Dynamic Discrete Choice Structural Models in Empirical IO

Lecture 3: Dynamic Games with Out-of-Equilibrium Beliefs: Identification, Estimation, and Applications

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Outline

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(b) Full identification of the model

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(a) Entry in unexplored markets

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Dynamic games of oligopoly competition

- Firms compete not only in prices or quantities but also in other dimensions such as market entry, capacity, quality, advertising, R&D and innovation, or product design.
- This type of competition "at the extensive margin" is typically modelled using discrete choice games.
- Most of these choices involve sunk costs and are partly irreversible: dynamic decisions.
- During the last decade, there have been important developments in the estimation and solution of dynamic games and an increasing of empirical applications.

Examples of Empirical Applications

- Land use regulation (entry cost) and entry-exit and competition in the hotel industry: Suzuki (IER; 2013);

- Subsidies to entry in small markets for the dentist industry: Dunne et al. (RAND, 2013);

- Environmental regulation and entry-exit and capacity choice in cement industry: Ryan (ECMA, 2012);

- Demand uncertainty and firm investment in the concrete industry: Collard-Wexler (ECMA, 2013);

- Time-to-build, uncertainty, and dynamic competition in the shipping industry: Kalouptsidi (AER, 2014);

Examples of Empirical Applications [2]

- Fees for musical performance rights and the choice of format (product design) of radio stations: Sweeting (ECMA, 2013);

- Hub-and-spoke networks, route entry-exit and competition in the airline industry: Aguirregabiria and Ho (JoE, 2012);

- Competition in R&D and product innovation between Intel and AMD: Goettler and Gordon (JPE, 2011);

- Product innovation of incumbents and new entrants in the hard drive industry: Igami (JPE, 2017);

- Cannibalization and preemption strategies in the Canadian fast-food industry: Igami and Yang (QE, 2016)

Examples of Empirical Applications [3]

- Release date of a movie: Einav (EI, 2010).

- Dynamic price competition: Kano (IHIO, 2013); Ellickson, Misra, and Nair (JMR, 2012).

- Endogenous mergers: Jeziorski (RAND, 2014).

- Exploitation of a common natural resource (fishing): Huang and Smith (AER, 2014).

Equilibrium beliefs: a common assumption

- In games, the best response of a player depends on her beliefs about the behavior of her opponents.
- The most common assumption is that players' beliefs about other players' actions are in equilibrium, i.e., **they are unbiased expectations of the actual behavior of other players**.
- Dynamic games: Markov Perfect Equilibrium (MPE). Maskin & Tirole (1988); Ericson & Pakes (1995).
- There are good reasons to impose assumption of equilibrium beliefs:

  (a) This assumption has identification power.
  (b) Counterfactual analysis: model predicts how beliefs change endogenously.

Sources of out-of-equilibrium beliefs

- Sometimes, the assumption of equilibrium beliefs can be unrealistic.
- There are different sources of bias in players beliefs:

(a) **Bounded rationality**: *Limited capacity to process information / compute;*

(b) **Asymmetric information** *about other players´ permanent characteristics (learning);*

(c) **Strategic uncertainty:** *With multiple equilibria, players can have different beliefs about the selected equilibrium. Some players believe that they are playing equilibrium A, other players believe they are playing equilibrium B, . . .*

Strategic Uncertainty and oligopoly competition

- In the context of games of oligopoly competition, Strategic Uncertainty seems particularly plausible, especially when firms are **competing in new markets, or after substantial regulatory changes**.

- Dynamic games are typically characterized by multiple equilibria, and some equilibria are better for some firms, i.e., firms do not have incentives to coordinate their beliefs.

- Firms are very secretive about their own strategies and face significant uncertainty about the strategies of their competitors.

- Implications for the evaluation of a new policy.

Empirical applications of games with biased beliefs

- In **experimental economics**, researchers have used data from games played in laboratory experiments to study whether players have biased beliefs and whether these beliefs converge over time to an equilibriums,
  e.g.,Van Huyck et al. (AER, 90); Camerer & Ho (ECMA, 99); Heinemann et al. (REStud, 09).

