ECO 310: Empirical Industrial Organization

Lecture 5: Demand Systems: Introduction

Victor Aguirregabiria (University of Toronto)

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Introduction

- Estimation of demand equations is a fundamental component to answer many empirical questions in economics.
- **[1] Determination of firms' optimal prices or quantities:** Optimal price or quantity implies MR $=$ MC. To know the MR, we need to know the demand elasticity**.**
- **[2] Measuring firms' market power (P - MC):** In the absence of data on firms MCs, demand estimation give us MR and therefore (under profit maximization) the MC.
- **[3] Measures of consumer welfare.** Demand is a representation of consumers' valuations for products. As such, it is fundamental in the evaluation of the consumer welfare gains or losses associated to taxes, subsidies, the introduction of a new product, mergers, etc.

Introduction: Demand systems

- Consumers do not demand a single product.
- In most industries, firms sell different varieties of a differentiated product.
  - Airlines; Tablets; Smartphones; Restaurants; Movies; etc
- Demands of different products are inter-connected in a **demand system**. With two products:

$$q_1 = f_1(p_1, p_2, y) \qquad \text{and} \qquad q_2 = f_2(p_1, p_2, y)$$

- Firms (and researchers) are interested in the estimation of elasticities (own and cross) in demand systems.

Outline on this topic (Weeks 5 to 7)

1. **Introduction**
2. **Demand systems in product space**
   - **2.1.** Model
   - **2.2.** Data and Estimation
   - **2.3.** Limitations
   - **2.4.** An Application
3. **Demand systems in characteristics space**
   - **3.1.** Model
   - **3.2.** Estimation
   - **3.3.** An Application

1. Demand Systems in Product Space: Model

Demand systems in product space: Products

- There are J different products that a consumer can buy. We index products by $j \in \{1, 2, ..., J\}$.
- These J products may include all the product categories that an individual may consume (e.g., food, transportation, clothing, entertainment, etc) and all the specific variety products within each category (e.g., every possible variety of computers, or of automobiles).
- This means that the J can be of the order of millions of products.

Demand systems in product space: Products [2]

- We will see later how, under some conditions, we can aggregate the demand of a group of products "as if" they were a single product. This can reduce the dimensionality of this large product space.

- For this purpose, it is convenient to introduce **"product zero"** that we denote as **"the outside product"** and it represents **the aggregation** of all the other products which are nor product 1 to J.

- Example: Products 1 to J are all the models/brands of automobiles in the market (hundreds of models). Product 0 represents all the other goods in the economy.

Consumer preferences

- Let q_j be the amount of product j that a consumer buys and consumes. And let c be the amount of the outside product.
- The utility function is: $U(c, q_1, q_2, ..., q_J)$.
- If we could observe the utility that a consumer obtains from consuming a bundle of products $(c, q_1, q_2, ..., q_J)$, then we could estimate the utility function using a **direct approach** as we have done with the estimation of the production function.
- For instance, given a Cobb-Douglas utility, $U = c \; q_1^{\alpha_1} \; q_2^{\alpha_2} ... \; q_J^{\alpha_J}$, and a sample of individuals with consumptions bundles $(c_i, q_{1i}, q_{2i}, ..., q_{Ji})$ and utils U_i, we could estimate the α parameters using the linear regression model:

$$\ln U_i = \alpha_1 \; \ln q_{1i} + \alpha_2 \; \ln q_{2i} + ... + \alpha_J \; \ln q_{Ji} + \ln c_i$$

Consumer problem

- However, we cannot use this direct approach because (typically) we do not have direct observations of consumers' utility, or satisfaction.
- Instead, we will estimate consumer preferences by **estimating demand equations**: i.e., by measuring how changes in prices affect the purchased quantities of products.
- For this purpose, we need to solve the consumer problem to obtain demand equations.

Consumer problem [2]

- Consumer problem:

$$\max_{\{c, q_1, q_2, \ldots, q_J\}} \quad U(c, q_1, q_2, \ldots, q_J)$$

$$\textit{subject to}: \quad c + p_1 \; q_1 + p_2 \; q_2 + \ldots + p_J \; q_J \leq y$$

- Note that the price of the outside good is normalized to 1, such that c represents the \$ expenditure in goods other than 1 to J.

