CHAPTER 4

Static Models of Competition in Prices and Quantities

1. Introduction

In most industries, the main source of strategic interactions between firms comes from firms' price and quantity decisions. In a differentiated product industry, consumer demand for a firm's product depends on the prices of other products sold by other firms in the industry. In the case of an homogeneous good, the market price for this good depends on the total quantity produced by all the firms in the industry. This type of strategic interactions have first order importance to understand competition and outcomes in most industries. For this reason, models of competition where firms choose prices or quantities are at the core of Industrial Organization.

From an empirical point view, there are two main purposes in the estimation of models of competition in prices/quantities. First, the estimation of these models provides estimates of firms' marginal costs, and of the structure of these costs such economies of scale or different forms of economies of scope. This is the most common approach to obtain estimates of variable costs, since researches rarely have direct information on firms' costs. Given an assumption about competition (e.g., Cournot, Bertrand, Stackelberg, Monopolistic Competition, Collusion), the model predicts that a firm's marginal cost should be equal to a particular (model specific) marginal revenue. This is the key condition that is used to estimate firms' marginal costs in this class of models. Typically, the first step in the econometric analysis of these models consists in the estimation of the demand function or demand system. Given the estimated demand, we can construct an estimate of the realized marginal revenue for every observation in the sample. This measure of marginal revenue provides, directly, an estimate of the realized marginal cost at each sample observation. Finally, we use this sample of realized marginal costs to estimate the marginal cost function, and in particular how the marginal cost depends on firm's output of different products (i.e., economies of scale and scope), and possibly on other firm's characteristics such as historical cumulative output (i.e., learning by doing), maximum capacity, R&D investments, or geographic distance between the firm's production plants (i.e., economies of density).

As we described in the previous paragraph, the estimation of marginal costs is typically based on an assumption about competition, or what it is typically described as *the nature* *of competition* in an industry. In fact, the researcher's selection of a model of competition typically implies the choice of an specific answer on each of the following questions: (a) is the product homogeneous or differentiated; (b) do firms compete in prices or in quantities?; (c) is there collusion between some or all the firms in the industry?; and (d) what does a firm believe about the behavior of other firms in the market? For instance, if the model is Nash-Cournot, the researcher assumes that the product is homogenous, firms compete in quantities, there is no collusion in the industry, and firms choose their levels of output under the belief that the other firms will not change their respective output levels (i.e., Nash assumption). In principle, some of these assumptions may be supported by the researcher's knowledge of the industry. However, in general, some of these assumptions are difficult to justify. Furthermore, they may have important implications on our estimates of firms' costs, and perhaps more importantly, on our interpretation of competition in an industry and on our predictions about the effects of hypothetical changes in structural parameters or public policies. Ideally. we would like to learn from our data about the nature of competition. This is the main purpose of the so called *conjectural variation approach*. This approach tries to estimate simultaneously firms' costs and a set of parameters (i.e., conjectural variation parameters) that represent firms' believes and that describe the nature of competition in the industry. There are results in this literature that establish conditions for the joint identification of firms' costs and the conjectural variation parameters.

In this Chapter, we describe the specification and estimation of empirical models of Cournot competition in an homogenous product industry, Bertrand competition in a differentiated product industry, and the conjectutal variation approach both in homogenous and differentiated product industries.

2. Empirical models of Cournot competition

2.1. Model. Consider the industry of an homogenous product such as sugar. There are N firms active in the industry, that we index by $i \in \{1, 2, ..., N\}$. In this chapter we abstract from firms' decisions to be active or not in the market, and therefore we ignore fixed costs. We incorporate market entry decisions in Chapter 5. The variable profit of firm i is $\Pi_i = P\ q_i - C(q_i; Z_i, \omega_i)$, where P represents the market price of the homogeneous product, q_i is the amount of output produced by firm i, and $C_i(q_i; Z_i, \omega_i)$ is the variable cost of this firm. Z_i and ω_i are exogenous variables that affect the cost of firm i. Z_i contains variables that are observable to the researcher, such as input prices, the firm's maximum capacity, years of experience, or the geographic location of the firms' production plants. The variable ω_i captures heterogeneity in cost efficiency across firms that is unobservable to the researcher. The inverse demand function is $P = p\left(Q; X^D, \varepsilon^D\right)$, where $Q \equiv \sum_{i=1}^{N} q_i$ is market total output, and X^D and ε^D represent exogenous market characteristic that affect demand, where X^D are observable to the researcher and ε^D are unobservable. Define the marginal revenue function:

