Industrial Organization II (ECO 2901)

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Problem Set #1. Static Entry Models

Due on Thursday, February 11, 2010

The Stata datafile `eco2901_problemset_01_chiledata_2010.dta` contains a panel dataset of 167 local markets in Chile with annual information over the years 1994 to 1999 and for five retail industries: Restaurants ('Restaurantes,' product code 63111); Gas stations ('Gasolineras,' product code 62531); Bookstores ('Librerias,' product code 62547); Shoe Shops ('Calzado,' product code 62411); and Fish shops ('Pescaderias,' product code 62141). The 167 "isolated" local markets in this dataset have been selected following criteria similar to the ones in Bresnahan and Reiss (1991). This is the list of variables in the dataset with a brief description of each variable:

`comuna_code`	:	Coder of local market
`comuna_name`	:	Name of local market
`year`	:	Year
`procode`	:	Code of product/industry
`proname`	:	Name of product/industry
`pop`	:	Population of local market (in # people)
`areakm2`	:	Area of local market (in square Km)
`expc`	:	Annual expenditure per capita in all retail products in the local market
`nfirm`	:	Number of firms in local market and industry at current year
`nfirm_1`	:	Number of firms in local market and industry at previous year
`entries`	:	Number of new entrants in local market and industry during current year
`exits`	:	Number of exiting firms in local market and industry during current year

Consider the following static entry model in the spirit of Bresnahan and Reiss (JPE, 1991, hereinafter *BR-91*). The profit of an active firm in market m at year t is:

$$\Pi_{mt} = S_{mt}\ v(n_{mt}) - F_{mt}$$

where S_{mt} is a measure of market size; n_{mt} is the number of firms active in the market; $v(.)$ is the *variable profit per capita* and it is a decreasing function; and F_{mt} represents fixed operating costs in market m at period t. The function $v(.)$ is nonparametrically specific. The specification of market size is:

$$S_{mt} = POP_{mt}\ \exp\left\{\beta_0^S + \beta_1^S\ expc_{mt} + \varepsilon_{mt}^S\right\}$$

where POP_{mt} is the population in the local market; $expc_{mt}$ is per capita sales in all retail industries operating in the local market; β_0^S and β_1^S are parameters; and ε_{mt}^S is an unobservable component of market size. The specification of the fixed cost is:

$$F_{mt} = \exp\left\{\beta^F + \varepsilon_{mt}^F\right\}$$

where β^F is a parameter, and ε_{mt}^F in an unobservable component of the fixed cost. Define the unobservable $\varepsilon_{mt} \equiv \varepsilon_{mt}^S - \varepsilon_{mt}^F$. And let $X_{mt} \equiv (\ln POP_{mt}, expc_{mt})$ be the vector with the observable characteristics of the local market. We assume that ε_{mt} is independent of X_{mt} and iid over (m,t) $N(0,\sigma^2)$.

Question 1. [**10 points**] Show that the model implies the following probability distribution for the equilibrium number of firms: let $n_{\max}$ be the maximum value of n_{mt}, then for any $n \in \{0, 1, ..., n_{\max}\}$:

$$\begin{aligned}
\Pr(n_{mt} = n \mid X_{mt}) &= \Pr\left(cut(n) \leq X_{mt} \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\beta_1^S}{\sigma} \end{bmatrix} + \frac{\varepsilon_{mt}}{\sigma} \leq cut(n+1) \right) \\
&= \Phi\left(cut(n+1) - X_{mt} \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\beta_1^S}{\sigma} \end{bmatrix} \right) - \Phi\left(cut(n) - X_{mt} \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\beta_1^S}{\sigma} \end{bmatrix} \right)
\end{aligned}$$

where $cut(0)$, $cut(1)$, $cut(2)$, ... are parameters such that for $n \in \{1, 2, ..., n_{\max}\}$, $cut(n) \equiv (\beta^F - \beta_0^S - \ln v(n))/\sigma$, and $cut(0) \equiv -\infty$, and $cut(n_{\max}+1) \equiv -\infty$.

Question 2. [**20 points**] Given the Ordered Probit structure of the model, estimate the vector of parameters $\{1/\sigma,\ \beta_1^S/\sigma,\ cut(1),\ cut(2),\ ...,\ cut(n_{\max})\}$ for each of the five industries separately. Given these estimates, obtain estimates of the parameters $\dfrac{v(n+1)}{v(n)}$ for $n \in \{1, 2, ..., n_{\max}\}$. Present a figure of the estimated function $\dfrac{v(n+1)}{v(n)}$ for each of the five industries. Interpret the results. Based on these results, what can we say about *the nature of competition* in each of these industries?

Question 3. [**20 points**] Repeat the same exercise as in Question 3 but using the following specification of the unobservable ε_{mt}:

$$\varepsilon_{mt} = \gamma_t + \delta_m + u_{mt}$$

where γ_t are time effects that can be captured by using time-dummies; δ_m are fixed market effects that can be captured by using market-dummies; and u_{mt} is independent of X_{mt} and iid over (m,t) $N(0,\sigma^2)$. Comment the results.