Consumer problem [3]

- We can define the Lagrange problem:

$$\max_{\{c, q_1, q_2, \dots, q_J\}} \quad U(c, q_1, q_2, \dots, q_J) + \lambda \left[y - c - p_1 \ q_1 - \dots - p_J \ q_J \right]$$

- The first order conditions are:

$$\begin{array}{rcll} U_j - \lambda \ p_j & = & 0 & \quad \text{for } j = 1, 2, \dots, J \\ U_0 - \lambda & = & 0 & \\ y - c - p_1 \ q_1 - \dots - p_J \ q_J & = & 0 & \end{array}$$

where U_j represents the marginal utility of product j.

Solving consumer problem

- The solution to this system of equations give us the **System of Marshallian demand equations**:

$$
\begin{aligned}
q_1 &= f_1\left(p_1, p_2, ..., p_J, y\right) \\
q_2 &= f_2\left(p_1, p_2, ..., p_J, y\right) \\
\vdots & \qquad\qquad \vdots \\
q_J &= f_J\left(p_1, p_2, ..., p_J, y\right)
\end{aligned}
$$

Demand systems

- Different utility functions imply different demand systems.
- Not every system of equations that relates quantities and prices is a demand system. We should be able to derive it as the solution of the consumer problem for a given utility function (**invertibility of a demand system**).
- A substantial part of the empirical literature on demand is based on finding utility functions which imply demand systems with the following properties:
  - Simple enough to be estimable using standard econometric methods such as linear regression.
  - Flexible enough such that it allows for flexible patterns in the elasticities of substitution between products.

Demand systems [2]

- For consumer demand models in product space, we are going to consider three different demand systems.
- [1] Linear expenditure (or Stone-Geary) demand system.
- [2] Constant Elasticity of Substitution (CES) demand system.
- [3] Deaton-Muellbauer (or 'Almost Ideal') demand system.
- They are shorted and chronological order.

Linear Expenditure (Stone-Geary) Demand System

- Consider the utility function:

$$U = c\ (q_1 - \gamma_1)^{\alpha_1}\ (q_2 - \gamma_2)^{\alpha_2} ...\ (q_J - \gamma_J)^{\alpha_J}$$

  where α's and γ's are positive parameters.

- $\gamma_j \geq 0$ can be interpreted as the minimum amount of consumption of good j that a consumer needs to "survive" ($U > 0$).
- $\alpha_j > 0$ represents the "intensity" of product j in generating utility.
- The marginal utilities are:

$$U_j = \alpha_j \frac{U}{q_j - \gamma_j} \quad \text{and} \quad U_0 = \frac{U}{c}$$

- Note that all the products are complements in this utility function:

$$U_{jk} = \alpha_j \alpha_k \frac{U}{(q_j - \gamma_j)(q_k - \gamma_k)} > 0$$

Linear Expenditure Demand System [2]

- $U_0 - \lambda = 0$ implies that $\dfrac{U}{c} - \lambda = 0$, and $c = \dfrac{U}{\lambda}$
- $U_j - \lambda \ p_j = 0$ implies that $\alpha_j \dfrac{U}{q_j - \gamma_j} = \lambda \ p_j$, and $\alpha_j \dfrac{1}{q_j - \gamma_j} \dfrac{U}{\lambda} = p_j$.
- Combining these equations, we the **Linear Expenditure System**:

$$q_j = \gamma_j + \alpha_j \frac{c}{p_j} \quad \text{for } j = 1, 2, ..., J$$

  and $c = \dfrac{y - \sum_{j=1}^{J} p_j \ \gamma_j}{1 + \sum_{j=1}^{J} \alpha_j}$.

- Note that for any $j \neq k$, we have that:
  $\dfrac{\partial q_j}{\partial p_k} = \dfrac{\alpha_j}{p_j} \dfrac{\partial c}{\partial p_k} = \dfrac{-\alpha_j \gamma_k}{p_j \left[1 + \sum_{i=1}^{J} \alpha_i \right]} < 0$. All the products are complements.