$$MR_i \equiv p\left(Q; X^D, \varepsilon^D\right) + \frac{\partial p\left(Q; X^D, \varepsilon^D\right)}{\partial Q} q_i \tag{2.1}$$

Firms in this industry compete a la Nash-Cournot. Firm i takes as given the quantity produced by the rest of the firms, $\widetilde{Q}_{-i}$, and chooses his own output q_i to maximize his profit. That is, firm i believes that $\frac{\partial \widetilde{Q}_{-i}}{\partial q_i} = 0$. We assume that the profit function $\Pi_i(q_i, \widetilde{Q}_{-i})$ is globally concave in q_i for any positive value of $\widetilde{Q}_{-i}$ such that there is a unique value of q_i that maximizes the firm's profit, and it is fully characterized by the marginal condition of optimality that establishes that marginal revenue equals marginal cost:

$$MR_i = MC(q_i; Z_i, \omega_i) \tag{2.2}$$

where $MC(q_i; Z_i, \omega_i) \equiv \partial C(q_i; Z_i, \omega_i)/\partial q_i$ represents the marginal cost for firm i. These conditions determine the amount of output of each firm as a function of the exogenous variables $(X^D, \varepsilon^D, Z_i, \omega_i : i = 1, 2, ...N)$.

2.2. Estimation. The researcher has data from M markets, indexed by m, with information on firms output, market price, and exogenous market and firms' characteristics:

$$Data = \left\{q_{im},\ Z_{im},\ X_m^D : i = 1, 2, ..., N_m;\ m = 1, 2, ..., M\right\} \tag{2.3}$$

Suppose that the demand function has been estimated in a fist step, such that there is a consistent estimate $\widehat{p}(.)$ of the demand function, and an estimated residual $\widehat{\varepsilon_m^D}$ of the error term in each market. Therefore, the researcher can construct consistent estimates of marginal revenues as:

$$\widehat{MR}_{im} = \widehat{p}\left(Q_m; X_m^D, \widehat{\varepsilon_m^D}\right) + \frac{\partial \widehat{p}\left(Q_m; X_m^D, \widehat{\varepsilon_m^D}\right)}{\partial Q} q_{im} \tag{2.4}$$

Consider a power specification of a firm's variable cost function: $C(q_i; Z_i, \omega_i) = \frac{1}{\gamma+1} q_i^{\gamma+1}$ $\exp\{Z_i\alpha + \omega_i\}$, such that the marginal cost function is $MC(q_i; Z_i, \omega_i) = q_i^{\gamma}$ $\exp\{Z_i\alpha + \omega_i\}$. Then, the econometric model can be described in terms of the following system linear regression functions:

$$\ln\left(\widehat{MR}_{im}\right) = \gamma\ \ln\left(q_{im}\right) + Z_{im}\ \alpha + \omega_{im} \tag{2.5}$$

We are interested in the estimation of the parameters α and γ. In particular, γ measures the degree of scale diseconomies (if $\gamma < 1$) or economies (if $\gamma > 1$). The estimation of firms' relative efficiency, $Z_{im}\ \alpha + \omega_{im}$, and of the sources of this efficiency (i.e., the value of each parameter in the vector α) is also of interest.

