a_{it}, \mathbf{n}_t)\right\}}{\sum_{a\in A(n_{it})} \exp\left\{v_i^{\mathbf{P}}(a, \mathbf{n}_t)\right\}} \tag{23}$$

The iid extreme value distribution is restrictive because it implies no spatial correlation between private information shocks. However, it is very convenient from a computational point of view because it avoids numerical integration over the space of ε_{it}. ■

This dynamic game can have multiple equilibria. This is an issue when we use this model for comparative statics. In principle, a researcher may be willing to deal with multiplicity by imposing an equilibrium selection mechanism such that, for different values of the model parameters, the same equilibrium type is always selected. We illustrated in Section II(iii) how imposing an equilibrium selection mechanism is relatively simple in

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static supermodular games. Unfortunately, it is generally difficult to establish supermodularity in dynamic games.[20]

To deal with multiple equilibria in the dynamic game, we use different approaches when estimating the model and when making counterfactual experiments. In Section IV, we describe different estimation methods that do not require solving for an equilibrium of the dynamic game and that can deal with multiplicity at the estimation stage. In Section III(iii), we present a computationally simple homotopy method to deal with multiplicity of equilibria in the implementation of counterfactual experiments. The method involves the computation, or approximation, of only two equilibria, one under the factual scenario and other under the counterfactual.

II(vii). *Extensions of the Benchmark Model*

(a) Positive spillover effects from other stores. In order to reduce their searching/shopping costs, consumers may be attracted to locations with multiple stores selling the same differentiated product. This argument implicitly assumes that when consumers decide which location to visit they have some uncertainty about product availability (stockouts) or quality of service at stores and this uncertainty disappears only when the consumer visits the store. We do not model explicitly this consumer uncertainty. Instead, we extend our specification of consumer utility by including a new term that accounts for a positive spillover effect from the presence of other stores. The indirect utility of consumer (z, v) patronizing store (i, ℓ) is:

$$u_{i\ell} = \omega_{i\ell} + \delta h\left(\sum_{j \neq i} n_{j\ell}\right) - p_{i\ell} - \tau g(||z - z_\ell||) + v_{i\ell} \tag{24}$$

where δ is a positive parameter, $h(.)$ is an increasing function, and $\sum_{j \neq i} n_{j\ell}$ is the number of stores at location ℓ from firms other than i. This extension of the model does not have any effect on the basic structure of the demand model and of the Bertrand equilibrium. All the results above remain by only replacing $\omega_{i\ell}$ with $\omega_{i\ell}^*$, where $\omega_{i\ell}^* \equiv \omega_{i\ell} + \delta\ h(\sum_{j \neq i} n_{j\ell})$. However, this extension can have substantial effects on the spatial configuration of stores in equilibrium, and on the relationship between equilibrium prices and the number of stores in a location.

In our benchmark model of price competition, the number of stores in a location only has a competition effect in the sense that, *ceteris paribus*, prices are lower in locations with more stores. In this extended model with spillovers, the number of stores in a location still has a competition effect, but it

[20] Curtat [1996] provides a useful overview of the problem. More recently, Bernstein and Kök [2009] have obtained conditions for the supermodularity of a dynamic game of innovation and cost reduction in assembly networks.

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also has a positive effect on prices due to the increase in consumer traffic. The net effect on prices can be either positive or negative depending on the relative magnitude of the spillover parameter δ and the parameter μ that represents the degree of (non-spatial) horizontal product differentiation.

Our benchmark dynamic model of store location includes some features that can generate agglomeration of stores in some locations, e.g., minimization of consumer transportation costs if function $g(.)$ is convex; or locations with high (low) values of the amenities $\omega_{i\ell}$ (costs). The extended model with positive spillover effects introduces an additional centripetal force of store agglomeration.

(b) Store experience increases quality. It is well known in the empirical IO literature that firms or stores that have been longer in the market are more productive and have larger market shares, on average, than younger stores. There are multiple (not mutually exclusive) explanations for this empirical evidence, e.g., dynamic selection, active or passive learning. Here we present a simple extension of our benchmark model that incorporates store passive learning. Stores that stay active accumulate experience, and this experience has a positive effect on the service quality the store provides to customers. The specification of consumer utility is:

$$u_{i\ell t}=\omega_{i\ell}+\gamma k(e_{i\ell t})-p_{i\ell t}-\tau g(||z-z_\ell||)+v_{i\ell} \tag{25}$$

where γ is a positive parameter that captures the magnitude of the experience effect on quality, $k(.)$ is an increasing function, and $e_{i\ell t}$ is a discrete variable that measures the experience of a store at period t, e.g., a dummy variable that is equal to zero if the store is brand new and equal to one otherwise; or the number of consecutive periods that the store has been active in the market, with a maximum value $\bar{e}$.

This extension has no implications on the basic structure of the Bertrand equilibrium model and the computation of an equilibrium, but it has relevant implications on its predictions. The extended model predicts that, in equilibrium, younger stores charge lower prices. This extension implies a more substantial modification in the structure of the dynamic game. Now, the vector of endogenous state variables describing retail chain networks is $\{\mathbf{n}_t, \mathbf{e}_t\}$ where $\mathbf{e}_t \equiv \{e_{i\ell t} : i \in \Upsilon; \ell=1,2,\ldots,L\}$ is the vector of store experiences. Similar to $\mathbf{n}_t$, the vector $\mathbf{e}_t$ has also a deterministic transition rule conditional on firms' entry-exit decisions. Otherwise, the description of a Markov Perfect Equilibrium does not change. However, this extension can have substantial implications on the predictions of the dynamic game. The positive effect of experience on store quality introduces a *first-mover advantage*, creates incentives for 'early' entry, and discourages entry in locations with experienced incumbents.

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(c) Economies of scale and density at the chain level. The specification of fixed costs can be extended to take into account that the fixed cost of operating a network of stores may depend on the number of stores (e.g., economies of scale) and on the distance between the stores (e.g., economies of density). Economies of density are spatial scope economies enjoyed by clustered stores belonging to the same firm, in the sense that the closer the stores of a firm are to each other in space, the more scope economies there are. For example, such density economies may include reduced distribution costs, easier product quality inspection and employee monitoring, and sharing of advertising, inventory, and personnel among same-firm stores. The recent empirical IO literature on retail chains has emphasized the importance of these economies of scale and density (Holmes [2011], Jia [2008], Nishida [2014]). Here we present an extended version of our benchmark model that includes economies of density in fixed costs using a specification similar to Jia [2008]. Other specifications are possible, as well as incorporating similar network effects in other components of the model such as variable costs, entry costs, or even consumer demand.

The fixed cost of operating store (i,ℓ) is:

$$\theta_{i\ell}^{FC} n_{i\ell t} - \frac{\theta^{ED}}{2} \sum_{\ell' \neq \ell} \frac{n_{i\ell' t}}{||z_{\ell'} - z_{\ell}||} \tag{26}$$

where θ^{ED} is a parameter that represents the magnitude of the economies of density. The effect on this fixed cost of stores of the same firm at other locations is weighted by the inverse of the distance to location ℓ. This term is multiplied by one-half to avoid double counting in the total fixed cost of the retail chain.

This extension does not alter the basic structure of the dynamic game, and it does not have any implication on the pricing game (i.e., conditional on market structure $\mathbf{n}_t$, equilibrium prices are the same as in the benchmark model). However, economies of density can have very substantial implications on the predictions of the model. For obvious reasons, spatial agglomeration of stores of the same chain should be stronger with economies of density. It seems plausible that this extension can generate an equilibrium with the flavour of spatial entry deterrence, i.e., if chain A is an incumbent in location ℓ, then this reduces the probability that chain B opens a store in locations close to ℓ. This implies that store agglomeration becomes stronger for stores of the same chain but weaker for stores of different chains. Moreover, economies of density could potentially cause such within-firm agglomeration of stores even if there is no spatial entry deterrence motive. For example, under density economies a firm with a store in location ℓ would likely prefer to open a new store in a location that is near location ℓ (instead of in a similar but more distant location) simply because its overall fixed operating costs would be lower due to the density economies.

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(d) Scarce business space. Some business locations are unique in the sense that they offer a limited number of slots for retail stores. For example, it is not unusual for open-air 'strip malls' to have a small number of lots that can accommodate large retailers such as supermarkets or department stores. Such restriction can be even stronger if one considers older and/or denser cities. Here we present an extension of our benchmark model that incorporates a simple version of this feature. Suppose that a location ℓ can accommodate a maximum of N_ℓ stores. The values N_ℓ at different locations are common knowledge to firms. However, the incomplete information nature of our dynamic game implies that, when firms make their entry-exit decisions, they do not know how many other firms want to operate a store in a location. Therefore, we distinguish between firms 'applications' for store entry and the actual realization of entry decisions.

