Industrial Organization II (ECO 2901)

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Problem Set

Market Entry-Exit and Misallocation

Due of Friday, March 27, 2015

TOTAL NUMBER OF POINTS: 200

Model. Consider an homogeneous product industry. Time is discrete and indexed by t, and firms are indexed by i. Market demand is characterized by the inverse demand function $P_t = A_t - B\ Q_t$, where P_t is the market price, Q_t is the aggregate industry output, B is a parameter, and A_t is a demand shifter that follows an exogenous Markov process. Every firm uses the same production technology characterized by the (CRS) Cobb-Douglas production function, $q_{it} = L_{it}^{\alpha}\ K_{it}^{1-\alpha}\ e_{it}$, where L_{it} and K_{it} represent labor and capital, respectively, and e_{it} is the (potential) efficiency of firm i at period t that follows an exogenous Markov process. We assume that the production function parameter α is equal to $1/2$. There are perfectly competitive input markets with prices w_L and w_K, that we assume constant over time. In order to be active in the market, an inactive firm should purchase one unit of capital (e.g., a specific equipment) that remains constant during the time the firm stays active in the market. Therefore, for every active firm we have that $K_{it} = 1$, and for every inactive firm $K_{it} = 0$. At the beginning of period t, a firm is characterized by its productivity e_{it}, and by its incumbent status, $K_{it} \in \{0, 1\}$. Active firms at period t compete in quantities a la Cournot, and this competition determines firms' outputs and market price. At the end of the period, every firm (incumbents and potential entrants) decide whether to be active or not at next period, i.e., decide $K_{i,t+1}$. Note that there is one-period time-to-build in the decisions of market entry and exit, i.e., the decision is taken at period t but it is not effective until period $t + 1$. The period-profit of a firm is:

$$\pi_{it} = P_t\ q_{it} - w_L\ L_{it} - w_K\ (1 - K_{it})\ K_{i,t+1} + \varphi\ K_{it}\ (1 - K_{i,t+1})$$

where $w_K\ (1 - K_{it})\ K_{i,t+1}$ is the cost of entry for a firm that is currently inactive (i.e., $K_{it} = 0$ and $K_{i,t+1} = 1$), and $\varphi\ K_{it}\ (1 - K_{i,t+1})$ is the value of exit for a firm that is currently active and decides to exit (i.e., $K_{it} = 1$ and $K_{i,t+1} = 0$), where φ is a constant that represents the scrap value of used capital.

Problem 1 [Static Cournot Equilibrium] [25 points]

(a) [5 points] Define the variable profit function of an active firm. Write this function in terms of the firm's output.

(b) [10 points] For notational simplicity, define and use $w_{it} \equiv w_L/e_{it}^2$ as the *effective wage* of firm i. Describe the static Cournot model of competition. Obtain firms' best response functions. Derive the equations for the equilibrium industry output and for the equilibrium price in terms of exogenous variables and parameters.

[Note: Let N_t be the number of active firms at period t. To describe the Cournot equilibrium values, it is convenient to use the variable $N_t^*(w) \equiv \sum_{i=1}^{N_t} \frac{B+2w_L}{B+2w_{it}}$ that can defined as the effective number of active firms.]

(c) [5 points] Derive the Cournot equilibrium expressions for a firm's output and variable profit.

(d) [5 points] Suppose that $A_t = 100$, $B = 1$, $\alpha = 1/2$, and $w_L = 1$. There are $N_t = 4$ active firms in the market with productivities $e_{1t} = 0.9$, $e_{2t} = 1.0$, $e_{3t} = 1.1$, and $e_{4t} = 1.2$. Calculate the equilibrium values for market price, aggregate output, firms' market shares, and firms' variable profits.

Problem 2 [Dynamic Entry-exit decision with price taking firms] [100 points] We first consider the case where firms behave as price takers in the product market such that the price P_t follows an exogenous Markov process, and the optimal output of an active firm is $q_{it} = P_t/2w_{it}$, i.e., Price = Marginal Cost. Firms are forward-looking and make their entry and exit decisions to maximize their discounted stream of current and future expected profits. The time discount factor is $\beta \in (0, 1)$. For simplicity, suppose that a firm that exits from the market will never enter again.

(a) [10 points] Describe the dynamic decision problem of an incumbent firm ($K_{it} = 1$). Describe the vector state variables, and the Bellman equation that implicitly defines the value of an incumbent firm. [Note: Keep in mind that the value if an active firm is equal to its current profit plus the option value of staying in the market].

(b) [5 points] Suppose that potential entrants have only one chance to enter in the market, such that if they decide not to enter their future value is zero. Describe the dynamic decision problem of a potential entrant ($K_{it} = 0$).

(c) [25 points] Suppose that P_t can take two values, $P_L = 40$ and $P_H = 60$, and the transition probabilities between these states are $\Pr(P_{t+1} = P_L \mid P_t = P_L) = 0.8$, and $\Pr(P_{t+1} = P_H \mid P_t = P_H) = 0.8$. Similarly, suppose that w_{it} can take 9 values: $w_{it} \in \{0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.0,\ 1.2,\ 1.4,\ 1.6,\ 1.8\}$, and the transition probability $\Pr(w_{it+1}|w_{it})$ is defined by the following conditions: (i) for any value w, $\Pr(w_{it+1} = w|w_{it} = w) = 0.8$; (ii) for $w \in \{0.4,\ 0.6,\ 0.8,\ 1.0,\ 1.2,\ 1.4,\ 1.6\}$, $\Pr(w_{it+1} = w + 0.2|w_{it} = w) = 0.1$, and $\Pr(w_{it+1} = w - 0.2|w_{it} = w) = 0.1$; (iii) for $w = 0.2$, $\Pr(w_{it+1} = w + 0.2|w_{it} = w) = 0.2$; and (iv) for $w = 1.8$, $\Pr(w_{it+1} = w - 0.2|w_{it} = w) = 0.2$. The values of the rest of the parameters of the model are:

$$\alpha = 0.5;\ w_L = 1;\ \beta = 0.95;\ w_K = 500;\ \varphi = 350;$$

Given these parameters, write a computer program that solves for the value function of an incumbent firm (i.e., a vector of $2 * 9 = 18$ values).