- The empirical literature **using field data** is more recent:

  Goldfarb & Xiao (AER, 2011);
  Aguirregabiria and Magesan (2015);
  Doraszelski, Lewis, & Pakes (NBER wp, 2016);
  Asker, Fershtman, Jeon, & Pakes (NBER wp, 2016);
  Jeon (2016)

Dynamic Game

- Two players i and j. Dynamic game over T periods.
- Every period t, each player takes an action $Y_{it} \in \{0, 1\}$.
- One-period payoff function is:

$$\Pi_{it} = \pi_i(Y_{it}, Y_{jt}, \mathbf{X}_t) + \varepsilon_{it}(Y_{it})$$

- $\mathbf{X}_t$ is a vector of common knowledge state variables with transition $f\left(\mathbf{X}_{t+1} \mid Y_{it},\ Y_{jt},\ \mathbf{X}_t\right)$.
- Payoff functions $\pi_i(.)$ and $\pi_j(.)$ are nonparametrically specified.
- $\varepsilon_{it} = \{\varepsilon_{it}(0), \varepsilon_{it}(1)\}$ is private info of player i and unobservable to researcher. It is i.i.d. over time and players with CDF Λ.

Example: Dynamic game of market entry-exit

- The average profit of an inactive firm, $\pi_{it}(0, \mathbf{Y}_{-it}, \mathbf{X}_t) + \varepsilon_{it}(0)$, is normalized to zero.

- The profit of an active firm is $\pi_{it}(1, \mathbf{Y}_{-it}, \mathbf{X}_t) + \varepsilon_{it}(1)$ where:

$$\pi_{it}(1, Y_{jt}, \mathbf{X}_t) \quad = \quad H_t \left(\theta_i^M - \theta_i^D Y_{jt} \right) - \theta_{i0}^{FC} - \theta_{i1}^{FC} Z_{it} - 1\{ Y_{it-1} = 0 \}$$

- $H_t \left(\theta_i^M - \theta_i^D \sum_{j \neq i} Y_{jt} \right)$ is the variable profit of firm i. H_t represents market size (e.g., market population) and it is an exogenous state variable. θ_i^M and θ_i^D are parameters.
- $\theta_{i0}^{FC} + \theta_{i1}^{FC}\ Z_{it}$ is the fixed cost of firm i, where θ_{i0}^{FC} and θ_{i1}^{FC} are parameters, and Z_{it} is an exogenous firm-specific characteristic.
- $1\{ Y_{it-1} = 0 \}\ \theta_i^{EC}$ is the entry cost.
- In this model, $\mathbf{X}_t = (H_t, Z_{it}, Z_{jt}, Y_{i,t-1}, Y_{jt-1})$.

Markov Perfect Equilibrium

- The concept of Markov Perfect Equilibrium (MPE) can be described as a combination of .

ASSUMPTION 1: *Players' strategy functions depend only on payoff relevant state variables:* $\mathbf{X}_t$ and ε_{it}.

ASSUMPTION 2: *Players have beliefs about the behavior of other players and about their own behavior in the future. Given these beliefs, they maximize expected intertemporal payoffs.*

ASSUMPTION 3: *Players have rational expectations on their own behavior in the future.*

ASSUMPTION EQUIL: *Players have rational expectations about other players' behavior now and in the future.*

Relaxing MPE

- Following the concept of Rationalizability:
- We maintain Assumptions 1 to 3;
- We relax Assumption EQUIL of "equilibrium beliefs".

Strategies, Choice Probabilities, and Beliefs

- Let $\sigma_{it}(\mathbf{X}_t, \varepsilon_{it})$ be the strategy function for player i at period t.
- $P_{it}(\mathbf{X}_t) \equiv \Pr(\sigma_{it}(\mathbf{X}_t, \varepsilon_{it}) = 1 | \mathbf{X}_t)$ is the choice probability of player i.
- Player i's beliefs at period t about the behavior (choice probability) of player j at period $t+s$ is $B_{i,t+s}^{(t)}(\mathbf{X}_{t+s})$.
- Equilibrium (unbiased) beliefs: $B_{i,t+s}^{(t)}(\mathbf{x}) = P_{j,t+s}(\mathbf{x})$ at any t, $s \geq 0$, and $\mathbf{x}$.

Best Response Functions

- Given his beliefs at period t, $\mathbf{B}_i^{(t)} = \left\{ B_{i,t+s}^{(t)}(\mathbf{X}) : \mathbf{X} \in \mathcal{X},\ s \geq 0 \right\}$, a player best response is the solution of a single-agent Dynamic Programming problem with some specific expected payoffs and expected transition probabilities.