Let $a_{it}^* \in A(n_{it})$ be the decision of firm i at period t, where the set $A(n_{it})$ has exactly the same definition as in the benchmark model. However, a_{it}^* represents an 'application' and not necessarily the actual opening-closing of stores of the firm. Let a_{it} be the variable that represents the actual change in the store network of firm i, such that $n_{it+1}=n_{it}+1\{a_{it}\}$ as in the benchmark model. We consider the following relationship between the decisions/applications $\{a_{it}^*\}$ and the actual realizations of store entry-exit $\{a_{it}\}$. First, if the application is for closing an existing store (i.e., $a_{it}^*=\ell_- \in A(n_{it})$) or for doing nothing (i.e., $a_{it}^*=0$), then this application is realized with probability one such that $a_{it}=a_{it}^*$. Second, if the application is for opening a new store (i.e., $a_{it}^*=\ell_+ \in A(n_{it})$), then this request is not automatically implemented and there is some uncertainty. Let $N_{\ell t+1}^{inc}$ be the total number of incumbent stores in location ℓ at period t that decide to stay in the market, i.e., $N_{\ell t+1}^{inc} \equiv \sum_{i=1}^{I} n_{i\ell t}\ 1\{a_{it}^* \neq \ell_-\}$. And let $N_{\ell t+1}^{new}$ be the number of new applicants for opening a store, i.e., $N_{\ell t+1}^{new} \equiv \sum_{i=1}^{I} 1\{a_{it}^*=\ell_+\}$. The allocation of the N_ℓ slots follows two simple rules: (1) if there is not excess demand for business slots at location ℓ (i.e., if $N_{\ell t+1}^{inc}+N_{\ell t+1}^{new} \leq N_\ell$), then all the applications are accepted; and (2) if there is excess demand (i.e., if $N_{\ell t+1}^{inc}+N_{\ell t+1}^{new} > N_\ell$), then all the incumbents can keep their stores with probability one and the new applicants enter into a lottery for the allocation of the remaining $N_\ell - N_{\ell t+1}^{inc}$ slots. Different lotteries may be considered. The simplest one is a sequential lottery of each slot where all the applicants have the same probability of winning a slot. Other lotteries might be considered to try to capture that firms may have different willingness to pay for a business slot.

This extension of the benchmark model introduces an additional source of uncertainty in the dynamic game. Firms' strategy functions and Conditional Choice Probabilities are defined in terms of the application decisions $\{a_{it}^*\}$ in a very similar way as in our benchmark model. The main difference appears in the transition rule of store networks that becomes stochastic.

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The implications of this new source of uncertainty on the predictions of the model are not obvious. Of course, the scarcity of business space should reduce entry and store agglomeration in some locations.

III. ALGORTHIMS FOR SOLUTION AND COMPARATIVE STATICS

III(i). *Computation of a Nash-Bertrand equilibrium*

To compute a Nash-Bertrand equilibrium of the static pricing game, we iterate in the best response function. More specifically, we use a *Gauss-Seidel* version of the algorithm that iterates in the best response mapping, such that players take turns in best-responding instead of jointly best-responding in each iteration. Topkis [1998] has shown that the Lemma that we have presented in Section II(iii) also applies to the Gauss-Seidel version of the algorithm. In fact, Topkis shows that for supermodular games the Gauss-Seidel algorithm is faster (see also Echenique [2007]).

For a given value of the state variables, we have defined the best response mapping as $\mathbf{b}(\mathbf{p}) \equiv \mathbf{c}+\boldsymbol{\Lambda}(\mathbf{p})^{-1} \cdot \mathbf{s}(\mathbf{p})$. Let $\mathbf{b}_{(i)}(\mathbf{p})$ be the elements of $\mathbf{b}(\mathbf{p})$ associated with the prices of firm i. Similarly, let $\mathbf{p}_{(i)}$ be the elements of the vector $\mathbf{p}$ associated with firm i. To obtain the equilibrium with smallest prices we initialize the algorithm with prices equal to marginal costs.

Step 0: Start with the vector of prices $\mathbf{p}^0$ such that $\mathbf{p}^0_{(i)}=c_i$ for any $i \in \Upsilon$.

Step 1: Compute aggregate demands $\mathbf{s}(\mathbf{p}^0)$ and the matrix of partial derivatives $\Lambda(\mathbf{p}^0)$ using quadrature integration (see below).

Step 2 (Gauss-Seidel iteration): Starting with firm 1, obtain a new vector $\mathbf{p}^1_{(1)}$ as $\mathbf{p}^1_{(1)}=\mathbf{b}_{(1)}(\mathbf{p}^0)$. Then, for firm 2, $\mathbf{p}^1_{(2)}=\mathbf{b}_{(2)}(\mathbf{p}^1_{(1)}, \mathbf{p}^0_{(2)}, \ldots, \mathbf{p}^0_{(I)})$, and so on for firm i, $\mathbf{p}^1_{(i)}=\mathbf{b}_{(i)}(\mathbf{p}^1_{(1)}, \ldots, \mathbf{p}^1_{(i-1)}, \mathbf{p}^0_{(i)}, \ldots, \mathbf{p}^0_{(I)})$.

Step 3: If $||\mathbf{p}^1-\mathbf{p}^0||$ is smaller than a pre-fixed small constant, then $\mathbf{p}^*=\mathbf{p}^1$. Otherwise, proceed to step 1 with $\mathbf{p}^0=\mathbf{p}^1$.

Once the price equilibrium is computed, we encode the equilibrium current variable profits of a firm given a particular state, $R^*_i(\mathbf{n}, \phi)$.

Given the logit assumption on the idiosyncratic tastes, the local demands have the closed form expression in (2). However, to obtain the vector of aggregate demands $\mathbf{s}(\mathbf{p})$ and the matrix of partial derivatives $\Lambda(\mathbf{p})$ we have to integrate local demands over consumers' addresses in the two-dimensional city $\mathbb{C}$. We use a quadrature method with midpoint nodes (see Judd [1998], ch. 7). We first divide $\mathbb{C}$ into a pre-specified number of mutually exclusive and adjacent rectangular cells, with each cell k having a representative node point $z_{(k)}$ in its center. For each location z in cell k we approximate the local demand $\sigma_{i\ell}(z, \mathbf{n}_t, \mathbf{p}_t)$ and the density $\phi_t(z)$ using $\sigma_{i\ell}(z_{(k)}, \mathbf{n}_t, \mathbf{p}_t)$ and $\phi_t(z_{(k)})$, respectively. Therefore, we calculate aggregate demand for store (i, ℓ) as:

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$$s_{i\ell}(\mathbf{n}_t, \mathbf{p}_t, \phi_t) = \sum_k \sigma_{i\ell}(z_{(k)}, \mathbf{n}_t, \mathbf{p}_t)\phi_t(z_{(k)})area(k) \tag{27}$$

where *area*(*k*) is the area of the rectangular cell *k*.

III(ii). *Computation of an MPE*

Consider the example where the private information variables are extreme value distributed. A MPE is a vector of probabilities $\mathbf{P}^*$ such that $\mathbf{P}^* = \mathbf{\Psi}(\mathbf{P}^*)$, where the fixed-point mapping $\mathbf{\Psi}(\mathbf{P})$ is $\{\Psi_i(a_{it}|\mathbf{n}_t, \mathbf{P}) : (i, a_{it}, \mathbf{n}_t) \in \Upsilon \times A(n_{it}) \times \{0,1\}^{IL}\}$ with

$$\Psi_i(a_{it}|\mathbf{n}_t, \mathbf{P}) = \frac{\exp\left\{\pi_i(a_{it}, \mathbf{n}_t) + \beta \sum_{a_{-it}} \bar{V}_i^{\mathbf{P}}(\mathbf{n}_t + 1[a_{it}, a_{-it}]) \left[\prod_{j \neq i} P_j(a_{jt}|\mathbf{n}_t)\right]\right\}}{\sum_{a_i \in A(n_{it})} \exp\left\{\pi_i(a_i, \mathbf{n}_t) + \beta \sum_{a_{-it}} \bar{V}_i^{\mathbf{P}}(\mathbf{n}_t + 1[a_i, a_{-it}]) \left[\prod_{j \neq i} P_j(a_{jt}|\mathbf{n}_t)\right]\right\}} \tag{28}$$

and the value function $\bar{V}_i^{\mathbf{P}}$ solves the Bellman equation

$$\bar{V}_i^{\mathbf{P}}(\mathbf{n}_t) = \log\left(\sum_{a_{it} \in A(n_{it})} \exp\left\{\pi_i(a_{it}, \mathbf{n}_t) + \beta \sum_{a_{-it}} \bar{V}_i^{\mathbf{P}}(\mathbf{n}_t + 1[a_{it}, a_{-it}]) \left[\prod_{j \neq i} P_j(a_{jt}|\mathbf{n}_t)\right]\right\}\right) \tag{29}$$

To obtain an MPE we iterate in the best response function $\mathbf{\Psi}$ using Gauss-Seidel iterations. The algorithm proceeds as follows.

Step 0: Initialize the algorithm with a vector of probabilities $\mathbf{P}^0$.

Step 1 [Value function]: Starting with firm 1, and given $(\mathbf{P}_2^0, \mathbf{P}_3^0, \ldots, \mathbf{P}_I^0)$ fixed, we obtain the value function $\bar{V}_1^{\mathbf{P}^0}$ by applying value function iterations in the Bellman equation (29).

Step 2 [Best response]: Given $\bar{V}_1^{\mathbf{P}^0}$, we use the best response probability mapping in (28) to obtain a new vector of CCP's for firm 1: $\mathbf{P}_1^1 = \{\Psi_1(a_{it}|\mathbf{n}_t, \mathbf{P}^0)\}$.

