

**INDUSTRIAL ORGANIZATION II (EC 2901)**

**University of Toronto. Department of Economics. Winter 2015  
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**FINAL EXAM (Take Home)  
Due on Thursday, April 9th, before midnight**

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**INSTRUCTIONS:** The exam consists of nine questions. All the questions deal with the same model. Please, answer all the questions.

**TOTAL MARKS = 125**

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Consider the industry of a differentiated product. A product in this industry can be described in terms of a vector of time-invariant characteristics:  $X_j$  is the vector of characteristics of product  $j$ . We refer to this observable vector of characteristics as the *product design*. The vector  $X_j$  belongs to a discrete and finite set  $\mathbb{X} = \{X_1, X_2, \dots, X_J\}$ , where  $J$  is total number of possible product designs. There is a fixed number  $N$  of firms in the industry, that we index by  $n$ . Each firm in the industry produces one or several products/designs. A firm's *product portfolio* is the set of designs that the firm produces. The researcher observes this industry over  $T$  periods of time, where  $T$  is large such that we can consider asymptotics as  $T \rightarrow \infty$ .

**Question 1 (10 points).** The demand of these products at some period  $t$  depends on observable characteristics  $X_j$ , on prices  $\{p_{jt}\}$ , and on some demand shocks  $\{\xi_{jt}\}$  that are unobservable to the researcher. Describe a discrete choice static model of consumer demand in this industry and its main assumptions. Explain how to estimate demand parameters in this model using aggregate market level data on quantities, prices, and observable product characteristics for all the products in the industry,  $\{q_{jt}, p_{jt}, X_j : j = 1, 2, \dots, J; t = 1, 2, \dots, T\}$ .

**Question 2 (10 points).** The (constant) marginal cost of producing product/design depends on its characteristics  $X_j$ , and on some cost shocks  $\{\omega_{jt}\}$  that are unobservable to the researcher. Describe a static model of price competition in this industry. Derive the expression for the Nash-Bertrand equilibrium conditions. Explain how to estimate the parameters in the marginal cost function using aggregate market level data  $\{q_{jt}, p_{jt}, X_j : j = 1, 2, \dots, J; t = 1, 2, \dots, T\}$ .

**Question 3 (20 points).** Suppose that the researcher observes that, during the sample period, new products enter in the market and some products exit. A firm's decision to introduce a new product, or to stop producing an existing product, is endogenous and responds to the firm's intention of maximizing profits. Explain how the endogeneity of the set of available products affects the estimation of demand in Question 1 and the estimation of marginal costs in Question 2. Describe an estimation method, and the necessary assumptions, to deal with this endogeneity problem.

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The product portfolio of firm  $n$  at period  $t$  can be described using a  $J \times 1$  vector  $\mathbf{a}_{nt} = \{a_{njt} : j = 1, 2, \dots, J\}$  with the binary indicators for the designs that the firm produces. For instance,  $J = 5$  and  $\mathbf{a}_{nt} = (0, 1, 0, 1, 0)$  means that firm  $n$  produces designs 2 and 4. Define  $\mathbf{a}_t = \{\mathbf{a}_{nt} : n = 1, 2, \dots, N\}$  as the vector that describes the product portfolios of all the firms at period  $t$ . Similarly, let  $\boldsymbol{\xi}_t = \{\xi_{jt} : j = 1, 2, \dots, J\}$  and  $\boldsymbol{\omega}_t = \{\omega_{jt} : j = 1, 2, \dots, J\}$  be the unobservable exogenous shocks in demand and marginal costs, respectively, for all the active products.

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**Question 4 (5 points).** Let  $VP_n(\mathbf{a}_t, \boldsymbol{\xi}_t, \boldsymbol{\omega}_t)$  be the Nash-Bertrand equilibrium variable profit for firm  $n$ , given the predetermined state variables variables  $(\mathbf{a}_t, \boldsymbol{\xi}_t, \boldsymbol{\omega}_t)$ . Explain how to compute these equilibrium variable profits for different values of the state variables given estimates of parameters in demand and marginal cost.