- This DP problem has expected payoffs:

$$\pi_{i,t+s}^{\mathbf{B}(t)}(Y_{i,t+s}, \mathbf{X}_{t+s}) \equiv B_{i,t+s}^{(t)}(\mathbf{X}_{t+s})\ \pi_i(Y_{i,t+s}, 1, \mathbf{X}_{t+s})$$

$$+(1 - B_{i,t+s}^{(t)}(\mathbf{X}_{t+s}))\ \pi_i(Y_{it}, 0, \mathbf{X}_{t+s})$$

Best Response Functions (2)

- The expected transition of the state variables:

$$f_{i,t+s}^{\mathbf{B}(t)}(\mathbf{X}_{t+s+1}|Y_{i,t+s}, \mathbf{X}_{t+s}) \equiv$$

$$B_{i,t+s}^{(t)}(\mathbf{X}_{t+s})\ f(\mathbf{X}_{t+s+1}|Y_{i,t+s}, 1, \mathbf{X}_{t+s})$$

$$+(1 - B_{i,t+s}^{(t)}(\mathbf{X}_{t+s}))\ f(\mathbf{X}_{t+s+1}|Y_{i,t+s}, 0, \mathbf{X}_{t+s})$$

Best Response Functions (3)

- Given $\left\{\pi_{i,t+s}^{\mathbf{B}(t)}\right\}$ and $\left\{f_{i,t+s}^{\mathbf{B}(t)}\right\}$ the DP problem can be represented using the Bellman equation:

$$V_{it}^{\mathbf{B}(t)}(\mathbf{X}_t, \varepsilon_{it}) = \max_{Y_{it}} \left\{ \pi_{i,t+s}^{\mathbf{B}(t)}(Y_{it}, \mathbf{X}_t) + \varepsilon_{it}(Y_{it}) \right.$$

$$+\beta \int V_{i,t+1}^{\mathbf{B}(t)}(\mathbf{X}_{t+1}, \varepsilon_{it+1}) d\Lambda(\varepsilon_{it+1})\ f_{it}^{\mathbf{B}(t)}(\mathbf{X}_{t+1} | Y_{it},$$

Best Response Functions (3)

- The best response function for player i is:

$$\{Y_{it} = 1\} \;\Leftrightarrow\; \left\{ \varepsilon_{it}(0) - \varepsilon_{it}(1) \leq \widetilde{\pi}_{it}^{\mathbf{B}(t)}(\mathbf{x}_t) + \widetilde{v}_{it}^{\mathbf{B}(t)}(\mathbf{x}_t) \right\}$$

- $\widetilde{\pi}_{it}^{\mathbf{B}(t)}(\mathbf{x}) \equiv \pi_{it}^{\mathbf{B}(t)}(1, \mathbf{x}) - \pi_{it}^{\mathbf{B}(t)}(0, \mathbf{x})$.
- $\widetilde{v}_{it}^{\mathbf{B}(t)}(\mathbf{x}) \equiv v_{it}^{\mathbf{B}(t)}(1, \mathbf{x}) - v_{it}^{\mathbf{B}(t)}(0, \mathbf{x})$, i.e., difference of continuation values.

- The best response probability function is:

$$P_{it}(\mathbf{x}_t) = \Lambda\left(\widetilde{\pi}_{it}^{\mathbf{B}(t)}(\mathbf{x}_t) + \widetilde{v}_{it}^{\mathbf{B}(t)}(\mathbf{x}_t) \right)$$

Data

- We have a random sample of M markets, indexed by m, where we observe
$$\{Y_{imt},\ Y_{jmt},\ \mathbf{X}_{mt} : t = 1, 2, ..., T^{data}\}$$
for every market $m = 1, 2, ..., M$.
- T^{data} is small and M is large.
- Model can include **time-invariant common-knowledge market characteristics** ω_m which are **unobservable to the researcher**. Nonparametric finite mixture structure of ω_m.

Identification question

- Given this model and data, is it possible to fully identify all the structural functions of the model, i.e.,
  players' payoff functions $\pi_i(y_i, y_j, \mathbf{x})$ and players' belief functions $B^{(t)}_{i,t+s}(\mathbf{x})$?

- Is it possible to test for the null hypothesis that players´ beliefs are in equilibrium?

- Before we address these questions for this model, it is helpful to review the identification of the model when we impose the restriction of equilibrium beliefs (MPE).