Now, consider the following static entry model of incomplete information. There are N_{mt} potential entrants in market m at period t. The profit of an active firm in market m at year t is:

$$\Pi_{imt} = S_{mt}\ v(n_{mt}) - F_{imt}$$

Market size, S_{mt}, has the same specification as in Question 2. The firm-specific fixed cost, F_{mt}, has the following specification:

$$F_{imt} = \exp\left\{\beta^F + \varepsilon_{mt}^F + \xi_{imt}\right\}$$

The random variables ε_{mt}^S, ε_{mt}^F, and ξ_{imt} are unobservable to the researcher. From the point of view of the firms in the market, the variables ε_{mt}^S and ε_{mt}^F are common knowledge, while ξ_{imt} is private information of firm i. We assume that ξ_{imt} is independent of X_{mt} and iid over (m,t) $N(0,\sigma_\xi^2)$.

The number of potential entrants, N_{mt}, is assumed to be proportional to population: $N_{mt} = \lambda\ POP_{mt}$, where the parameter λ is industry specific.

Question 4. [5 points] Consider the following estimator of the number of potential entrants:

$$\hat{N}_{mt} = \text{integer}\left\{ \max_{\text{over all}\{m',t'\}} \left[\frac{entrants_{m't'} + incumbents_{m't'}}{POP_{m't'}} \right] \ POP_{mt} \right\}$$

where $entrants_{m't'}$ and $incumbents_{m't'}$ are the number of new entrants and the number of incumbents, respectively, in market m' at period t'. Show that $\hat{N}_{mt}$ is a consistent estimator of $N_{mt} = \lambda$ POP_{mt}.

Question 5. [15 points] Let $P(X_{mt}, \varepsilon_{mt})$ be the equilibrium probability of entry given the common knowledge variables $(X_{mt}, \varepsilon_{mt})$. And let $G(n|X_{mt}, \varepsilon_{mt})$ be the distribution of the number of active firms in equilibrium conditional on $(X_{mt}, \varepsilon_{mt})$ and given that one of the firms is active with probability one. (i) Obtain the expression of the probability distribution $G(n|X_{mt}, \varepsilon_{mt})$ in terms of the probability of entry $P(X_{mt}, \varepsilon_{mt})$. (ii) Derive the expression for the expected profit of an active firm in terms of the probability of entry. (iii) Obtain the expression of the equilibrium mapping that defines implicitly the equilibrium probability of entry $P(X_{mt}, \varepsilon_{mt})$.

NOTE: *For Questions 6 and 7, consider the following approximation to the function* $\ln E(v(n_{mt})$ $| X_{mt}, \varepsilon_{mt}, 1$ *sure):*

$$\ln E(v(n_{mt})|X_{mt}, \varepsilon_{mt}, 1\text{sure}) \simeq \ln v(1) + \sum_{n=1}^{N_{mt}} G(n|X_{mt}, \varepsilon_{mt}) \ \left[\frac{v(n) - v(1)}{v(1)} \right]$$

This is a first order Taylor approximation to $\ln E(v(n_{mt})|X_{mt}, \varepsilon_{mt}, 1sure)$ *around the values* $v(1) = v(2) = ... = v(N)$, *i.e., no competition effects. The main advantage of using this approximation for estimation is that it is linear in the parameters* $\left[\frac{v(n)-v(1)}{v(1)}\right]$.

Question 6. [20 points] Suppose that $\varepsilon_{mt} \equiv \varepsilon_{mt}^S - \varepsilon_{mt}^F$ is just an aggregate time effect, $\varepsilon_{mt} = \gamma_t$. Use a two-step pseudo maximum likelihood method to estimate the vector of parameters:

$$\theta \equiv \left\{ \frac{1}{\sigma_\xi}, \ \frac{\beta_1^S}{\sigma_\xi}, \ \frac{\ln v(1) + \beta_0^S - \beta^F}{\sigma_\xi}, \ \frac{v(n) - v(1)}{\sigma_\xi \ v(1)} : n = 2, 3, ... \right\}$$

for each of the five industries separately. Given these estimates, obtain estimates of the parameters $\dfrac{v(n+1)}{v(n)}$ for $n \in \{1, 2, ..., n_{\max}\}$. Present a figure of the estimated function $\dfrac{v(n+1)}{v(n)}$ for each of the five industries. Interpret the results. Based on these results, what can we say about *the nature of competition* in each of these industries? Compare these results to those from the estimation of the *BR-91* models in Questions 2 and 3.

Question 7. [10 points] Repeat the same exercise as in Question 7 but using the following specification of the unobservable ε_{mt}:

$$\varepsilon_{mt} = \gamma_t + \delta_m$$

where γ_t are time effects that can be captured by using time-dummies; and δ_m are fixed market effects that can be captured by using market-dummies. Comment the results. Compare these results to those in Questions 2, 3, and 6.