Linear Expenditure Demand System [3]

- It is very convenient because its simplicity.
- Suppose that we have data of individual purchases and prices over T periods of time ($t = 1, 2, ..., T$): $\{c_t, q_{1t}, q_{2t}, ..., q_{Jt}\}$ and $\{p_{1t}, p_{2t}, ..., p_{Jt}\}$.
- The model implies a system of J linear regressions. For product j:

$$q_{jt} = \gamma_j + \alpha_j \; x_{jt} + \xi_{jt}$$

with $x_{jt} = c_t / p_{jt}$, and the error term ξ_{jt} could be measurement error in q_{jt}.

- But it is very restrictive. It imposes the restriction that **all the goods are complements in consumption**.
- This is not realistic in most applications, particularly when the goods under study are varieties of a differentiated product.

CES Demand System

- Consider the CES utility function:

$$U = c \left(\sum_{j=1}^{J} q_j^{\sigma} \right)^{1/\sigma}$$

where $\sigma \in [0, 1]$ is a parameter that represents the degree of substitution between the J products.

- The marginal utilities are:

$$U_j = q_j^{\sigma-1} \frac{U}{Q_\Sigma} \quad \text{and} \quad U_0 = \frac{U}{c}$$

where $Q_\Sigma \equiv \sum_{i=1}^{J} q_i^{\sigma}$.

- It is simple to verify that for any two pairs of products, j and k, we have that $\frac{\partial^2 U}{\partial q_j \partial q_k} \equiv \frac{\partial U_j}{\partial q_k} < 0$. **All the products are substitutes in consumption**.

CES Demand System [2]

- $U_0 - \lambda = 0$ implies that $\dfrac{U}{c} - \lambda = 0$, and $c = \dfrac{U}{\lambda}$
- $U_j - \lambda\ p_j = 0$ implies that $q_j^{\sigma-1}\dfrac{U}{Q_\Sigma} = \lambda\ p_j$, and $\dfrac{q_j^{\sigma-1}}{Q_\Sigma}\dfrac{U}{\lambda} = p_j$.
- Combining these equations, we the **CES System**:

$$q_j = \left[\frac{c}{p_j\ Q_\Sigma}\right]^{\dfrac{1}{1-\sigma}} \quad \text{for } j = 1, 2, ..., J$$

- For any pair of products, j and k, we have that:

$$\ln\left(\frac{q_j}{q_k}\right) = \frac{-1}{1-\sigma} \ln \ln\left(\frac{p_j}{p_k}\right)$$

CES Demand System [3]

- The CES is also very convenient because its simplicity.
- Suppose that we have data of individual purchases and prices over T periods of time: $\{c_t, q_{1t}, q_{2t}, ..., q_{Jt}\}$ and $\{p_{1t}, p_{2t}, ..., p_{Jt}\}$.
- The model implies a system of J linear regressions. For product $j > 1$:

$$\ln(q_{jt}/q_{1t}) = \beta \ \ln(p_{jt}/p_{1t}) + \xi_{jt}$$

where $\beta = \dfrac{-1}{1-\sigma}$, and the error term ξ_{jt} could be measurement error in $\ln q_{jt}$.

- But it is very restrictive. It imposes the restriction that **the substitution between any pair of products is exactly the same.**.

Demand System [4]

- The CES system imposes strong restrictions on cross-price elasticities. For any three varieties, say j, k, and l:

$$Elasticity_{k,j} = \frac{\partial \ln q_k}{\partial \ln p_j} = \frac{\partial \ln q_l}{\partial \ln p_j} = Elasticity_{l,j}$$

- Suppose that we use this system to study the demand of different varieties of automobiles.

- Suppose that products j and k are "similar" luxury cars, and product l is an basic and expensive variety.

- The CES model implies that a reduction in the price of the luxury car j implies the same proportional increase in the demand of the other luxury car k and the basic car l.

- This is very unrealistic.