Identification with equilibrium beliefs (1)

- **ASSUMPTION EQUIL:** Beliefs are in equilibrium (unbiased): $B_{i,t+s}^{(t)}(\mathbf{x}) = P_{j,t+s}(\mathbf{x})$ at any t, $s \geq 0$, and $\mathbf{x}$.
- **ASSUMPTION ID-1.** Players play the same equilibrium in markets with the same $\mathbf{x}$ variables, i.e., $P_{imt}(\mathbf{x}) = P_{it}(\mathbf{x})$ for any market m.
- Under condition ID-1, we can use our sample of markets to estimate consistently CCPs $P_{it}(\mathbf{x})$ for every player, t, and value $\mathbf{x}$.
- **Hotz-Miller inversion**: Define $q_{it}(\mathbf{x}) \equiv \Lambda^{-1}(P_{it}(\mathbf{x}))$ where Λ^{-1} is the inverse of the CDF Λ. This distribution is assumed known to the researcher (this condition can be relaxed) such that $q_{it}(\mathbf{x})$ is identified.

Identification with equilibrium beliefs (2)

- The model implies:

$$
\begin{aligned}
q_{it}(\mathbf{x}) &= \widetilde{\pi}_{it}^{\mathbf{B}(t)}(\mathbf{x}_t) + \widetilde{v}_{it}^{\mathbf{B}(t)}(\mathbf{x}_t) \\
&= B_{it}^{(t)}(\mathbf{x}) \left[\widetilde{\pi}_i(1,\mathbf{x}) + \widetilde{c}_{it}^{\mathbf{B}(t)}(1,\mathbf{x})\right] \\
&+ \left(1 - B_{it}^{(t)}(\mathbf{x})\right) \left[\widetilde{\pi}_i(0,\mathbf{x}) + \widetilde{c}_{it}^{\mathbf{B}(t)}(0,\mathbf{x})\right]
\end{aligned}
$$

- And with equilibrium beliefs:

$$
\begin{aligned}
q_{it}(\mathbf{x}) &= P_{jt}(\mathbf{x}) \left[\widetilde{\pi}_i(1,\mathbf{x}) + \widetilde{c}_{it}(1,\mathbf{x})\right] \\
&+ (1 - P_{jt}(\mathbf{x})) \left[\widetilde{\pi}_i(0,\mathbf{x}) + \widetilde{c}_{it}(0,\mathbf{x})\right]
\end{aligned}
$$

Identification with equilibrium beliefs (3)

- Bajari et al. (2010) provide conditions for the nonparametric identification of dynamic games under MPE.
- In addition to Assumption ID-1, and Equilibrium beliefs, there is an important exclusion restriction.
- **ASSUMPTION ID-2 (Exclusion Restriction)**: $\mathbf{X}_t = (S_{it}, S_{jt}, \mathbf{W}_t)$ such that S_{it} enters in the payoff function of player i but not in the payoff of the other player.

$$\pi_{it}\left(Y_{it}, Y_{jt}, S_{it}, S_{jt}, \mathbf{W}_t\right) = \pi_{it}\left(Y_{it}, Y_{jt}, S_{it}, \mathbf{W}_t\right)$$

Examples of exclusion restrictions

- The exclusion restriction in Assumption ID-2 appears naturally in many dynamic games of oligopoly competition.
- Entry-exit game: Lagged incumbent status of the competitor: $Y_{j,t-1}$.
- Game of investment in capital, capacity, quality, etc. The competitor´s capital stock at previous period, $K_{j,t-1}$.

Identification results with equilibrium beliefs

- Under **ID-1 & Equil. Beliefs:** $\Rightarrow$ Beliefs are identified (from behavior of other players) but **payoffs are NOT identified**.
- Under **ID-1 & ID-2 & Equil. Beliefs:** $\Rightarrow$ Beliefs and **payoffs are identified**.

Identification when beliefs are out-of-equilibrium

- We maintain Assumptions ID-1 & ID-2, and relax Assump. EQUIL.
- Model implies:

$$\begin{aligned} q_{it}(\mathbf{x}) &= B_{it}^{(t)}(\mathbf{x}) \left[\widetilde{\pi}_i(1,\mathbf{x}) + \widetilde{c}_{it}^{\mathbf{B}(t)}(1,\mathbf{x}) \right] \\ &= \left(1 - B_{it}^{(t)}(\mathbf{x})\right) \left[\widetilde{\pi}_i(0,\mathbf{x}) + \widetilde{c}_{it}^{\mathbf{B}(t)}(0,\mathbf{x}) \right] \end{aligned}$$

- **ASSUMPTION ID-3**: *The probability distribution over states tomorrow* $\mathbf{X}_{t+1}$ *is independent of* $\mathbf{S}_t$ *conditional on the vector* $\mathbf{W}_t$ *and the choices* $\mathbf{Y}_t$*:* $f(\mathbf{X}_{t+1}|\mathbf{Y}_t, \mathbf{X}_t) = f(\mathbf{X}_{t+1}|\mathbf{Y}_t, \mathbf{W}_t)$.