A main econometric issue in the estimation of this model is the endogeneity of a firm's output. The model implies that $E(\ln(q_{im})\ \omega_{im}) \neq 0$, and more specifically there is a negative correlation between a firm's output and its unobserved inefficiency. Therefore, and OLS estimator that ignores this correlation will generate downward biased estimates of the parameter γ, and the researcher may conclude that there are strong diseconomies of scale in the industry when really that is not the case. Under the assumption that the vector of firm characteristics in Z are exogenous, i.e., $E(Z_{jm}\ \omega_{im}) = 0$ for any (i,j), a natural approach to estimate this model is using GMM based on moment conditions that use the characteristics of other firms as an instrument for output. For instance, the moment conditions can be:

$$E\left(\begin{bmatrix} Z_{im} \\ \sum_{j\neq i} Z_{jm} \end{bmatrix} \left[\ln\left(\widehat{MR}_{im}\right) - \gamma\ \ln(q_{im}) - Z_{im}\ \alpha\right]\right) = \mathbf{0} \tag{2.6}$$

An alternative specification could consider that there is a market fixed effect, such that $\omega_{im} = \omega_m^{(1)} + \omega_{im}^{(2)}$, where the market fixed effect $\omega_m^{(1)}$ may be correlated with the observable exogenous variables Z, e.g., more profitable markets may attract firms with different variables Z (more efficient firms). In this model, the moment conditions shows be constructed for the equation in deviations with respect to the market means, i.e., $\widetilde{\ln\left(\widehat{MR}_{im}\right)} = \gamma\ \widetilde{\ln(q_{im})} + \widetilde{Z}_{im}$ $\alpha +\ \omega_{im}^{(2)}$.

2.3. An application.

3. Bertrand competition in a differentiated product industry

3.1. Model. Consider the industry of a differented. There are N firms active in the industry, and for the moment we consider that each firm produces only one variety of the differentiated product. The profit of firm i is $\Pi_i = p_i\ q_i(\mathbf{p}, \mathbf{X}, \boldsymbol{\xi}) - C(q_i(\mathbf{p}, \mathbf{X}, \boldsymbol{\xi}); X_i, \omega_i)$, where p_i is the price of product i, $q_i(\mathbf{p}, \mathbf{X}, \boldsymbol{\xi})$ represents the demand for this product that depends on vector of prices, $\mathbf{p}$, and other product characteristics, $\mathbf{X}$ and $\boldsymbol{\xi}$, for all the products in the industry. $C_i(q_i; X_i, \omega_i)$ is the variable cost of this firm, where X_i is the vector of observable product attributes, and ω_i captures heterogeneity in cost efficiency across firms that is unobservable to the researcher. Define the marginal revenue function:

$$MR_i \equiv p_i + \left[\frac{\partial q_i(\mathbf{p}, \mathbf{X}, \boldsymbol{\xi})}{\partial p_i}\right]^{-1} q_i(\mathbf{p}, \mathbf{X}, \boldsymbol{\xi}) \tag{3.1}$$

Firms in this industry compete a la Nash-Bertrand. Firm i takes as given the prices fixed by the rest of the firms, $\mathbf{p}_{-i}$, and chooses his own price p_i to maximize his profit. That is, firm i believes that $\frac{\partial \mathbf{p}_{-i}}{\partial p_i} = 0$. We assume that the profit function $\Pi_i(p_i, \mathbf{p}_{-i})$ is globally concave in p_i for any positive vector $\mathbf{p}_{-i}$ such that there is a unique value of p_i that maximizes the firm's profit, and it is fully characterized by the marginal condition of optimality that establishes that marginal revenue equals marginal cost:

$$MR_i = MC(q_i; X_i, \omega_i) \tag{3.2}$$

where $MC(q_i; X_i, \omega_i) \equiv \partial C(q_i; X_i, \omega_i)/\partial q_i$ represents the marginal cost for firm i. These conditions determine the amount of output of each firm as a function of the exogenous variables $(X_i, \xi_i, \omega_i : i = 1, 2, ...N)$.