Deaton-Muellbauer ('Almost Ideal') Demand System

- They propose the utility function:

$$U = c \left[\prod_{j=1}^{J} \left(q_j - \gamma_j \right)^{\alpha_j} \right] + \sum_{j=1}^{J} \sum_{k=1}^{J} \delta_{jk} \; q_j \; q_k$$

  where $\gamma' s$, $\alpha' s$ and $\delta' s$ are parameters. $\alpha' s$ and $\gamma' s$ are positive, but $\delta' s$ can be positive or negative.

- This utility allows for complementarity and substitutability between products, and for a flexible pattern of substitution between different products.

- The marginal utilities are:

$$U_j = \alpha_j \frac{U^*}{q_j - \gamma_j} + \sum_{k=1}^{J} \delta_{jk} \; q_k \quad \text{and} \quad U_0 = \frac{U^*}{c}$$

  where $U^* \equiv c \prod_{i=1}^{J} \left(q_i - \gamma_i \right)^{\alpha_i}$.

'Almost Ideal' Demand System [2]

- For any pair of products, j and k, we have that:

$$U_{jk} = \alpha_j \alpha_k \frac{U^*}{\left(q_j - \gamma_j\right)\left(q_k - \gamma_k\right)} + \delta_{jk} \gtreqless 0$$

that can be positive (complements) or negative (substitutes) and take different values for each pair of products.

'Almost Ideal' Demand System [4]

- $U_0 - \lambda = 0$ implies that $\dfrac{U^*}{c} - \lambda = 0$, and $c = \dfrac{U^*}{\lambda}$
- $U_j - \lambda \ p_j = 0$ implies that $\alpha_j \dfrac{U^*}{q_j - \gamma_j} + \sum_{k=1}^{J} \delta_{jk} \ q_k - \lambda p_j = 0$, and

  $\alpha_j \dfrac{c}{q_j - \gamma_j} + \sum_{k=1}^{J} \delta^*_{jk} \ q_k = p_j$.
- Combining these equations, it is possible to derive the following system of Marshallian demand equations:

$$w_j = \beta_j^{(0)} + \beta_j^{(y)} \ \ln(y) + \sum_{k=1}^{J} \beta_{jk}^{(p)} \ \ln(p_k)$$

- $w_j \equiv \dfrac{p_j q_j}{y}$ is the expenditure share of product j;

- $\{\beta_j^{(0)}, \beta_j^{(y)}, \beta_{jk}^{(p)}\}$ are parameters which are known functions of the utility parameters $\{\alpha_j, \gamma_j, \delta_{jk}\}$.

'Almost Ideal' Demand System [5]

$$w_j = \beta_j^{(0)} + \beta_j^{(y)} \ \ln(y) + \sum_{k=1}^{J} \beta_{jk}^{(p)} \ \ln(p_k)$$

- The model implies the symmetry conditions $\beta_{jk}^{(p)} = \beta_{kj}^{(p)}$ (Slutsky's symmetry condition).
- Therefore, the number of free parameters is: $2J + \frac{J(J+1)}{2}$, that increases quadratically with the number of products.

'Almost Ideal' Demand System [6]

- Suppose that we have data on individual purchases, income, and prices over T periods of time: $\{c_t, q_{1t}, q_{2t}, ..., q_{Jt}\}$, y_t, and $\{p_{1t}, p_{2t}, ..., p_{Jt}\}$.
- For each product j, we can estimate the regression equation:

$$w_{jt} = \beta_j^{(0)} + \beta_j^{(y)} \ln(y_t) + \beta_{j1}^{(p)} \ln(p_{1t}) + ... + \beta_{jJ}^{(p)} \ln(p_{Jt}) + \xi_{jt}$$

where the error ξ_{jt} can be interpreted as measurement error in w_{jt}.

Multi-stage Budgeting

- For products with many (> 100) varieties (automobiles, smartphones, cereals, beer, etc) the number of parameters to estimate in the AI demand system can be very large, even larger than the #observations.

- Deaton and Muellbauer propose using a multi-stage budgeting approach.