Examples of models that satisfy Assumption ID-3

- Market entry-exit: In this case, $S_{jt} = Y_{j,t-1}$. It is clear that Y_{jt} contains all the information about S_{jt+1}.
- More generally, multinomial dynamic games where Y_{jt} can be capital, capacity, quality, product design, etc, and $S_{jt} = Y_{j,t-1}$.
- Dynamic games of multi-armed bandit decisions.

Identification when beliefs are out-of-equilibrium

- Under **ID-1 to ID-3 & Unrestricted Beliefs:** $\Rightarrow$
  Null hypothesis of **Equilibrium Beliefs is testable.**

- Under **ID-1 to ID-3 & beliefs** are restricted at two points in $support(S)$**:** $\Rightarrow$
  Payoffs and contemporaneous beliefs are fully identified.

Test of Equilibrium Beliefs

- Model under assumptions ID-2 & ID-3 (I omit **W**):

$$
\begin{aligned}
q_{it}(s_i, s_j) &= B_{it}^{(t)}(s_i, s_j) \left[\widetilde{\pi}_i(1, s_i) + \widetilde{c}_{it}^{\mathbf{B}(t)}(1) \right] \\
&= \left(1 - B_{it}^{(t)}(s_i, s_j)\right) \left[\widetilde{\pi}_i(0, s_i) + \widetilde{c}_{it}^{\mathbf{B}(t)}(0) \right]
\end{aligned}
$$

- $\widetilde{\pi}_i$ depends on s_i but not on s_j.
- The continuation values $\widetilde{c}_{it}^{\mathbf{B}(t)}$ do not depend on either s_i or s_j.

Test of Equilibrium Beliefs (2)

- Given three values of S_j, say s_j^a, s_j^b, and s_j^c, we have that the function $q_{it}(.)$ identifies the following **function of contemporaneous beliefs**:

$$\frac{q_{it}(s_i, s_j^c) - q_{it}(s_i, s_j^a)}{q_{it}(s_i, s_j^b) - q_{it}(s_i, s_j^a)} = \frac{B_{it}^{(t)}(s_i, s_j^c) - B_{it}^{(t)}(s_i, s_j^a)}{B_{it}^{(t)}(s_i, s_j^b) - B_{it}^{(t)}(s_i, s_j^a)}$$

- The behavior of player i identifies a function of his beliefs.

- Under null hypothesis equilibrium beliefs, $B_{it}^{(t)}(\mathbf{x}) = P_{jt}(\mathbf{x})$ such that:

$$\left(\frac{q_{it}(s_i, s_j^c) - q_{it}(s_i, s_j^a)}{q_{it}(s_i, s_j^b) - q_{it}(s_i, s_j^a)}\right) - \left(\frac{P_{jt}(s_i, s_j^c) - P_{jt}(s_i, s_j^a)}{P_{jt}(s_i, s_j^b) - P_{jt}(s_i, s_j^a)}\right) = 0$$

Test of Equilibrium Beliefs (3)

- We can test the null hypothesis of equilibrium beliefs using a **Likelihood Ratio Test** in a **nonparametric multinomial model** for the CCPs P_i and P_j.
- The log-likelihood function of this multinomial model is:

$$\begin{aligned} \ell\left(\mathbf{P}_i, \mathbf{P}_j\right) &= \sum_{m,t} y_{imt} \ln P_{it}(\mathbf{x}_{mt}) + (1 - y_{imt}) \ln\left[1 - P_{it}(\mathbf{x}_{mt})\right] \\ &+ \sum_{m,t} y_{jmt} \ln P_{jt}(\mathbf{x}_{mt}) + (1 - y_{jmt}) \ln\left[1 - P_{jt}(\mathbf{x}_{mt})\right] \end{aligned}$$

- Test statistic:

$$LR = 2\left[\ell\left(\widehat{\mathbf{P}_i^u}, \widehat{\mathbf{P}_j^u}\right) - \ell\left(\widehat{\mathbf{P}_i^c}, \widehat{\mathbf{P}_j^c}\right)\right]$$

Test of Equilibrium Beliefs (4)

- The unconstrained MLE $\left(\widehat{\mathbf{P}_i^u}, \widehat{\mathbf{P}_j^u}\right)$ is just the frequency estimator of CCPs.

