Empirical Industrial Organization (ECO 310)
University of Toronto. Department of Economics
Fall 2018. Instructor: Victor Aguirregabiria

FINAL EXAM

Tuesday, December 18th, 2018. From 7pm to 9pm (2 hours)

Exam Reminders:

- Fill out your name and student number on the top of this page.
- Do not begin writing the actual exam until the announcements have ended and the Exam Facilitator has started the exam.
- As a student, you help create a fair and inclusive writing environment. If you possess an unauthorized aid during an exam, you may be charged with an academic offence.
- Turn off and place all cell phones, smart watches, electronic devices, and unauthorized study materials in your bag under your desk. If it is left in your pocket, it may be an academic offence.
- When you are done your exam, raise your hand for someone to come and collect your exam. Do not collect your bag and jacket before your exam is handed in.
- If you are feeling ill and unable to finish your exam, please bring it to the attention of an Exam Facilitator so it can be recorded before leaving the exam hall.
- In the event of a fire alarm, do not check your cell phone when escorted outside.

Special Instructions:

- This is a closed-book exam.
- No study aids, including calculators, are allowed.
- Please, try to answer all the questions.

Exam Format and Grading Scheme:

- The exam has two parts, and each part is divided into 5 questions.
- Each question counts 10 points for your final grade.
- The total number of points is 100.

PART 1 [50 points]. Consider a Logit model for the demand of a differentiated product, e.g., automobiles.

Question 1. [10 points] Write the equation for the utility of purchasing a product, say product j.

(a) Explain the different components of this utility.

ANSWER: The equation for the consumer utility of purchasing product j is: $u_j = x_j\beta - \alpha p_j + \xi_j + \varepsilon_j$, where: x_j is a vector of characteristics of product j other than price; p_j is the price of the product; β and α are parameters; ξ_j represents product characteristics which are observable to consumers but not to researcher; ε_j represents consumer idiosyncratic taste for product j. Under the logit model, $\varepsilon_j's$ are independently and identically distributed with Extreme Value type I distribution.

(b) What is the interpretation of the slope parameter associated to a product characteristic, e.g., an automobile fuel consumption?

ANSWER: Let x_{1j} be the fuel consumption of product j, measured in liters of fuel per 100 km. And let β_1 be the parameter associated to x_{1j}. This parameter measures the marginal utility of fuel consumption. That is, it measures the number of "utils" per one additional liter of fuel consumption (per 100 km).

Question 2. [10 points] Every consumer chooses the product that maximizes her utility. The market share of a product, say product j, is the proportion of consumers buying that product. The market share of product 0 is the proportion of consumers not purchasing any of the J products.

(a) Write the demand equation in the Logit model that relates the market share of product j with the prices and the characteristics of all the products.

ANSWER: The demand equation is:

$$s_j = \frac{\exp\left\{x_j\beta - \alpha\ p_j + \xi_j\right\}}{\sum_{k=0}^{J}\exp\left\{x_k\beta - \alpha\ p_k + \xi_k\right\}}$$

(b) Show that we can combine these demand equations for product j and for product 0 to obtain a linear regression equation. Write the expression for this regression equation.

ANSWER: The average utility of product 0 (outside alternative) is normalized to zero: $\delta_0 = x_0\beta - \alpha\ p_0 + \xi_0 = 0$. Therefore, the equation for the market share of product 0 is:

$$s_0 = \frac{1}{\sum_{k=0}^{J}\exp\left\{x_k\beta - \alpha\ p_k + \xi_k\right\}}$$

The ration between s_j and s_0 is:

$$\frac{s_j}{s_0} = \exp\left\{x_j\beta - \alpha\ p_j + \xi_j\right\}$$

And taking logs:

$$\ln\left(\frac{s_j}{s_0}\right) = x_j\ \beta - \alpha\ p_j + \xi_j$$

This equation has the form of a linear regression equation where the regressors are x_j and p_j, the parameters are β and α, and the error term is ξ_j.

Question 3. **[10 points]** Consider the linear regression equation that you have obtained in Question 2.

(a) Explain why OLS estimation does not provide a consistent (unbiased) estimator of the parameters of this demand model.

ANSWER: The OLS estimator is consistent only if the regressors are not correlated with the error term. However, expect the price p_j to be correlated with the error term ξ_j. Products with higher value of ξ_j have higher demand, and this implies a higher price in equilibrium.

(b) Consider an instrumental variables estimator of this equation. Propose specific instrumental variables. Explain why it is plausible to argue that your proposed instruments are valid.

ANSWER: Suppose that the observable characteristics x, other than price, are not correlated with the error terms ξ. Then, a possible instrumental variable for price in this regression equation are the characteristics of other products. For instance, the vector of instruments $z_j = (J-1)^{-1}\sum_{k\neq j} x_k$. This variable z_j satisfies the different conditions for a valid instrument: (1) it does not enter as an explanatory variable in the demand equation of product j; (2) we expect z_j to be correlated with the price p_j, because in equilibrium this price depends on the characteristics of competing products; and (3) under the assumption above, z_j is not correlated with the error term ξ_j.