- Suppose that the utility function is separable in the utility from G groups of products:

$$U = v_1 \left(\widetilde{\mathbf{q}}_1 \right) + v_2 \left(\widetilde{\mathbf{q}}_2 \right) + \ldots + \; v_G \left(\widetilde{\mathbf{q}}_G \right)$$

$\widetilde{\mathbf{q}}_g =$ Vector of quantities of varieties in group g;
$v_g \left(\widetilde{\mathbf{q}}_g \right) =$ Sub-utility from group g

Multi-stage Budgeting (under AIDS)

- Then, the demand system at the lower stage (**within-group stage**) is:

$$w_{jt} = \beta_j^{(0)} + \beta_j^{(y)} \ \ln \left(\frac{y_{gt}}{P_{gt}} \right) + \sum_{k \in \mathcal{J}_g} \beta_{jk}^{(p)} \ \ln(p_{kt})$$

y_{gt} = Expenditure in group g;
P_{gt} = Price index for group g.

- According to the model, this price index depends on the parameters of the model in group g. Non-linear system. Typically applications use "short-cuts": e.g., $\ln P_{gt} = \sum_{j \in \mathcal{J}_g} w_{jt} \ \ln(p_{jt})$.

- Number of parameters increases quadratically with J_g but not with J.

Multi-stage Budgeting (under AIDS) [2]

- The demand system at the **group stage** is:

$$\frac{y_{gt}}{y_t^*} = \beta_g^{(0,2)} + \beta_g^{(y,2)} \ \ln\left(\frac{y_t^*}{P_t^*}\right) + \sum_{g'=1}^{G} \beta_{g,g'}^{(p,2)} \ \ln(P_{gt})$$

y_t^* = Total expenditure in the large category (e.g., cereals);
P_t^* = Price index for the large category (e.g., cereals).

- Finally, at the top-stage, the **demand for the category** is:

$$\frac{y_t^*}{y_t} = \beta^{(0,3)} + \beta^{(y,3)} \ \ln\left(y_t\right) + \beta^{(p,3)} \ \ln(P_t^*)$$

2. Demand Systems in Product Space: Data & Estimation

Data

- Ideally, we would like to have data on consumer decisions at the consumer or household level.
- This type of data is available from consumer surveys but at the level of large product categories:
  - e.g., clothing (different types), food (different types).
- In Empirical IO, we are interested in estimating demand system between very specific type of products: e.g.,
  - the cross demand elasticity between Iphone-7 and Iphone-8; or between Iphone-8 and Samsung Galaxy;
  - or between a Toyota Corolla and VW Jetta (or for that matter, with any other automobile product).
- For these specific purchases, consumer level data is not so commonly available.

Data [2]

- The most common type of data available for the estimation of demand systems is aggregate market level data.

- Data on aggregate quantities and prices from all the consumers in a particular period t (year, month):

$$q_{jt}, \ p_{jt} \quad \text{for } j = 1, 2, ..., J \text{ and } t = 1, 2, ..., T$$

- In some cases, we have this information for M different geographic markets (e.g., cities), that we index by m:

$$q_{jmt}, \ p_{jmt} \quad \text{for } j = 1, 2, ..., J, \ t = 1, 2, ..., T, \text{ and } m = 1, 2, ..., M$$

- This is the type of data that we will consider for most of this topic.

Estimation: OLS

- Given the linear regression:

$$w_{jt} = \beta_j^{(0)} + \beta_j^{(y)} \ \ln(y_t) + \beta_{j1}^{(p)} \ \ln(p_{1t}) + ... + \beta_{jJ}^{(p)} \ \ln(p_{Jt}) + \xi_{jt}$$

- We can estimate the model parameters using OLS, separately for each product j (using the T observations for this product).
- Under the assumption that the error terms is ξ_{jt} just measurement error in the dependent variable, w_{jt}, and this measurement error is not correlated with prices or income, then the OLS estimator is consistent (unbiased as T is large).
- However, these conditions are very restrictive / unrealistic.