- And the constrained MLE $\left(\widehat{\mathbf{P}_i^c}, \widehat{\mathbf{P}_j^c}\right)$ is the estimator subject to the equlibrium constraints:

$$\left(\frac{\Lambda^{-1}(P_{it}(s_i, s_j^{(c)})) - \Lambda^{-1}(P_{it}(s_i, s_j^{(a)}))}{\Lambda^{-1}(P_{it}(s_i, s_j^{(b)})) - \Lambda^{-1}(P_{it}(s_i, s_j^{(a)}))}\right) - \left(\frac{P_{jt}(s_i, s_j^c) - P_{jt}(s}{P_{jt}(s_i, s_j^b) - P_{jt}(s}\right.$$
$$= \quad 0$$

Estimation

- Suppose that the null hypothesis is rejected.
- It is possible to use the estimated beliefs $\dfrac{B_{it}^{(t)}(s_i, s_j^c) - B_{it}^{(t)}(s_i, s_j^a)}{B_{it}^{(t)}(s_i, s_j^b) - B_{it}^{(t)}(s_i, s_j^a)}$ and estimated actual behavior of the other player, $\dfrac{P_{jt}(s_i, s_j^c) - P_{jt}(s_i, s_j^a)}{P_{jt}(s_i, s_j^b) - P_{jt}(s_i, s_j^a)}$, to study different hypotheses/models of learning.

Estimation (2)

- Alternatively, we can keep our agnostic approach about the structure of beliefs but impose the restriction that these beliefs are unbiased at two values of S_j, i.e., for s_j^a and s_j^b we have $B_{it}^{(t)}(s_i, s_j^a) = P_{jt}(s_i, s_j^a)$ and $B_{it}^{(t)}(s_i, s_j^b) = P_{jt}(s_i, s_j^b)$.

- Then, for any other value of S_j is identified.

- Given the identification of beliefs, the estimation of the payoff function proceeds in a very similar way as in the estimation of single-agent dynamic structural model.

How two choose points where to impose unbiased beliefs?

- *(a) Applying the test of equilibrium beliefs.*
- *(b) Testing for the monotonicity of beliefs and using this restriction.*
- *(c) Minimization of the player's beliefs bias.*
- *(d) Most visited states.*

Monte Carlo Experiments

- Main purposes of these experiments:

[1] To evaluate the power the power of the test.

[2] To assess the price, in terms of precision of our estimates, of relaxing the assumption of equilibrium beliefs?

[3] Evaluate the bias induced by imposing the assumption of equilibrium beliefs when this assumption does not hold in the DGP.

Monte Carlo Experiments: Model

- *Dynamic game of market entry and exit.*

$$\pi_{1mt}(1, Y_{2mt}, \mathbf{X}_{mt}) = \alpha_1 - \delta_1 \ Y_{2mt} + Y_{1mt-1} \ \theta_1^{EC}$$

$$\pi_{2mt}(1, Y_{2mt}, \mathbf{X}_{mt}) = \alpha_2 - \delta_2 \ Y_{1mt} - \theta_S \ S_{2m} + Y_{2mt-1} \ \theta_2^{EC}$$

- S_{2m} has a discrete uniform distribution with support $\{-2, -1, 0, 1, 2\}$.

Monte Carlo Experiments: Design

Table 3
Summary of DGPs in the Monte Carlo Experiments

For all the experiments: $\alpha = 2.4$; $\delta = 3.0$; $\theta^{EC} = 0.5$; $\beta = 0.95$

$S_{2m} \sim Uniform\ \{-2, -1, 0, +1, +2\}$

$M = 2,000$; $T = 5$; $MC\ \ rep = 10,000$

Experiment	1U:	$\theta_S = -0.5$;	Unbiased beliefs
Experiment	1B:	$\theta_S = -0.5$;	Biased beliefs
Experiment	2U:	$\theta_S = -1.0$;	Unbiased beliefs
Experiment	2B:	$\theta_S = -1.0$;	Biased beliefs