Then, we proceed with firm 2. Given $(\mathbf{P}_1^1, \mathbf{P}_3^0, \ldots, \mathbf{P}_I^0)$ fixed, we obtain the value function $\bar{V}_2^{(\mathbf{P}_1^1, \mathbf{P}_3^0, \ldots, \mathbf{P}_I^0)}$ by using value function iterations in the Bellman equation (29). Then, we update firm 2's CCP's as $\mathbf{P}_2^1 = \{\Psi_2(a_{it}|\mathbf{n}_t, \mathbf{P}_1^1, \mathbf{P}_3^0, \ldots, \mathbf{P}_I^0)\}$. We proceed in this way to update the CCP's of the I firms.

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Step 3: If $||\mathbf{P}^1 - \mathbf{P}^0||$ is smaller than a pre-fixed small constant, then $\mathbf{P}^* = \mathbf{P}^1$. Otherwise, proceed to step 1with $\mathbf{P}^0 = \mathbf{P}^1$.

The most serious burden for the computation of an equilibrium in our model comes from the space-memory requirements.[21] Value functions and choice probabilities should be stored in high-speed memory because they are required for value function iteration and for the calculation of best response probabilities. Given that the vector of state variables $\mathbf{n}_t$ is discrete, value functions and choice probability functions can be described as vectors in Euclidean spaces: i.e., $\bar{V}_i^{\mathbf{P}} \in \mathbb{R}^{2^{IL}}$ and $\mathbf{P} \in [0,1]^{IL(2^{IL})}$. In applications with many locations (or firms), the dimension of these Euclidean spaces can be very large. For instance, in a duopoly model with 40 locations we have that $2^{IL} \simeq 10^{12}$. This magnitude of memory space is rarely available.

There are two general approaches to deal with this computational issue. One is to approximate the value function using a parametric family of surfaces, such as polynomials or nonlinear basis functions derived from neural networks (see Bertsekas and Tsitsiklis [1996]). The other approach is to store $\bar{V}_i^{\mathbf{P}}$ and $\mathbf{P}$ only over a subset of the state space and use interpolation to obtain values of these functions at other points. In this paper we consider the interpolation approach.[22]

Let $S = \{\mathbf{n}_1, \mathbf{n}_2, \ldots, \mathbf{n}_{|S|}\}$ be a subset of the actual state space $\{0,1\}^{IL}$. The number of elements in the subset S is given by the amount of high-speed memory in our computer. The selection of the grid points in S can be done in different ways. For instance, we can take $|S|$ random draws from a uniform distribution over the set $\{0,1\}^{IL}$. Let $\bar{\mathbf{V}}_i^{\mathbf{P}|\mathbf{S}}$ be a vector of values restricted to the subset S: i.e., $\bar{\mathbf{V}}_i^{\mathbf{P}|\mathbf{S}} = \{\bar{V}_i^{\mathbf{P}|\mathbf{S}}(\mathbf{n}_t) : \mathbf{n}_t \in S\}$. The vector $\bar{\mathbf{V}}_i^{\mathbf{P}|\mathbf{S}}$ is the unique fixed point of the following Bellman equation: for any $\mathbf{n}_t \in S$,

$$\bar{V}_i^{\mathbf{P}|\mathbf{S}}(\mathbf{n}_t) = \log\left(\sum_{a_{it}\in A(n_{it})} \exp\left\{\pi_i(a_{it}, \mathbf{n}_t) + \beta \sum_{a_{-it}} \Gamma_i^{\bar{\mathbf{V}}^{\mathbf{P}|\mathbf{S}}}(\mathbf{n}_t + \mathbf{1}[a_{it}, a_{-it}]) \left[\prod_{j\neq i} P_j(a_{jt}|\mathbf{n}_t)\right]\right\}\right) \tag{30}$$

[21] Under the assumption that firms cannot open or close more than one store per period, the computation of the expected value of next period's value function is not a serious computational issue if the number of firms is small, e.g., no greater than four. To calculate the expected value $\sum_{a_{-it}} \bar{V}_i^{\mathbf{P}}(\mathbf{n}_t + 1[a_{it}, a_{-it}]) \prod_{j\neq i} P_j(a_{jt}|\mathbf{n}_t)$, we have to perform only $(1+L)^I$ sums and products, instead of the much larger number of 2^{IL} operations which are required in the general case. For instance, in a city with 40 locations we have that $(1+L)^I$ is equal to 1,681 in a model with two firms, and 2, 825, 761 in a model with four firms.

[22] This interpolation approach goes back at least to Larson and Casti [1982]. More recently, Rust [1997] has proposed a method of interpolation that exploits randomization in the selection of the grid points. See Rust [1996] for an excellent survey on numerical methods for dynamic programming that includes a discussion of interpolation techniques.

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where $\Gamma_i^{\bar{\mathbf{V}}^{\mathbf{P|S}}}(\mathbf{n})$ is the interpolation function. Different interpolating functions may be considered. However, given that the indirect variable profit function of a retail chain is an important component of the firm's value, it seems reasonable to use this function as a way of aggregating the information in the state variables. An example of this type of interpolation function is:

$$\Gamma_i^{\bar{\mathbf{V}}^{\mathbf{P|S}}}(\mathbf{n}) = \begin{cases} \bar{V}_i^{\mathbf{P|S}}(\mathbf{n}) & \text{if} \quad \mathbf{n} \in S \\ \gamma_i^{(0)} + \gamma_i^{(1)} R_i^*(\mathbf{n}) + \gamma_i^{(2)} [R_i^*(\mathbf{n})]^2 & \\ + \sum_{\ell=1}^{L} \left(\sum_{j=1}^{I} \gamma_{ij\ell}^{(3)} n_{j\ell} + \sum_{j \neq i} \gamma_{ij\ell}^{(4)} n_{i\ell} n_{j\ell} \right) & \text{if} \quad \mathbf{n} \notin S \end{cases} \tag{31}$$

where $\gamma's$ are the parameters that describe the interpolation function. For the interpolation function of a firm, the number of γ parameters is equal to $3+L(2I-1)$, and the total memory requirements to store the value function $\bar{V}_i^{\mathbf{P|S}}(.)$ is $|S| + I(3+L(2I-1))$. For instance, with $L=40$ locations, $I=4$ firms, we have to keep in memory $|S|+1,132$ values, that represents a very substantial reduction with respect to the $2^{4*40} \simeq 10^{48}$ values in the state space of the model. At each iteration of the solution method, we recalculate the γ parameters by running an OLS regression of $\bar{V}_i^{\mathbf{P|S}}(\mathbf{n})$ on $[1,\ R_i^*(\mathbf{n}),\ R_i^*(\mathbf{n})^2,\ \{n_{j\ell}\},\ \{n_{i\ell} * n_{j\ell}\}]$ for values of $\mathbf{n}$ in the set S.

Let us describe in more detail the procedure to approximate a firm's best response function. We start with an arbitrary initial guess of the vector $\bar{\mathbf{V}}_i^{\mathbf{P|S}}$, that we represent as $\bar{\mathbf{V}}_i^{[\mathbf{0}]}$, e.g., $\bar{\mathbf{V}}_i^{[\mathbf{0}]}(\mathbf{n}) = \log\left(\sum_{a_{it}} \exp\{\pi_i(a_{it}, \mathbf{n})\}\right)$. Then, we run an OLS regression of $\bar{\mathbf{V}}_i^{[\mathbf{0}]}(\mathbf{n})$ on $[1,\ R_i^*(\mathbf{n}),\ R_i^*(\mathbf{n})^2,\ \{n_{j\ell}\},\ \{n_{i\ell} * n_{j\ell}\}]$ using the values of $\mathbf{n}$ in the set S such that we determine the vector of parameters in the interpolation function, $\gamma_i^{[0]}$, as the OLS estimates in this regression. Next, we iterate in equation (30) to obtain a new vector of values, $\bar{\mathbf{V}}_i^{[1]}$. We iterate until convergence in the vector of values. Note that equations (30) and (31) define a contraction mapping. Finally, given the vector $\bar{\mathbf{V}}_i^{\mathbf{P|S}}$ we compute an approximation to the best response function of firm i as:

$$\Psi_i^{(S)}(a_{it}|\mathbf{n}, \mathbf{P}) = \frac{\exp\left\{\pi_i(a_{it}, \mathbf{n}) + \beta \sum_{a_{-it}} \bar{V}_i^{\mathbf{P|S}}(\mathbf{n} + \mathbf{1}[a_{it}, a_{-it}]) \left[\prod_{j \neq i} P_j(a_{jt}|\mathbf{n})\right]\right\}}{\sum_{a_i \in A(n_{it})} \exp\left\{\pi_i(a_i, \mathbf{n}) + \beta \sum_{a_{-it}} \bar{V}_i^{\mathbf{P|S}}(\mathbf{n} + \mathbf{1}[a_i, a_{-it}]) \left[\prod_{j \neq i} P_j(a_{jt}|\mathbf{n})\right]\right\}} \tag{32}$$

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An approximation to the MPE is a vector of probabilities $\mathbf{P}^* = \{P_i(a_{it}|\mathbf{n}_t) : (i, a_{it}, \mathbf{n}_t) \in \Upsilon \times A(n_{it}) \times S\}$ such that $\mathbf{P}^* = \mathbf{\Psi}^{(S)}(\mathbf{P}^*)$, where the fixed-point mapping $\mathbf{\Psi}^{(S)}(\mathbf{P})$ is $\{\Psi_i^{(S)}(a_{it}|\mathbf{n}_t, \mathbf{P}) : (i, a_{it}, \mathbf{n}_t) \in \Upsilon \times A \times S\}$. That is, the vector of choice probabilities and the equilibrium mapping are restricted to the subspace S of the state space. As the original equilibrium mapping, the mapping $\mathbf{\Psi}^{(S)}$ is continuously differentiable on the compact set of CCP's.