Endogeneity (Simultaneity)

$$w_{jt} = \beta_j^{(0)} + \beta_j^{(y)} \ \ln(y_t) + \beta_{j1}^{(p)} \ \ln(p_{1t}) + ... + \beta_{jJ}^{(p)} \ \ln(p_{Jt}) + \xi_{jt}$$

- At least part of the error term ξ_{jt} represents consumers' taste/preference for product j that is known to consumers but unknown to us as researchers.
- Advertising, promotions, changes in taste over time, changes in utility parameters are behind the error term ξ_{jt}.
- We will represent all these factors under the term: **unobserved demand shock** (for product j).
- This unobserved demand shock is known to consumers but unknown to us as researchers.

Endogeneity (Simultaneity) [2]

$$w_{jt} = \beta_j^{(0)} + \beta_j^{(y)} \ \ln(y_t) + \beta_{j1}^{(p)} \ \ln(p_{1t}) + \ldots + \beta_{jJ}^{(p)} \ \ln(p_{Jt}) + \xi_{jt}$$

- The unobserved demand shock (or part of it) can be also known by firms selling product j.
- Therefore, they will take into account ξ_{jt} when making their pricing decision.
- A higher ξ_{jt} implies a higher Marginal Revenue and therefore a higher price.
- We expect ξ_{jt} to be correlated with $\ln(p_{jt})$.
- The OLS estimator is inconsistent because the regressors are correlated with the error term.

Endogeneity (Simultaneity) [3]

$$w_{jt} = \beta_j^{(0)} + \beta_j^{(y)} \ln(y_t) + \beta_{j1}^{(p)} \ln(p_{1t}) + ... + \beta_{jJ}^{(p)} \ln(p_{Jt}) + \xi_{jt}$$

- We expect prices of products other than j also being correlated with ξ_{jt}.
- This is (mainly) because the demand shocks $\xi's$ of the different products can be correlated with each other: ξ_{kt} is correlated with p_{kt}, and ξ_{kt} is correlated with ξ_{jt}; typically, ξ_{jt} will be correlated with p_{kt}.
- Also firms other than j can observe ξ_{jt} and respond strategically to this shock in their pricing decisions.
- We will see in more detail in the context of models of price competition.

Endogeneity of prices: Solutions

$$w_{jt} = \beta_j^{(0)} + \beta_j^{(y)} \ \ln(y_t) + \beta_{j1}^{(p)} \ \ln(p_{1t}) + ... + \beta_{jJ}^{(p)} \ \ln(p_{Jt}) + \xi_{jt}$$

- We need to deal with the endogeneity problem due to $E(\ln(p_t) \ \xi_{jt}) \neq 0$.
- The most common approaches: (1) Instrumental variables; (2) Control function.
- **[1] Instrumental variables:**
  - (a) Input prices, costs
  - (b) Arellano-Bond (Dynamic Panel Data)
  - (c) Hausman-Nevo
- **[2] Control Function**
  - (a) Fixed effects
  - (b) Fixed effects - Cochrane-Orcutt

3. Empirical Application: Hausman on Cereals

An Application: Hausman (1996) on cereals

- Hausman (1996) presents an application of demand in product space to an industry with many varieties: ready-to eat (RTE) cereals in US.
- This industry has been characterized by the proliferation of many varieties. Period 1980-92: 190 new brands were added to the pool of existing 160 brands.
- He deals with the limitations mentioned above by using:
  (a) Multi-stage budgeting (and focusing on most popular varieties);
  (b) Data from many periods (weekly data) and multiple geographic markets (cities), and assuming that parameters are constant across weeks-markets (up to fixed effects in the intercepts).
  (c) Exploiting assumptions on the geographic structure of demand/supply shocks to generate instruments for prices.
  (d) Evaluates the introduction of a new brand (Cheerios).