(d) [10 points] Define the following threshold values for the effective wage of a firm: $w_{entry}(P_t)$ is the maximum wage such that a potential decides to enter in the market given that current price is P_t; $w_{stay}(P_t)$

is the maximum wage such that an incumbent decides to stay in the market given that current price is P_t. (i) Provide a theoretical characterization of $w_{entry}(P_t)$ and $w_{stay}(P_t)$. (ii) Provide a formal proof of the following the following result: for $w_K \geq \varphi$, we have that $w_{stay}(P_t) \geq w_{entry}(P_t)$, and $w_{entry}(P_t) = w_{stay}(P_t)$ if and only if $w_K = \varphi$. (iii) Interpret the distance $w_{stay}(P_t) - w_{entry}(P_t)$ as a measure of resource misallocation.

(e) [10 points] (i) Using the numerical solution of the model in Problem 2(c), compute the values $w_{stay}(P_t)$ and $w_{entry}(P_t)$ for $P_t = P_L$ and $P_t = P_H$. Comment the result. (ii) Repeat the exercise for values of w_K and φ that are 50% larger. Comment the result.

(f) [10 points] Suppose that we extend the model to incorporate i.i.d. idiosyncratic shocks in the costs of entry and in the scrap value such that $w_{K,it} = w_K + \varepsilon_{it}^{EC}$ and $\varphi_{it} = \varphi + \varepsilon_{it}^{SV}$, where ε_{it}^{EC} and ε_{it}^{SV} are independent and i.i.d. normal random variables, independent of (P_t, w_{it}) and with distributions $N(0, \sigma_{EC}^2)$ and $N(0, \sigma_{SV}^2)$, respectively. Define the following probabilities of entry and exit:

$$\begin{aligned} P_{entry}(P_t) &= \Pr\left(w_K + \varepsilon_{it}^{EC} \leq \beta\, \mathbb{E}\left[V(P_{t+1}, w_{i,t+1}) \mid P_t, w_{it}\right]\right) \\ P_{stay}(P_t, w_{it}) &= \Pr\left(\varphi + \varepsilon_{it}^{SV} \leq \beta\, \mathbb{E}\left[V(P_{t+1}, w_{i,t+1}) \mid P_t, w_{it}\right]\right) \end{aligned}$$

where $V(P_t, w_{it})$ is the value of an incumbent firm. Suppose that $\sigma_{SV} = \sigma_{EC} = 10$. Using the numerical solution of the model in Problem 2(c), obtain the probabilities of entry and stay for every value of the state variables. Note that the probability of entry is defined by integrating both over ε_{it}^{EC} and over the current w_{it} of the potential entrant. We assume that potential entrants take draws from the ergodic (steady-state) distribution of w_{it}. You have to define and compute that ergodic distribution from the transition probability of w_{it}.

(g) [30 points] Using the numerical solution of the model in Problem 2(c), and the entry-exit probabilities in Problem 2(f), write computer to generate simulations of the model for 1000 firms over 20 periods. Use these simulated data to estimate structurally the parameters of the model. You can select the estimation method that you prefer, but you need to explain the method and provide the computer code.

Problem 3 [Dynamic Entry-exit decision with Cournot competition] [75 points] Now, consider the dynamic model entry-exit but where every period firms compete a la Cournot as in Problem 1. The demand shifter A_t follows an exogenous Markov process. As before, we assume that a firm that exits from the market will never enter again, and that a potential entrant has only one chance to enter the market.

(a) [15 points] Describe the dynamic decision problem of an incumbent firm ($K_{it} = 1$). Describe the vector state variables, and the Bellman equation that implicitly defines the value of an incumbent firm. Explain the differences with Problem 2(a).

(b) [10 points] Describe the dynamic decision problem of a potential entrant ($K_{it} = 0$). Describe the equation that defines the value of a potential entrant. Explain the differences with Problem 2(b).

(c) [20 points] Now, consider a simplified version of this dynamic oligopoly game. Note that, in the static Cournot equilibrium, the profit of a firm is a known function $\pi(P_t, N_t^*(w), w_{it})$ where $N_t^*(w)$ is the effective number of active firms as defined in Problem 1. In particular, the effect on the profit of firm i of other firms' productivities (wages) takes place through the variable $N_t^*(w)$. The simplifying assumption is the following: when a firm makes its prediction about next period state variables to obtain its own next period value as an incumbent, it assumes that next period value of $N_t^*(w)$ is the same as current value, i.e., $N_{t+1}^*(w) = N_t^*(w)$. For this restricted model, describe the dynamic decision problem of an incumbent firm and the Bellman equation that implicitly defines the value of an incumbent firm. Describe the dynamic decision problem of a potential entrant.

(d) [30 points] Consider the simplified Dynamic Cournot model in Problem 3(c). Suppose that A_t can take two values, $A_L = 80$ and $A_H = 120$, and the transition probabilities between these states are $\Pr(A_{t+1} = A_L \mid A_t = A_L) = 0.8$, and $\Pr(A_{t+1} = A_H \mid A_t = A_H) = 0.8$. The rest of the values of the primitives are the same in Problem 2(c). Given these parameters, write a computer program that solves for the value function of an incumbent firm and for the value function of a potential entrant. Note that $N_t^* \in (0, 1, 2, ...)$ should be included as a state variable.