MCs: Cost of Relaxing Equil Beliefs

Monte Carlo Experiment 1U

Parameter (True value)	Estimation WITH equil. rest. Bias (%)	Std (%)	Estimation WITHOUT equil. rest. Bias (%)	Std (%)
Payoffs				
α (2.4)	-0.0992 (4.13)	0.2208 (9.20)	0.1412 (5.88)	0.3702 (15.42)
δ (3.0)	-0.1004 (3.35)	0.2349 (7.83)	0.1448 (4.83)	0.3763 (12.54)
θ^{EC} (0.5)	-0.0021 (0.42)	0.0665 (13.30)	-0.0760 (15.20)	0.1118 (22.35)

MCs: Benefits of Relaxing Equil Beliefs

Monte Carlo Experiment 1B

Parameter (True value)	Estimation WITH equil. rest. Bias (%)	Std (%)	Estimation WITHOUT equil. rest. Bias (%)	Std (%)
Payoffs				
α (2.4)	-0.3332 (13.88)	0.2666 (11.11)	-0.0081 (0.34)	0.2829 (11.79)
δ (3.0)	0.2979 (9.93)	0.2746 (9.15)	0.1543 (5.14)	0.3071 (10.24)
θ^{EC} (0.5)	-0.3277 (65.55)	0.0778 (15.56)	0.0134 (2.68)	0.1482 (29.63)

EMPIRICAL APPLICATION

- Dynamic game of store location by McDonalds (MD) and Burger King (BK) using data for United Kingdom during the period 1990-1995.
- Panel of 422 local markets (districts) and six years, 1990-1995.
- Information on the number of stores of McDonalds (MD) and Burger King (BK) in United Kingdom.
- Information on local market characteristics such as population, density, income per capita, age distribution, average rent, local retail taxes, and distance to the headquarters of each firm in UK.

Table 1
Descriptive Statistics for the Evolution of the Number of Stores
Data: 422 markets, 2 firms, 5 years = 4,220 observations

	Burger King					McDonalds			
	1991	1992	1993	1994	1995	1991	1992	1993	1994
# Markets	98	104	118	131	150	213	220	237	248
Δ # Markets	17	6	14	13	19	7	7	17	11
# of stores	115	128	153	181	222	316	344	382	421
Δ # of stores	36	13	25	28	41	35	28	38	39
stores per mark	1.17	1.23	1.30	1.38	1.48	1.49	1.56	1.61	1.70

Model

- $K_{imt} \in \{0, 1, ..., |\mathcal{K}|\}$ number of stores of firm i in market m at period $t-1$.
- $Y_{imt} \in \{0, 1\}$ decision of firm i to open a new store.
- $Y_{imt} + K_{imt} = \#$ stores of firm i at period t.
- Firm i's total profit function is equal to:

$$\Pi_{imt} = VP_{imt} - EC_{imt} - FC_{imt}$$

Model (2)

- Variable profit function:

$$VP_{imt} = (\mathbf{W}_m\ \gamma)\ (Y_{imt} + K_{imt}) \left[\begin{array}{c} \theta_{0i}^{VP} + \theta_{can,i}^{VP}(K_{imt} + Y_{imt}) \\ +\theta_{com,i}^{VP}(K_{jmt} + Y_{jmt}) \end{array} \right]$$

- Entry cost:

$$EC_{imt} \quad = \quad 1\{Y_{imt} > 0\} \left[\theta_{0i}^{EC} + \theta_{K,i}^{EC}\ 1\{K_{imt} > 0\} + \theta_{S,i}^{EC}\ S_{imt} + \varepsilon_{it}\right]$$

- Fixed cost:

$$FC_{imt} \quad = \quad 1\{(K_{imt} + Y_{imt}) > 0\} \left[\begin{array}{c} \theta_{0i}^{FC} + \theta_{lin,i}^{FC}(K_{imt} + Y_{imt}) \\ +\theta_{qua,i}^{FC}(K_{imt} + Y_{imt})^2 \end{array} \right]$$

Tests of Unbiased Beliefs

Data: 422 markets, 5 years = 2,110 observations

BK: $\widehat{D}$ (p-value)	66.841	(0.00029)
MD: $\widehat{D}$ (p-value)	42.838	(0.09549)

- We can reject hypothesis that BK beliefs are unbiased (p-value 0.00029).
- Restriction is more clearly rejected for large values of the state variable (distance to chain network) S_{MD}.