Hausman (1996) on cereals: Data

- Supermarket scanner data: period 1990-1992.
- 137 weeks ($T = 137$); 7 geographic markets ($M = 7$) or standard metropolitan statistical areas (SMSAs), including Boston, Chicago, Detroit, Los Angeles, New York City, Philadelphia, and San Francisco.
- Though the data includes information from hundred of brands, the model and the estimation concentrates in 20 brands classified in three segments: adult (7 brands), child (4 brands), and family (9 brands).
- $\{p_{jmt}, q_{jmt} : j = 1, 2, ..., 20;\ m = 1, 2, ..., 7;\ t = 1, 2, ..., 137\}$.
- Quantities are measured is physical units.
- There are not observable cost shifters.

Hausman (1996) on cereals: Model

- Almost-Ideal-Demand-System

$$w_{jmt} = \alpha_{jm} + \gamma_t + \beta_j^{(y)} \ln\left(\frac{y_{gmt}}{P_{gmt}}\right) + \sum_{k \in J_g} \beta_{jk}^{(p)} \ln(p_{kmt}) + \xi_{jmt}$$

- The terms $\alpha_{jm} + \gamma_t$ represent brand-city and time "fixed effects".
- Possible instruments:
  - Arellano-Bond instruments
  - Hausman-Nevo instruments

Hausman (1996) on cereals: Instruments

- The identification assumption is that demand shocks are not (spatially) correlated across markets: for any pair of markets $m \neq m$ it is assumed that:

$$E(\xi_{jmt}\ \xi_{jm't}) = 0 \qquad \text{for any } m, m'$$

- After controlling for brand-city fixed effects, all the correlation between prices at different locations comes from correlation in costs, and not from spatial correlation in demand shocks.

- Under these assumptions we can use average prices in other local markets, $\overline{P}_{j(-m)t}$, as instruments, where:

$$\overline{P}_{j(-m)t} = \frac{1}{M-1} \sum_{m' \neq m} p_{jm't}$$

4. Some Limitations of Demand Systems in Product Space

Some Limitations of Demand Systems in Product Space

- For the type of empirical questions in which we are interested in Empirical IO, demand systems in product space have several practical limitations.

    1. **Representative consumer assumption.**
    2. **Too many parameters.**
    3. **Finding instruments for prices.**
    4. **Problems to predict demand of new varieties.**

[1] Representative consumer assumption

- Very unrealistic. Propensity to substitute between different products is very heterogeneous across consumers.
- Ignoring this heterogeneity can generate substantial biases.
- In principle, the model can be applied to consumer/household level data. However:
  - Household-level data is often not available for some products / industries.
  - At the lower-stage, observed household choices seem discrete (only one variety) and this is at odds with this "continuous choice" model.

[2] Too many parameters

- The number of parameters is $2J + \frac{J(J+1)}{2}$, i.e., J intercept parameters (α); J income elasticities (γ); and $\frac{J(J+1)}{2}$ free price elasticities (β).
- It is not possible to estimate demand systems for differentiated products with many varieties.
- For instance, demand system for car models. With $J = 100$, the $\#$parameters $= 5,250$.
- We need many thousands of observations (markets or/and time periods) to estimate this model. This type of data is typically not available.

[3] Finding instruments for prices

- Most applications of this class of models have ignored the potential endogeneity of prices.
- However, it is well known and simultaneity and endogeneity are potentially important issues in any demand estimation.
- The typical solution to this problem is using instrumental variables.
- In this model, the researcher needs at least as many instruments as prices, that is J.
- The ideal case is when we have information on production costs for each individual good. However, that information is very rarely available.

[4] Problems to predict demand of new varieties

- A problem that has received substantial attention is the prediction of the demand of a new product.
- Trajtenberg (1989), Hausman (1996), and Petrin (2002) are some prominent applications.
- In a demand system in product space, estimating the demand of a new good, say $J+1$, requires estimates of the parameters associated with that good: $\beta_{J+1}^{(0)}$, $\beta_{J+1}^{(y)}$ and $\{\beta_{J+1,j}^{(p)} : j = 1, 2, ..., J+1\}$.
- This makes it impossible to make counterfactual predictions, i.e., predict the demand of a product that has not been introduced in any market yet.
- It also limits the applicability of this model in cases where the new product has been introduced very recently or in very few markets, because we may not have enough data to estimate these parameters.