A key question is how well our interpolation method approximates the true value function. Unfortunately, we do not have general theoretical results on this point. It is always difficult to provide general results about the properties of approximation methods, other than simple properties like consistency, i.e., as S goes to the true state space, the approximated value function converges to the true one. It is also difficult to provide useful tests to evaluate an approximation in a dynamic programming model because, for the state space dimensions that we find in empirical applications, we can never calculate true values and approximation errors. For our interpolation method, a necessary condition to obtain a good approximation to the value function is that the method can approximate well the variable profit function. This idea provides a simple test for our approximation method. Given the values of the variable profit evaluated at the states in our grid S, we can use the interpolation method to approximate the variable profit at other points in a different grid S^*. Then, we can compare these approximated profits in S^* with the true values, and evaluate the goodness of fit.

A limitation of the framework and method above is the assumption that private information variables are independently extreme value distributed. This assumption is made for convenience because it avoids numerical integration to calculate the multiple integral $\int V_i^{\mathbf{P}}(\mathbf{n}_t, \varepsilon_{it}) dG_i(\varepsilon_{it})$. However, the assumption of no spatial correlation between shocks at different locations is not innocuous. For instance, this correlation can generate spatial agglomeration of stores. Relaxing the extreme value assumption requires one to use simulation techniques to approximate multiple dimensional integrals. This can increase substantially the computational cost in the implementation of the method.

III(iii). *Comparative Statics*

Let $\boldsymbol{\theta}$ be the vector of structural parameters of the model. We include this vector explicitly as an argument in the equilibrium mapping, $\Psi(\mathbf{P}, \boldsymbol{\theta})$. An equilibrium of the model associated with $\boldsymbol{\theta}$ is a solution to the fixed-point problem $\mathbf{P} = \Psi(\mathbf{P}, \boldsymbol{\theta})$. Let $\boldsymbol{\theta}_0$ and $\boldsymbol{\theta}_1$ be two values of $\boldsymbol{\theta}$. We want to study how the equilibrium of the model, described by $\mathbf{P}$, changes when we change the structural parameters from $\boldsymbol{\theta}_0$ to $\boldsymbol{\theta}_1$ but *keeping fixed the type of equilibrium*. The later condition is key. In the comparative statics exercise that we are interested in here, we control for the type of equilibrium.

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Before we describe the method, we first introduce the concepts of *equilibrium type* and *regular equilibrium* as defined in Doraszelski and Escobar [2010], and present a Lemma that establishes that the number of regular equilibria is finite.

Definition. [*Regular MPE*]. Let $f(\mathbf{P},\boldsymbol{\theta})$ be the function $\mathbf{P}-\Psi(\mathbf{P},\boldsymbol{\theta})$ such that an equilibrium of the game for given $\boldsymbol{\theta}$ can be represented as a solution to the system of equations $f(\mathbf{P},\boldsymbol{\theta})=0$. We say that an *MPE* $\mathbf{P}^*$ is regular if the Jacobian matrix $\partial f(\mathbf{P}^*,\boldsymbol{\theta})/\partial\mathbf{P}'$ is non-singular.

Definition. [*Equilibrium types*]. Let $\boldsymbol{\theta}_0$ and, $\boldsymbol{\theta}_1$ be two vectors of structural parameters in the Euclidean space. And let $\mathbf{P}_0^*$ and $\mathbf{P}_1^*$ be *MPEs* associated with $\boldsymbol{\theta}_0$ and $\boldsymbol{\theta}_1$, respectively. We say that $\mathbf{P}_0^*$ and $\mathbf{P}_1^*$ belong to the same type of equilibrium if and only if there is a continuous path $\{\mathbf{p}(\lambda):\lambda\in[0,1]\}$ that satisfies the condition

$$f(\mathbf{p}(\lambda),(1-\lambda)\boldsymbol{\theta}_0+\lambda\boldsymbol{\theta}_1)=0$$

for every $\lambda\in[0,1]$, such that $\mathbf{p}(0)=\mathbf{P}_0^*$ and $\mathbf{p}(1)=\mathbf{P}_1^*$, and this path connects in a continuous way the equilibria $\mathbf{P}_0^*$ and $\mathbf{P}_1^*$.

Lemma. [*Doraszelski and Escobar* [2010]].[23] Under the conditions of our model, for almost all games/payoffs $\boldsymbol{\theta}$: *(A)* all equilibria are regular; *(B)* the number of equilibria is finite; and *(C)* each equilibrium belongs to a particular type.

The model has a discrete and finite set of equilibrium types that we index by k. Let $P^{[k]}(\boldsymbol{\theta})$ be the vector of choice probabilities that represents equilibrium type k when the vector of parameters is $\boldsymbol{\theta}$, such that it satisfies the equilibrium restrictions $P^{[k]}(\boldsymbol{\theta})=\Psi(P^{[k]}(\boldsymbol{\theta}),\boldsymbol{\theta})$. Note that some equilibrium types may exist only for a subset of the parameter space Θ. Under our conditions on the mapping $\Psi(\mathbf{P},\boldsymbol{\theta})$, the equilibrium probability functions $P^{[k]}(\boldsymbol{\theta})$ are continuous in $\boldsymbol{\theta}$.

Let $\mathbf{P}_0$ be a regular equilibrium associated with $\boldsymbol{\theta}_0$. $\mathbf{P}_0$ belongs to an equilibrium type, say k, i.e., $\mathbf{P}_0=P^{[k]}(\boldsymbol{\theta}_0)$. For instance, $\mathbf{P}_0$ can be the equilibrium that we obtain using the solution method described in Section III(ii) above when we initialized the algorithm using a vector where all the probabilities are zero. The problem that we consider here is how to compute $\mathbf{P}_1=P^{[k]}(\boldsymbol{\theta}_1)$, i.e., how to calculate the equilibrium associated to $\boldsymbol{\theta}_1$ that belongs to the same type as $\mathbf{P}_0$. This seems a reasonable way of doing

[23] Doraszelski and Escobar [2010] study dynamic games of incomplete information. Our Lemma A comes from their Theorem 1. Lemma B corresponds to their Corollary 1. And Lemma C is a corollary of their Proposition 2.

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comparative statics if we are interested in how changes in $\boldsymbol{\theta}$ affect players' equilibrium behavior only via payoff-relevant factors and keeping constant the same equilibrium type. The following is a simple homotopy method that solves this problem.

The method is based on a first order Taylor approximation to $P^{[k]}(\boldsymbol{\theta}_1) = \Psi(P^{[k]}(\boldsymbol{\theta}_1), \boldsymbol{\theta}_1)$ around $\boldsymbol{\theta} = \boldsymbol{\theta}_0$. Note that the equilibrium $\mathbf{P}_0$ is regular, and then the Jacobian matrix $\mathbf{I} - \partial\Psi(\mathbf{P}_0, \boldsymbol{\theta}_0)/\partial\mathbf{P}'$ is non-singular. Therefore, the Taylor approximation is:

$$\text{(33)} \qquad \mathbf{P}_1 = P^{[k]}(\boldsymbol{\theta}_1) \simeq \mathbf{P}_0 + \left(\mathbf{I} - \frac{\partial\Psi(\mathbf{P}_0, \boldsymbol{\theta}_0)}{\partial\mathbf{P}'}\right)^{-1} \frac{\partial\Psi(\mathbf{P}_0, \boldsymbol{\theta}_0)}{\partial\boldsymbol{\theta}'} (\boldsymbol{\theta}_1 - \boldsymbol{\theta}_0)$$

When $\|\boldsymbol{\theta}_1 - \boldsymbol{\theta}_0\|^2$ is small, the right-hand-side of this expression provides a good approximation to $P^{[k]}(\boldsymbol{\theta}_1)$. To improve the accuracy of this approximation, we may combine this approach with iterations in the equilibrium mapping. Suppose that the equilibrium type k is Lyapunov stable, i.e., the Jacobian matrix $\partial\Psi(P^{[k]}(\boldsymbol{\theta}), \boldsymbol{\theta})/\partial\mathbf{P}'$ has all its eigenvalues in the unit circle. This implies that there is a neighborhood of $\mathbf{P}_1$, say $\mathcal{N}$, such that if we iterate in the equilibrium mapping $\Psi(., \boldsymbol{\theta}_1)$ starting with a $\mathbf{P} \in \mathcal{N}$, then we converge to $\mathbf{P}_1$. The neighborhood $\mathcal{N}$ is called the *dominion of attraction* of the stable equilibrium $\mathbf{P}_1$. Suppose that the Taylor approximation is precise enough such that it belongs to the dominion of attraction of $\mathbf{P}_1$. Then, by iterating in the equilibrium mapping $\Psi(., \boldsymbol{\theta}_1)$ starting with the Taylor approximation $\hat{\mathbf{P}}_1 = \mathbf{P}_0 + \left(\mathbf{I} - \frac{\partial\Psi(\mathbf{P}_0, \boldsymbol{\theta}_0)}{\partial\mathbf{P}'}\right)^{-1} \frac{\partial\Psi(\mathbf{P}_0, \boldsymbol{\theta}_0)}{\partial\boldsymbol{\theta}'} (\boldsymbol{\theta}_1 - \boldsymbol{\theta}_0)$, we will obtain the equilibrium $P^{[k]}(\boldsymbol{\theta}_1)$.