Question 4. **[10 points]** Suppose that you have a STATA dataset for one market and J products with the following variables: q (quantity sold of the product); p (price); x1, x2, ..., xK (K characteristics of the products); H (number of consumers in the market).

(a) Using STATA programming language, write the code lines to construct the dependent variable of the regression model in Question 3.

ANSWER: The code lines are:

```
gen share = q/H
egen sum_share = sum(share)
```

```
gen share_0 = 1 - sum_share
gen depvar = ln(share/share_0)
```

(b) Using STATA programming language, write the code line to implement the OLS estimator of this demand model.

ANSWER: The code lines are (suppose that K=4)

```
reg depvar x1 x2 x3 x4 p
```

(c) Using STATA programming language, write the code line to implement the IV estimator of this demand model.

ANSWER: The code lines are (including the construction of the vector of instruments $z_j = (J-1)^{-1} \sum_{k \neq j} x_k$).

```
egen x1_sum = sum(x1)
gen z1 = (x1_sum - x1)/(J-1)
```

... the same for the other instruments z2, z3, z4

```
ivregress 2sls depvar x1 x2 x3 x4 (p = z1 z2 z3 z4)
```

Question 5. [10 points] Suppose that there are J firms and each firm produces one and only one of the J products.

(a) Write the profit function of a firm, say firm j.

ANSWER: The profit function is: $\pi_j = p_j \ q_j - C_j(q_j)$.

(b) Suppose that firms compete ala Bertrand. Write the equation that describes the first order condition of profit maximization for firm j.

ANSWER: The first order conditions for profit maximization are:

$$q_j + p_j \frac{\partial q_j}{\partial p_j} - MC_j \ \frac{\partial q_j}{\partial p_j} = 0$$

where MC_j is the marginal cost. In the logit model (with price in levels), $\frac{\partial q_j}{\partial p_j} = -\alpha \ H \ s_j(1-s_j) = -\alpha \ q_j(1-s_j)$, such that the first order condition becomes:

$$q_j - [p_j - MC_j] \ \ \alpha \ q_j(1-s_j) = 0$$

or

$$p_j - MC_j = \frac{1}{\alpha(1-s_j)}$$

(c) Suppose that demand parameters have been estimated in a first step. Use the first order condition in (b) to write a linear regression model where you can estimate parameters in the marginal cost.

ANSWER: The first order condition implies:

$$MC_j = p_j - \frac{1}{\alpha(1 - s_j)}$$

Therefore, if the demand parameter α has been estimated, we have that $p_j - \frac{1}{\alpha(1-s_j)}$ provides an estimate of the realized marginal costs. Then, given an specification for the Marginal cost function we can use these realized marginal costs to estimate the parameters of the marginal cost function. For instance, if the marginal cost function is $x_j \ \gamma + \theta \ q_j + \omega_j$, where ω_j is and error term, we have the following linear regression equation:

$$p_j - \frac{1}{\alpha(1 - s_j)} = x_j \ \gamma + \theta \ q_j + \omega_j$$

PART 2 [50 points]. Consider the Conjectural Variation Model in the market for a homogeneous product, e.g., sugar.

Question 6. [10 points]

(a) Write the expression of the profit function of a firm in this industry.

ANSWER: The profit of firm j is: $\pi_j = P \ q_j - C_j(q_j)$, where P is the market price that is the same for all the firms because it is an homogeneous product.

(b) Define mathematically the "parameter" that represents the Conjectural Variation (CV) of a firm.

ANSWER: In an homogeneous product industry, the conjectural variation (CV) parameter of a firm, say j, represents this firm's belief about how all the rest of the firms will change their total amount of output when firm j changes marginally its own output. That is,

$$CV_j = \frac{\partial Q_{-j}}{\partial q_j}$$

where $Q_{-j} = \sum_{k \neq j} q_k$.

Question 7. [10 points]

(a) Obtain the equation that describes the marginal condition for profit maximization using a general value for the conjectural variation, CV.

ANSWER: The marginal condition for profit maximization is $\dfrac{\partial \pi_j}{\partial q_j} = 0$. Taking into account that $\pi_j = P \ q_j - C_j(q_j)$ and that $P = P(Q) = P(q_j + Q_{-j})$ where $P(.)$ is the inverse demand equation, we have that:

$$P + P'(Q) \ \ [1 + CV_j] \ \ q_j - MC_j = 0$$

where $P'(Q) = dP(Q)/dQ$ and MC_j is the marginal cost.