Where to impose unbiased beliefs?

- We propose three different criteria:
  [1] Minimize distance $\|B_i - P_j\|$
  [2] Test for monotonicity of beliefs: if not rejected, impose unbiased beliefs in extreme values of S_j.
  [3] Most visited values of S_j.

- In this empirical application, the three criteria have the same implication: impose unbiased beliefs at the lowest values for the distance S_j.

Estimation of Dynamic Game

Data: 422 markets, 2 firms, 5 years = 4,220 observations

	$\beta = 0.95$ (not estimated)			
	Equilibrium Beliefs		Biased Beliefs	
	BK	MD	BK	MD
Var Profits:				
θ_0^{VP}	0.5413	0.8632	0.4017	0.8271
	$(0.1265)^*$	$(0.2284)^*$	$(0.2515)^*$	$(0.4278)^*$
θ_{can}^{VP} cannibalization	-0.2246	0.0705	-0.2062	0.0646
	$(0.0576)^*$	$(0.0304)^*$	$(0.1014)^*$	(0.0710)
θ_{com}^{VP} competition	**-0.0541**	-0.0876	**-0.1133**	-0.0856
	(0.0226)*	(0.0272)	**(0.0540)***	(0.0570)
Log-Likelihood	-848.4		-840.4	

Estimation of Dynamic Game

Data: 422 markets, 2 firms, 5 years = 4,220 observations

	$\beta = 0.95$ (not estimated)			
	Equilibrium Beliefs		Biased Beliefs	
	BK	MD	BK	MD
Fixed Costs:				
θ_0^{FC} fixed	0.0350	0.0374	0.0423	0.0307
	(0.0220)	(0.0265)	(0.0478)	(0.0489)
θ_{lin}^{FC} linear	0.0687	0.0377	0.0829	0.0467
	$(0.0259)^*$	$(0.0181)^*$	$(0.0526)^*$	(0.0291)
θ_{qua}^{FC} quadratic	-0.0057	0.0001	-0.0007	0.0002
	(0.0061)	(0.0163)	(0.0186)	(0.0198)

Estimation of Dynamic Game

Data: 422 markets, 2 firms, 5 years = 4,220 observations

	$\beta = 0.95$ (not estimated)			
	Equilibrium Beliefs		Biased Beliefs	
	BK	MD	BK	MD
Entry Cost:				
θ_0^{EC} fixed	0.2378	0.1887	0.2586	0.1739
	$(0.0709)^*$	$(0.0679)^*$	$(0.1282)^*$	$(0.0989)^*$
θ_K^{EC} (K)	-0.0609	-0.107	-0.0415	-0.1190
	(0.043)	$(0.0395)^*$	(0.096)	$(0.0628)^*$
θ_S^{EC} (S)	0.0881	0.0952	0.1030	0.1180
	$(0.0368)^*$	$(0.0340)^*$	$(0.0541)^*$	$(0.0654)^*$

Other empirical applications

- **Goldfarb & Xiao (AER, 2011):** Entry in US local telephone markets after the 1996 Telecommunications Act of 1996. Level-K rationalizability. Find that more experienced, better-educated managers can predict better competitor behavior.
- **Doraszelski, Lewis, & Pakes (NBER wp, 2016):** Dynamic price competition in a new electricity market in UK. Evidence of substantial strategic uncertainty but also convergence over time. Tests of different models of learning (fictitious play, adaptive learning).
- **Asker, Fershtman, Jeon, & Pakes (NBER wp, 2016):** Dynamic bidding. Persistent asymmetric information as the source of biased beliefs and learning. Experience Based Equilibrium (EBE).
- **Jeon (2016):** Dynamic game of capacity investment in the shipping industry. Source of biased beliefs is Knightian uncertainty about the evolution of demand. Evidence of biased beliefs and slow learning.

Summary and Conclusions

- Strategic uncertainty can be important for competition in oligopoly markets. Under these conditions, the assumption of equilibrium beliefs can be too restrictive and it can bias our estimates and policy evaluations.

- There are reasonable conditions under which this restriction is testable. If rejected, it is also possible to identify the model under weaker conditions that unbiased beliefs everywhere.

- It will be interesting to see more empirical applications on evaluation of policy changes, deregulations, mergers, that take into account the possibility of temporary biased beliefs.

- Use of estimated contemporaneous beliefs to understand how firms´ learn over time about other firms´ strategies.