IV. STRUCTURAL ESTIMATION

IV(i). *Data*

We start this section on structural estimation with a discussion of the type of data that can be used to estimate the model. The ideal dataset provides information for a single city or metropolitan area on the store location decisions, prices, and quantities of the I retail chains in a retail industry. We partition the city into L locations where firms in this industry can operate stores. We also make a partition of the city into C small regions that represent consumer addresses, e.g., census tracts. For each of these regions (for businesses or consumers) indexed by ℓ, we construct a centroid z_ℓ that represents the address of that region. The dataset for a city can be described in terms of three sets of variables: (1) store locations $\{n_{i\ell t}\}$; (2) population density $\{\phi_t(z_\ell)\}$ and consumer demographics $\{x_{\ell t}\}$; and (3) prices and quantities for every active store $\{p_{i\ell t}, q_{i\ell t}\}$. We find this type of datasets in recent empirical applications of spatial competition such as, among others, Thomadsen [2005] for the fast food industry in Santa Clara

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County, California; Davis [2006] for movie theatres; Ho and Ishii [2011] for the retail banking industry, or Slade [1992] and Houde [2012] for gas stations in Vancouver and Quebec city, respectively.

For the statistical properties of the estimators presented below, the key dimension of the dataset is the number of locations where firms can operate stores, L. The number of locations L should be relatively large (e.g., $L \geq 200$) because the application of a *Law of Large Numbers* (LLN) and a *Central Limit Theorem* (CLT) to our estimator is for the number of locations L going to infinity, and I and T fixed. For the application of LLN and CLT using our spatial data, we need to impose some restrictions on the spatial correlation of the unobservables (see Jenish and Prucha [2012]). For instance, under the assumption that unobservables in the demand system are not spatially correlated, and provided that we have valid instruments, we can obtain consistent and asymptotically normal estimates of demand parameters using store-level prices and quantities with large L and fixed I and T. The restriction of no spatial correlation can be replaced with a weaker restriction such as a spatial autoregressive process. Under the same type of restriction on the spatial correlation of the unobservables in marginal costs, fixed costs, and entry costs, we can also obtain consistent estimates of the parameters in these functions when L goes to infinity and I and T are fixed. Note that the number of periods T and firms I in the data can be as small as two. Given the assumptions of time-homogeneity and stationarity in the primitives of the model and in the equilibrium concept, two periods of data are enough to identify choice probabilities and transition probabilities of the state variables. Note that, given these stationarity assumptions, we could also consider asymptotics when $L * T$ goes to infinity. We present below an example of a dataset from Vicentini [2013] where $L = 144$ and $T = 55$ years.

Store location information is easily available. However, prices and especially quantities at the store level can be more difficult to obtain for some industries. Therefore, a more common type of dataset contains only information on (1) store locations and (2) consumer demographics and other location characteristics. This is the case in applications of store location models such as Seim [2006] or Nishida [2014], among others. As we explain in Section IV(iii), this type of data can be used for the estimation of our model as long as the researcher is willing to calibrate or fix some of the parameters in the demand function or in variable costs.

Most of these applications consider data from multiple cities or regions. However, in Slade [1992], Thomadsen [2005], Houde [2012], or Vicentini [2013] the datasets include only one big city, as in our description above of the ideal dataset. With this type of data, the asymptotics of the estimators (i.e., law of large numbers and central limit theorems) is based on the number of locations L going to infinity and applies results in spatial econometrics (see Anselin [2010], and Pinkse and Slade [2010], for recent surveys, or

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Pinkse, Slade and Brett [2002], and Slade [2005], for other applications of these econometric methods in empirical IO).

Example. We now provide an example of an actual dataset that can be used to estimate a dynamic game of spatial competition with the methods proposed in this paper. The dataset is described in detail in Vicentini [2013], and is comprised of the location history of supermarkets and drug stores within the city and suburbs of Greensboro, North Carolina. In that paper, the author uses the dataset to estimate the extent to which density economies and spatial monopolization behavior are present in the supermarket and drug store industries, although without incorporating dynamics explicitly and structurally as we propose in this paper. Here we provide a brief description of the supermarket component of that dataset.

The supermarket dataset was collected from yearly published Greensboro city directories from 1955 to 2010 ($T = 56$ years). Each directory provides the name and address of each 'grocery store' present in Greensboro in the respective year. The addresses were then geocoded and the latitude and longitude coordinates obtained. Additionally, data was collected on the physical size of the building of each of the grocery stores; this allowed the author to rule out stores that were relatively small. Based on all of these researched items, the original list of grocery stores was narrowed down to 184 supermarket stores. For every year in the sample, there are no more than five chains that account for almost all the supermarket stores in the city. To illustrate the sequential location pattern followed by chain supermarkets, Figure 2A depicts the sequential spatial positioning of Winn-Dixie supermarkets over time. For instance, Winn-Dixie opened its first three stores within about two miles of each other in the central part of the city in the 1950's. It then opened its fourth store (labeled store '#4') in 1961 on the Northeast area, slightly isolated from the other stores, but then it opened its fifth store in 1962 almost in between stores number one and three. Figure 2B presents a discretized grid of possible business locations, i.e., a uniform grid of 12×12 locations ($L = 144$) with a 1.5 miles distance (vertical and horizontal) between locations. ■

IV(ii). *Specification of Primitive Functions*

To complete the specification of the econometric model, we need to make some assumptions about the primitives $\omega_{i\ell}$, $c_{i\ell}$, $\theta_{i\ell}^{FC}$, $\theta_{i\ell}^{EC}$, and $\theta_{i\ell}^{EV}$. We assume that these primitives vary freely across firms, but they vary over locations only according to the observable socioeconomic variables in x_ℓ.

$$(34) \qquad \omega_{i\ell} = \theta_i^{\omega} x_\ell; \quad c_{i\ell} = \theta_i^{c} x_\ell; \quad \theta_{i\ell}^{FC} = \theta_i^{FC} x_\ell; \quad \theta_{i\ell}^{EC} = \theta_i^{EC} x_\ell$$

where θ_i^{ω}, θ_i^{c}, θ_i^{FC}, θ_i^{EC}, and θ_i^{EV} are vectors of parameters. This assumption rules out the possibility of unobserved heterogeneity across locations

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Figure 2A
Sequential Location of Stores: Winn-Dixie
Notes: Solid gray line represents Greensboro 2000 urbanized area, and dotted gray line represents Greensboro 1950 city limits, both based on U.S. Census boundary files.

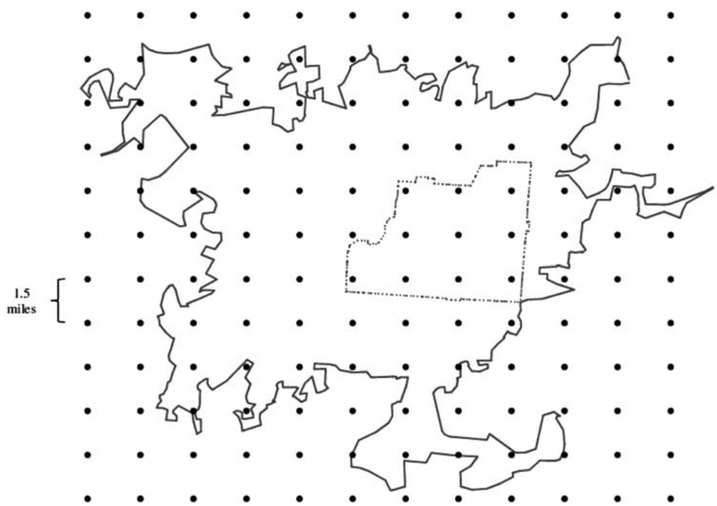

Figure 2B
Construction of Business Locations: Greensboro, North Carolina
Notes: Solid gray line represents Greensboro 2000 urbanized area, and dotted gray line represents Greensboro 1950 city limits, both based on U.S. Census boundary files.

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ℓ, and therefore it may be a strong assumption in applications without a rich set of observables in x_ℓ.[24]

Let $\mathbf{x}$ be the vector $\{x_\ell : \ell=1,2,\ldots,L\}$, that is a description of the 'landscape' of observable socioeconomic characteristics in the city. All the analysis is conditional on $\mathbf{x}$. Given $\mathbf{x}$, we can interpret the store networks in $\mathbf{n}_t$ as a single realization of a spatial stochastic process. In terms of the econometric analysis, this has similarities with time series econometrics in the sense that a time series is a single realization from a stochastic process. Though we observe a single realization of this stochastic process, we can still estimate consistently the parameters of that process as the number of locations L goes to infinity.