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Consider the following equilibrium model for the determination of firms' product designs. The period-profit for a firm is:

$$\Pi_{nt} = \pi_n(\mathbf{a}_t, \boldsymbol{\xi}_t, \boldsymbol{\omega}_t) - FC(\mathbf{a}_{nt}) - AC(\mathbf{a}_{nt}, \mathbf{a}_{nt-1}) - \varepsilon_{nt}(\mathbf{a}_{nt})$$

$FC(\mathbf{a}_{nt})$  represents fixed costs.  $\varepsilon_{nt}(\mathbf{a}_{nt})$  is an unobservable component of fixed costs that is private information of firm  $n$  at period  $t$  and is unobservable to the researcher. And  $AC(\mathbf{a}_{nt}, \mathbf{a}_{nt-1})$  represents adjustment costs, i.e., costs of introducing new products and removing existing products. The observed value of the vector  $\mathbf{a}_t$  is the result of a static Bayesian Nash equilibrium at period  $t$ . Every period  $t$ , firms observe previous period designs  $\mathbf{a}_{t-1}$ , the current state of demand and costs  $(\boldsymbol{\xi}_t, \boldsymbol{\omega}_t)$ , and their respective private information shocks  $\varepsilon_{nt}$ , and decide independently and simultaneously their product designs to maximize expected period-profits. The private information shocks  $\varepsilon_{nt}(a)$  are i.i.d. over  $(n, t, a)$  with type 1 extreme value distribution.

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**Question 5 (10 points).** Propose a parametric parsimonious specification of the fixed cost function  $FC(\mathbf{a}_{nt})$  with the following properties: (a) two firms with the same product portfolio have the same fixed cost; (b) it depends on observable product characteristics  $X_j$ ; (c) the fixed cost increases with the number of products, and the cost may increase more or less than proportionally with the number of products, i.e., (dis)economies of scope; (d) producing similar products (as described by the vector  $X_j$ ) may be cheaper than producing products with very different attributes, i.e., economies of density. Interpret the parameters of this function. [Note: it is important that the specification is flexible but parsimonious such that it can be precisely estimated using the available data. There is a specification with 3 parameters that satisfies conditions (c) and (d), and a specification with  $3 + \dim(X_j)$  parameters that satisfies conditions (a) to (d), where  $\dim(X_j)$  is the number of variables in the vector  $X_j$ .]

**Question 6 (10 points).** Propose a parsimonious specification of the adjustment cost function  $AC(\mathbf{a}_{nt}, \mathbf{a}_{nt-1})$  with the following properties: (a) two firms with the same values of  $(\mathbf{a}_{nt}, \mathbf{a}_{nt-1})$  have the same adjustment cost; (b) adjustment costs are zero if and only if there are no adjustments, i.e., if  $\mathbf{a}_{nt} = \mathbf{a}_{nt-1}$ ; (c) the cost of introducing a new product (and the cost of stopping an existing product) depends on its observable characteristics  $X_j$ ; (d) adjustment cost increases with the number of new products (and with the number of stopping products), and it may increase more or less than proportionally with the number of products; (e) introducing (stopping) similar new products may be cheaper than introducing new products with very different attributes. Interpret the parameters of this function. [Note: There is a specification with 6 parameters that satisfies conditions (d) and (e), and a specification with  $6 + \dim(X_j)$  parameters that satisfies conditions (a) to (d)].

**Question 7 (20 points).** Suppose that each firm has "adaptive beliefs" about the product portfolio decisions of other firms in the market. That is, at period  $t$ , firm  $n$  believes that for any other firm  $m \neq n$ , the decision is  $\mathbf{a}_{mt} = \mathbf{a}_{m,t-1}$ . Obtain the best response Conditional Choice Probabilities of a firm given these beliefs. Show the identification of parameters in your specification of the fixed cost and adjustment costs functions and under this restriction on firms' beliefs. Describe an estimator of these parameters.

**Question 8 (20 points).** Now, suppose that firms have rational (equilibrium) beliefs about the product portfolio decisions of other firms in the market. Describe a Bayesian Nash equilibrium in terms of firms' Conditional Choice Probabilities. Provide conditions for the identification of parameters in the fixed cost and adjustment cost functions. Describe an estimator of these parameters.

**Question 9 (20 points).** A researcher is interested in using the model estimated in Question 7 to predict the short-run and long-run effects of an hypothetical (counterfactual) merger between two firms, say firms  $A$  and  $B$ . More specifically, the researcher is interested in obtaining a prediction of the evolution after the merger of the product portfolio of the new firm  $AB$ , its prices, markets share, and profits. Describe a procedure to implement this counterfactual experiment.