(b) What is the value of the CV under Nash-Cournot competition?
ANSWER: Under Nash-Cournot competition, $CV_j = 0$ for every firm j, such that we have:

$$P + P'(Q)\ q_j - MC_j = 0$$

(c) What is the value of the CV under Perfect competition?
ANSWER: Under perfect competition, $CV_j = -1$ for every firm j, such that we have:

$$P - MC_j = 0$$

(d) What is the value of the CV under Cartel (collusion)?
ANSWER: Under Collusion $CV_j = N - 1$ for every firm j, such that we have:

$$P + P'(Q)\ N\ q_j - MC_j = 0$$

Question 8. [10 points] Consider the equation and the definition of CV from Question 7.
(a) Based on this equation, solve for the CV to represent it as a function of price, marginal cost, output, and demand. Interpret this equation.
ANSWER: Solving for CV_j we have:

$$CV_j = \frac{P - MC_j}{-P'(Q)\ q_j} - 1$$

Given the slope of the demand curve $P'(Q)$, to rationalize a price cost margin $(P - MC_j)$ and an output q_j, we need a conjectural variation that increases with the price cost margin and declines with the firm's output.

(b) Consider a firm with a market share of 0.50 (50%) in a market with demand elasticity (in absolute value) equal to 2. What should be the value of the Lerner Index to conclude that the CV is consistent with Cournot competition? What is the conclusion if the Lerner index exceeds this value?
ANSWER: To answer this question, it is convenient to rewrite the previous expression for the CV in terms of the Lerner Index $\dfrac{P - MC_j}{P}$ and the absolute demand elasticity $|\eta| = -\dfrac{\partial Q}{\partial P}\dfrac{P}{Q} = \dfrac{P}{-P'(Q)\ Q}$. We have:

$$CV_j = \left[\frac{P - MC_j}{P}\right] \frac{1}{|\eta|}\ \frac{1}{q_j/Q} - 1$$

The question provides the following data: $|\eta| = 2$, $q_j/Q = 0.5$, and $CV_j = 0$ (Nash-Cournot). Therefore,

$$0 = \left[\frac{P - MC_j}{P}\right] \frac{1}{2} \frac{1}{1/2} - 1$$

And solving for the Lerner index:

$$\frac{P - MC_j}{P} = 1$$

Question 9. [10 points] Suppose that the researcher has estimated the demand function and firms' marginal costs in a first step, and then has used the equation in Question 8 to construct the CVs for every firm and every time period in the sample. Let CV* be the mean value of the CVs over all the firms and all the time periods in the sample.

(a) Consider the null hypothesis that CV* is equal to zero, and the alternative hypothesis that CV* $>$ 0. What is the economic interpretation of these hypotheses?

ANSWER: The null hypothesis CV*=0 means that firms compete a la Nash-Cournot. And the alternative hypothesis of CV*>0 means that there is some degree of collusion between the firms in this market.

(b) Explain how to test this null hypothesis under this alternative.

ANSWER: This is a t-test for the sample mean. Let CV* be the sample mean of the CVs and let S be the standard deviation of the the CVs. Then, under the null hypothesis CV*/S has a distribution with degrees of freedom the number of observations minus one, n-1. Suppose that we choose a significance level of 5%. Let $t(n-1, 0.05)$ be the value in the distribution with n-1 degrees of freedom that leaves 5% of probability to the right. Then, we do not reject the null hypothesis if CV*/S $\leq t(n-1, 0.05)$; and we reject the null hypothesis and accept the alternative if CV*/S $> t(n-1, 0.05)$.

Question 10. [10 points] Suppose that the researcher has estimated the demand function but she does not have data on firms' marginal costs. The researcher is interested in using the marginal conditions of profit maximization to estimate firms' marginal costs and conjectural variations. Suppose that the CV is the same for all the firms, and the marginal cost function is the same for all the firms and is linear in output.

(a) Using the marginal condition for profit maximization, shows that we can obtain a linear regression equation where the CV and the parameters in the marginal cost function are the parameters of this regression.

ANSWER: Remember that the f.o.c. is:

$$P_t + P'(Q_t)\ [1 + CV]\ q_{jt} - MC_{jt} = 0$$

Suppose that the marginal cost function is $MC_{jt} = \beta_0 + \beta_1 \; q_{jt} + \omega_{jt}$. Define the observable regressor $X_{jt} = -P'(Q_t) \; q_{jt}$ and the parameter $\theta = 1 + CV$. Then, we have the regression model:

$$P_t = \beta_0 + \beta_1 \; q_{jt} + \theta \; X_{jt} + \omega_{jt}$$

(b) Explain how to estimate consistently these parameters in this regression equation.

ANSWER: In this regression, both regressors q_{jt} and X_{jt} are endogenous: they are correlated with the unobserved component of the firm's cost, ω_{jt}. To estimate this equation we need instruments. We can use as instruments observable variables that affect the demand but do not enter in the cost function. Let Z_t be the vector with these observable demand variables. Note that to identify the parameters β_1 and θ separately we need that the slope of the demand $P'(Q_t)$ varies exogenously with these Z_t demand variables. Otherwise, if the slope of demand $P'(Q)$ is constant over the sample, we can identify $\beta_1 +$ $P'(Q) \; \theta$ but we cannot identify β_1 and θ separately.

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