IV(iii). *Estimation*

For estimation purposes, it is convenient to distinguish three subvectors in $\boldsymbol{\theta}$. That is, $\boldsymbol{\theta}=\{\boldsymbol{\theta}_D, \boldsymbol{\theta}_{MC}, \boldsymbol{\theta}_{FC}\}$ where $\boldsymbol{\theta}_D$ represents demand parameters $\{\mu, \tau, \theta_i^{\omega} : i=1,2,\ldots,I\}$, $\boldsymbol{\theta}_{MC}$ contains parameters in marginal costs $\{\theta_i^c : i=1,2,\ldots,I\}$, and $\boldsymbol{\theta}_{FC}$ contains parameters in fixed costs, entry costs, and exit values, $\{\theta_i^{FC}, \theta_i^{EC}, \theta_i^{EV} : i=1,2,\ldots,I\}$.

The estimation of parameters in demand and marginal costs can follow the well known approach in Berry, Levinsohn and Pakes [1996] and Nevo [2001] for the estimation of demand of differentiated products, i.e., the so called BLP method. In fact, given the assumption of no unobserved location heterogeneity in $\omega_{i\ell}$, the estimation of $\boldsymbol{\theta}_D$ can be much simpler than in the BLP approach because there is no endogeneity of prices and no need to invert the relationship between market shares and average utilities. Therefore, a possible estimator of $\boldsymbol{\theta}_D$ is a Nonlinear Least Squares estimator:

$$\text{(35)} \qquad \hat{\boldsymbol{\theta}}_D=\arg\min_{\boldsymbol{\theta}_D} \sum_{i=1}^{I}\sum_{\ell=1}^{L}\left[s_{i\ell}-\sum_{k}\sigma_{i\ell}\left(z_{(k)},\mathbf{n},\mathbf{p};\boldsymbol{\theta}_D\right)\phi\left(z_{(k)}\right)\right]^2$$

where $s_{i\ell}$ is the observed market share of store (i,ℓ) (equals zero if that store does not exist), and $\sigma_{i\ell}(z_{(k)},\mathbf{n},\mathbf{p};\boldsymbol{\theta}_D)$ is the multinomial logit probability that provides the proportion of consumers in location $z_{(k)}$ patronizing store (i,ℓ). Given this estimation of demand, marginal costs parameters in θ_{MC} can be also estimated by least squares using the marginal conditions of optimality for prices in equation (5). And given consistent estimators $\hat{\boldsymbol{\theta}}_D$ and $\hat{\boldsymbol{\theta}}_{MC}$, and the algorithm for computing the Bertrand equilibrium, we can construct estimates of equilibrium variable profits, $\hat{R}_i(\mathbf{n}_t)$, for any observed or hypothetical market structure $\mathbf{n}_t$.

[24] This is a common assumption in some empirical applications, such as Seim [2006].

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The restriction of no unobserved location heterogeneity in demand is a strong assumption. To relax this assumption, we need some instruments for prices in the estimation of demand. For instance, suppose that some observable exogenous variables in the vector **x** vary both across firms and over locations, i.e., $x_{i\ell}$. With this type of data, we could use the so-called BLP instruments and method for the estimation of demand parameters. More specifically, we could obtain an equation that relates a known function of store market shares with the average utility of purchasing in store (i,ℓ). In this equation, the observable characteristics of stores other than (i,ℓ), i.e., $\{x_{i\ell} : j \neq i\}$, can be used to instrument for the price $p_{i\ell}$.

When data of prices and quantities at the store level is not available, the researcher may be willing to calibrate the parameters of a parsimonious specification of demand and variable costs. Suppose that the researcher has information on average price and aggregate quantity at the city-firm level, $\{P_i, Q_i\}$. These $2 * I$ data points can be used to calibrate the parameters $\{\mu,\ \tau,\ \omega,\ c\}$ under the assumption that firms have homogeneous qualities and unit costs. Another alternative is estimating jointly all the parameters of the model using the dynamic game and firms' store location decisions.

For the estimation of $\boldsymbol{\theta}_{FC}$, we exploit the restrictions imposed by the equilibrium of the dynamic game. Here we propose a two-step method for the estimation of the dynamic game. Let $\mathbf{P}^0$ be the vector of CCP's in the MPE in the city under study. We assume that there is a unique equilibrium in the data, i.e., the equilibrium does not vary over time or over regions within the city. We can use our approximation to firms' best response probability functions $\Psi_i^{(S)}(a_{it}|\mathbf{n}_t, \mathbf{P}, \boldsymbol{\theta})$ evaluated at the true equilibrium $\mathbf{P}^0$ to construct the following likelihood function:

$$
\begin{aligned}
Q(\mathbf{P},\boldsymbol{\theta}) = & \sum_{i,t,\ell} 1\{n_{i\ell t} = n_{i\ell t-1}\} \ln \Psi_i^{(S)}(0|\mathbf{n}_t, \mathbf{P}, \boldsymbol{\theta}) \\
& + \sum_{i,t,\ell} 1\{n_{i\ell t-1} = 0, n_{i\ell t} = 1\} \ln \Psi_i^{(S)}(\ell_+|\mathbf{n}_t, \mathbf{P}, \boldsymbol{\theta}) \\
& + \sum_{i,t,\ell} 1\{n_{i\ell t-1} = 1, n_{i\ell t} = 0\} \ln \Psi_i^{(S)}(\ell_-|\mathbf{n}_t, \mathbf{P}, \boldsymbol{\theta})
\end{aligned}
\tag{36}
$$

Let $\hat{\mathbf{P}}^0$ be a reduced-form nonparametric estimator of the population CCP's $\mathbf{P}^0$. We describe below how to obtain this estimator. Then, a consistent estimator of $\boldsymbol{\theta}_{FC}$ can be obtained by maximizing the pseudo likelihood function $Q(\hat{\mathbf{P}}^0, \boldsymbol{\theta}_{FC}, \hat{\boldsymbol{\theta}}_D, \hat{\boldsymbol{\theta}}_{MC})$ with respect to $\boldsymbol{\theta}_{FC}$. The estimator is root-L consistent and asymptotically normal. This two-step method has been used for the estimation of dynamic games in IO applications in Ellickson, Beresteanu and Misra [2010] or Sweeting [2013], among many others (see Aguirregabiria and Nevo [2013], for a survey of applications).

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A key aspect in the implementation of this two-step method is the reduced form nonparametric estimation of CCP's in the first step. Given the extreme value type I distribution of private information variables, the equilibrium CCP's have the following Conditional Logit structure: $P_i^0(a_{it}|\mathbf{n}_t) = \exp\{v_i(a_{it}, \mathbf{n}_t; \boldsymbol{\theta}^0)\} \ / \ [\sum_{a_i \in A(n_{it})} \exp\{v_i(a_i, \mathbf{n}_t; \boldsymbol{\theta}^0)\}]$, where $v_i(a_i, \mathbf{n}_t; \boldsymbol{\theta}^0)$ is the conditional choice value function evaluated at the true parameters, that is unknown to the researcher. For the reduced-form estimation of P_i^0, we propose approximating the value functions $v_i(a_{it}, \mathbf{n}_t; \boldsymbol{\theta}^0)$ with a polynomial in the variable profit $R_i^*(\mathbf{n}_t + 1\{a_{it}\})$ (i.e., the variable profit if current network $\mathbf{n}_t$ is changed according to choice a_{it}) and polynomials for each of the location-specific variables $\{n_{1\ell t}, \ldots, n_{I\ell t}\} * x_\ell$ that correspond to choice a_{it}, that we represent as $(n_{\ell t}[a_{it}] * x_{\ell t}[a_{it}])$. More specifically,

$$P_i^0(a_{it}|\mathbf{n}_t; \boldsymbol{\alpha}_i) = \frac{\exp\left\{f_1\left(R_i^*(\mathbf{n}_t + 1\{a_{it}\})\right)' \boldsymbol{\alpha}_{1i} + f_2'(n_{\ell t}[a_{it}] * x_\ell[a_{it}]) \boldsymbol{\alpha}_{2i}\right\}}{\sum_{a_i \in A(n_{it})} \exp\left\{f_1\left(R_i^*(\mathbf{n}_t + 1\{a_i\})\right)' \boldsymbol{\alpha}_{1i} + f_2'(n_{\ell t}[a_i] * x_\ell[a_i]) \boldsymbol{\alpha}_{2i}\right\}} \tag{37}$$

where $f_1\left(R_i^*(\mathbf{n}_t + 1\{a_{it}\})\right)$ and $f_2(n_{\ell t}[a_{it}] * x_\ell[a_{it}])$ represent vectors of polynomial terms in $R_i^*(\mathbf{n}_t + 1\{a_{it}\})$ and $(n_{\ell t}[a_{it}] * x_\ell[a_{it}])$, respectively, that form the basis for the polynomial approximation; and $\boldsymbol{\alpha}_{1i}$ and $\boldsymbol{\alpha}_{2i}$ are vectors of reduced form parameters. For instance, if the basis functions f_1 and f_2 corresponds to polynomials of order m_1 and m_2, respectively, then the dimension of the vector $\boldsymbol{\alpha}_i \equiv (\boldsymbol{\alpha}_{1i}, \boldsymbol{\alpha}_{2i})$ is $m_1 + m_2 \ I \ K$, where K is the dimension of vector x_ℓ.[25] These parameters are estimated by maximizing the reduced form likelihood function (for firm i):

$$\begin{aligned} Q_i^{RF}(\boldsymbol{\alpha}_i) = & \sum_{t,\ell} 1\{n_{i\ell t} = n_{i\ell t-1}\} \ln P_i^0(0|\mathbf{n}_t; \boldsymbol{\alpha}_i) \\ & + \sum_{t,\ell} 1\{n_{i\ell t-1} = 0, n_{i\ell t} = 1\} \ln P_i^0(\ell_+|\mathbf{n}_t; \boldsymbol{\alpha}_i) \\ & + \sum_{t,\ell} 1\{n_{i\ell t-1} = 1, n_{i\ell t} = 0\} \ln P_i^0(\ell_-|\mathbf{n}_t; \boldsymbol{\alpha}_i) \end{aligned} \tag{38}$$

For instance, with $I = 3$ firms, $K = 2$ exogenous variables x, and cubic polynomials ($m_1 = m_2 = 3$), we have 21 parameters to estimate in the vector α_i. These parameters may be estimated with enough precision using a dataset from a city with $L = 100$ locations and enough store turnover.

[25] Note that, though our specification of the reduced form CCP's is very flexible, we do not have a formal proof that, as the dimension of the polynomials increases, this function can approximate arbitrarily well any value function $v_i(a_{it}, \mathbf{n}_t)$. In particular, our specification apparently imposes the restriction that the dependence on variables at locations other that $\ell[a_{it}]$ occurs only through variable profit. This might not be really a restriction if the order of the polynomial $f_1\left(R_i^*(\mathbf{n}_t + 1\{a_{it}\})\right)$ is high enough, but we do not have a formal proof.

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V. ENTRY COSTS AND THE DYNAMICS OF STORE LOCATION

We apply our model to analyze how changes in the cost of setting-up a store (entry costs) affect firms' strategies, firm value and consumer welfare.

V(i). *Benchmark Model*

The following parameters are constant over our experiments.

a. *Market*. The market is a square city of 10×10 kilometers: $\mathbb{C}=[0,10]^2$. Consumers (households) are uniformly distributed on $\mathbb{C}$ and the population size is equal to 100,000 households. Population size and the geographical distribution of consumers are constant over time. There are 16 business locations ($L=16$) which form a uniform grid in the square city. The coordinates of business locations are the 16 points that result from the intersection of coordinates $(2,4,6,8)$ in the horizontal and vertical axes, i.e., $z_1=(2,2)$, $z_2=(2,4)$,..., $z_{15}=(8,6)$, and $z_{16}=(8,8)$. The unit transportation cost τ, which includes the opportunity cost of travel time, is \$5 per kilometer.[26]

b. *Product and firms*. Firms are supermarket chains. The product under consideration is the weekly shopping basket of a family in a supermarket. Therefore, ω_i represents the (average) willingness to pay for the weekly shopping basket in supermarket chain *i*. There are two firms in this city. These firms are identical in terms of the quality of their products. We fix $\omega_1=\omega_2=\$135$. One period in the model is a calendar year and the discount factor is set at $\beta=0.9$. Note that households' willingness to pay ($\omega=\$135$) and the transportation cost ($\tau=\$5/km$) correspond to weekly shopping, while the frequency of firms' decisions is annual. Therefore, the correct definition of market size should be the number of households times the number of weeks per year: $M=100,000 \times 52=5,200,000$ household-weeks. The parameter μ, that measures the degree of (non-spatial) horizontal product differentiation, is equal to \$2.

c. *Firms' costs*. Firms are also identical in their cost structures. A firm's marginal cost to provide the weekly shopping basket of a household is \$100. Fixed operating costs and exit values are set to zero. The common knowledge component of entry costs, θ_{EC}, is constant across locations. We compute an equilibrium of the dynamic game under 21 different values of θ_{EC} between \$0 and \$20 million. To give an idea of the relative magnitude of this range of entry costs, note that given the demand and variable cost parameters that we have fixed, the total variable profit of a monopolist with one store at every location is

[26] Suppose that most of the transportation cost comes from the opportunity cost of travel time. If the average hourly wage is $\$30/hour$ and the average transportation speed in this city is $6km/hour$, then $\tau=(\$30/hour)/(6km/hour)=\$5$ per kilometer.

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TABLE 1
NASH-BERTRAND EQUILIBRIUM(1)

Market Structure		Price-Cost Margin	Variable Profits(2)	Consumer Surplus(3)	Total Surplus(3)
Monopoly with:	*1 store*	15.2%	25.15	14.16	39.31
	2 stores	17.9%	45.89	19.65	65.54
	3 stores	18.5%	67.71	27.55	95.26
	4 stores	19.4%	87.06	31.65	118.7
Duopoly	*1 store*	3.7%	12.17	45.64	57.81
Same locations:	*2 stores*	3.8%	16.57	73.70	90.27
	3 stores	4.0%	19.93	99.51	119.4
	4 stores	4.0%	20.80	117.2	138.0
Duopoly	*Zero distance*	3.7%	12.17	45.64	57.81
Different locations	*Small distance*	16.2%	45.27	24.23	69.50
one store for each firm	*Large distance*	15.5%	49.77	27.15	76.93

Note 1: Parameter values: City [0,10] × [0,10]; four locations (2,2), (2,8), (8,2) and (8,8); $\mu=2$; $\tau=5$; $\omega=135$; $c=100$.
Note 2: Annual variable profits, in million dollars, of all firms, i.e., variable profits per person-week × 100,000 × 52.
Note 3: Annual surplus in million dollars.

close to \$2.5 million. Therefore, the range of values of θ_{EC} goes from 0% to 800% of the annual variable profits of that monopolist. Though we admit that an entry cost above 400% may be unrealistically high, we have included high values of the entry cost in our analysis in order to understand the effects of these costs in the limit when they generate almost zero entry. The private information parts of entry costs and exit values are independently and identically distributed with extreme value type 1 distribution.

Table 1 presents a summary of the Nash-Bertrand equilibrium outcome under different market structures. Profits and consumer surplus are concave functions of the number of stores, either for a monopolist or a duopolist. There are decreasing returns, in profits and in consumer surplus, of an additional store. Competition reduces profits per firm and increases consumer surplus. The closer the stores of the two firms, the smaller price-cost margins and profits, and the larger consumer surplus.

d. *Interpolation algorithm*. The number of points in the state space of this dynamic game is $2^{IL}=2^{32} \simeq 4.3 * 10^8$. We do not have enough memory space to compute exactly an equilibrium of this dynamic game. Instead, we use the interpolation method described in Section III(ii). We now explain our selection of the set of points S and of the interpolation function Γ. The grid S contains $20,000$ points which are random draws from a uniform distribution over the whole state space. More precisely, let $\mathbf{n}^{(S)} \equiv \{n_{i\ell}^{(S)} : i=1,2;\ \ell=1,2,\ldots,16\}$ be a point in the grid S. To obtain a point in this grid we generate the values $n_{i\ell}^{(S)} \in \{0,1\}$ as independent random draws from a Bernoulli distribution with probability 0.5. Our specification of the interpolation function exploits several

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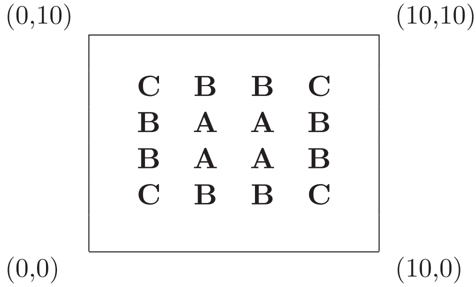

Figure 3
Subregions A, B and C used for Interpolation

features of our example. First, firms are identical and we impose symmetry in the equilibrium. Therefore, the interpolating function should be symmetric across firms. Second, given that consumers are uniformly distributed over the city, the key feature that represents the 'quality' of a business location is its distance to the center of the city. We can distinguish three regions, A, B and C, such that two locations within the same region have the same distance to the center of the city. Figure 3 shows these regions. Third, the average distance between the stores of a firm summarizes the degree of cannibalization between these stores. And fourth, the average distance between the stores of the two firms summarizes the degree of substitution between the two firms. Based on these ideas, we use the following interpolation function:

$$\Gamma_i^{\bar{\mathbf{V}}^{\mathbf{P|S}}}(\mathbf{n}) = \begin{cases} \bar{V}_i^{\mathbf{P|S}}(\mathbf{n}) & if \quad \mathbf{n} \in S \\ \text{Second order polynomial in the following variables} \\ \{n_{iA}, n_{iB}, n_{iC}, n_{-iA}, n_{-iB}, n_{-iC}, \\ \;d_{iA}, d_{iB}, d_{iC}, d_{-iA}, d_{-iB}, d_{-iC}, & if \quad \mathbf{n} \notin S \\ \;d_A, d_B, d_C\} \end{cases} \tag{39}$$

where n_{iR} is the number of stores that firm i has in region R; d_{iR} is the average distance between firm i's stores in region R; and d_R is the average distance between the stores of firm 1 and firm 2 in region R. The number of parameters in this interpolation function is 136.

We have made some sensitivity analysis to validate our approximation to the exact solution. To validate the number of grid points in S, we have solved the model using 5000, 10000, 15000 and 20000 grid points. While the solution with 5000 points presents some differences with respect to the solution with 20000 points, the other three solutions are almost identical.

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To validate the interpolation function (39) we have applied this function to similar but much smaller problems for which we can compare the approximation to the exact solution of the game. We have considered the same dynamic game but with four locations instead of 16. The market is still the square city of 10 × 10 kilometers, $\mathbb{C}=[0,10]^2$, but there are only four business locations which are located in a straight line along the city's main street with coordinates $z_1=(2,5)$, $z_2=(4,5)$, $z_3=(6,5)$, and $z_4=(8,5)$. For this simpler game the state space has $2^{10}=1024$ cells. For the interpolation function we consider only two regions (A and B) according to the distance to the city center, such that locations $z_2=(4,5)$ and $z_3=(6,5)$ are in region A (with distance to the center equal to 1), and locations $z_1=(2,5)$ and $z_4=(8,5)$ are in region B (with distance to the center equal to 3). The interpolation function is a second order polynomial in the variables $\{n_{iA}, n_{iB}, n_{-iA}, n_{-iB}, d_i, d_{-i}\}$ and it has 28 parameters. The experiment shows that this interpolation function provides an excellent approximation to the true value function and to the equilibrium choice probabilities in this example. More generally, we conjecture that for specifications where firms are symmetric, consumers are uniformly distributed and business locations have a symmetric spatial structure, this type of interpolation function may provide a good approximation to the exact solution of the game.

V(ii). *Results*

Figures 4 to 8 summarize the results of our numerical experiments. Each of these figures presents an outcome variable of the game (e.g., average number of stores) in the vertical axis as a function of the entry cost parameter θ_{EC} in the horizontal axis. To analyze the effects of competition we represent these functions both for a duopoly model and for a monopolist without threat of potential entrants. In the horizontal axis, the entry cost is measured as a percentage of the annual variable profit of a monopolist with stores at every business location. Some of the outcome variables (e.g., number of stores, consumer welfare, value of a firm, average distance between stores) are calculated using the steady-state distribution of state variables which is implied by the Markov perfect equilibrium. We have used the homotopy method described in Section III(iii) to guarantee that we select the same type of equilibrium when we vary the value of the entry cost.

a. *Figure 4: Store turnover: openings and shutdowns.*[27] Figure 4 presents the average number of store openings per year and firm. As one would expect, an increase in the entry cost always reduces store openings and shutdowns. For most values of the entry cost, store turnover is similar under monopoly and duopoly. Nevertheless, for entry costs

[27] In steady-state the number of store openings should be equal to the number of store shutdowns.

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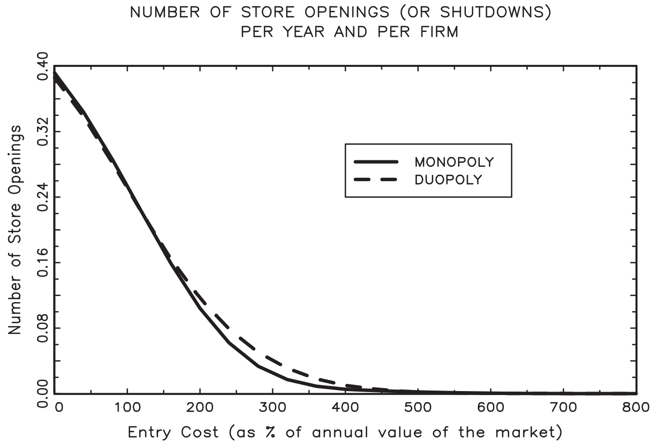

Figure 4

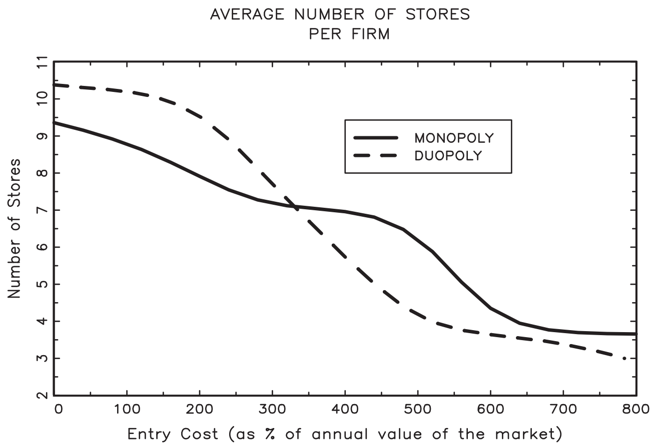

Figure 5

between 200% and 400% we observe that competition can generate some extra store turnover relative to monopoly.

b. *Figure 5: Number of stores.* Entry costs have a negative effect both on the creation of new stores and on store shutdowns. For our benchmark model we obtain that an increase in the entry cost has always a negative effect on the number of stores per firm. Interestingly, this negative effect is stronger under duopoly than under monopoly. In fact, this implies that the number of stores per firm is not always larger under monopoly than under duopoly. For entry costs larger than 350%, a monopolist has more stores than a duopolist.

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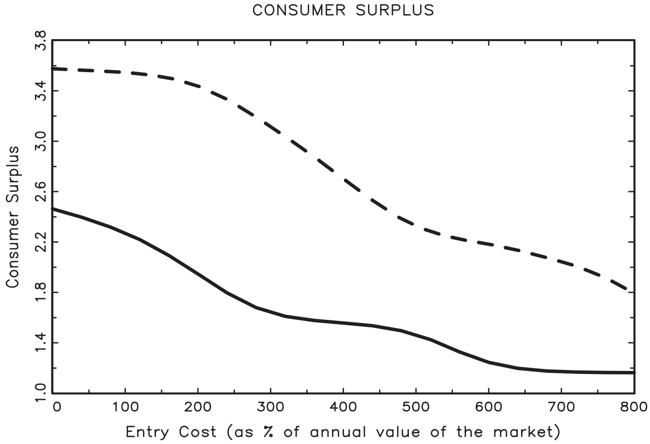

Figure 6

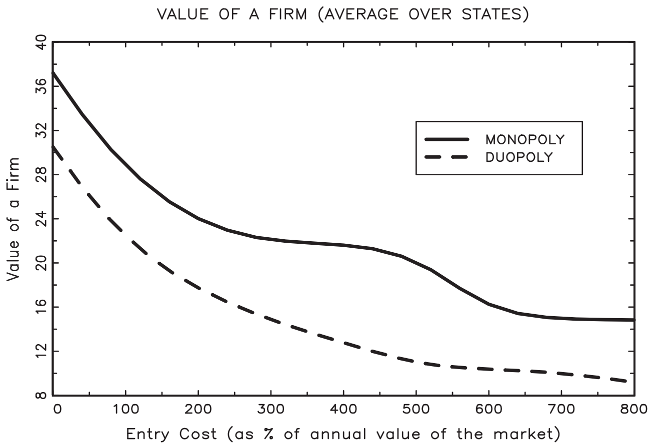

Figure 7

c. *Figures 6, 7 and 8: Consumer welfare and value of a firm*. Given the negative effect of entry costs on the number of stores, consumer welfare also declines with entry costs. Furthermore, despite entry costs' having a stronger effect on the number of stores under duopoly, consumer welfare is always around 50% larger under duopoly than under monopoly. This is because the monopolist offsets a reduction in consumer transportation costs with higher prices. The value of a firm, in Figure 7, and total welfare, in Figure 8, decline monotonically with entry costs. Welfare is always greater under duopoly.

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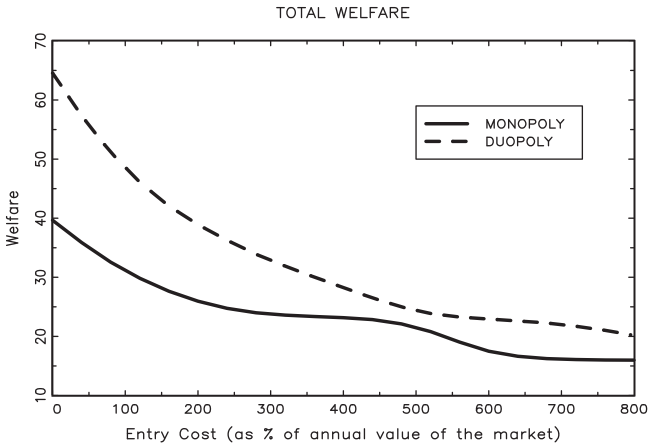

Figure 8

VI. CONCLUSION

This paper proposes a dynamic model of an oligopoly industry characterized by spatial competition between multi-store firms. Firms compete in prices and decide where to open or close stores depending on the spatial market structure. We define and characterize a Markov Perfect Equilibrium in this model. Our framework is a useful tool to study multi-store competition issues that involve spatial and dynamic considerations. An algorithm to compute an equilibrium of the model is proposed. The algorithm exploits interpolation techniques. We also propose a procedure for the consistent estimation of the parameters of the model using panel data on store location, prices and quantities from multiple locations in a single city. We illustrate the model and the algorithm with several numerical experiments that analyze the competitive and welfare effects of the sunk cost of setting-up a store.

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