3.2. Estimation. *** Very similar to Cournot case ****

*** Extend to multi-product case.

3.3. An application.

4. The Conjectural Variation Approach

4.1. Model: Homogeneous product.

- We first consider the CV model in an homogeneous product industry.
- The variable profit of firm i is

$$\Pi_i = p\left(Q; X^D, \varepsilon^D\right)\ q_i - C(q_i; Z_i, \omega_i)$$

- Suppose that firm i believes that $\dfrac{\partial \widetilde{Q}_{-i}}{\partial q_i} = \theta_i$, where θ_i is a parameter that represents the believes of firm i.
- The "perceived" MR of firm i is:

$$MR_i = p\left(Q; X^D, \varepsilon^D\right) + \frac{\partial p\left(Q; X^D, \varepsilon^D\right)}{\partial Q}\left[1 + \theta_i\right]\ q_i$$

- If we treat these beliefs θ_i as exogenous, we can define an equilibrium where q_i is a function of the exogenous variables $(X^D, \varepsilon^D, \theta_i, Z_i, \omega_i : i = 1, 2, ...N)$.
- F.O.C:

$$MR_i = MC(q_i; Z_i, \omega_i)$$

- where $MR_i = P + \frac{\partial p}{\partial Q}\left[1 + \theta_i\right] q_i$.
- The value of the parameters $\{\theta_i\}$ are related to the "nature of competition", i.e., Cournot, Perfect Competition, Bertrand, Stackelberg, or Cartel (Monopoly).

$$\text{PC / Bertrand:} \quad \theta_i = -1; \quad MR_i = P$$

$$\text{Cournot:} \quad \theta_i = 0; \quad MR_i = P + \frac{\partial p}{\partial Q} q_i$$

$$\text{Cartel } n < N \text{ firms:} \quad \theta_i = n - 1; \quad MR_i = P + \frac{\partial p}{\partial Q}\ n\ q_i$$

$$\text{Cartel all firms:} \quad \theta_i = N - 1; \quad MR_i = P + \frac{\partial p}{\partial Q}\ N\ q_i$$

********* THIS CHAPTER REVISED UP TO THIS POINT *******

In general, a firm's best response function (the decision that maximizes the firm's profit) is based on the condition that marginal revenue is equal to marginal cost. Under any form of (static) competition, the marginal revenue of a single firm that produces q units of output is:

$$MR = \frac{d(P\ q)}{dq} = P + \frac{dP}{dq}\ q$$

Given the linear (inverse) demand function $P = A - B\ Q$, and given that $Q = Q^{others} + q$, we have that:

$$\frac{dP}{dq} = -B\frac{dQ}{dq} = -B\left[1 + \frac{dQ^{others}}{dq}\right]$$

- The derivative $\frac{dQ^{others}}{dq}$ is called the **conjectural variations "parameter"**. It represents a firm's conjecture or belief about how other firms will respond when the firm changes his own amount of output. In CV approach $\frac{dQ^{others}}{dq}$ is treated as a constant parameter that we represent as cv.[1] Solving the expression of $\frac{dP}{dq}$ into the equation for the marginal revenue, we have:

$$MR = P - B\left[1 + cv\right]\ \frac{Q}{N}$$

The value of the parameter cv depends on the "nature" of competition, i.e., Cournot, Perfect Competition, Bertrand, Stackelberg, or Cartel (Monopoly).

$$\begin{aligned} \text{PC / Bertrand:} \quad & cv = -1; \quad MR = P \\ \text{Cournot:} \quad & cv = 0; \quad MR = P - \left(\tfrac{1}{N}\right)\ B\ Q \\ \text{Cartel } n < N \text{ firms:} \quad & cv = n - 1; \quad MR = P - \left(\tfrac{n}{N}\right)\ B\ Q \\ \text{Cartel all firms:} \quad & cv = N - 1; \quad MR = P - B\ Q \end{aligned}$$

Define the parameter $\theta \equiv \dfrac{1 + cv}{N}$. Then, we can write the marginal revenue function as:

$$MR = P - \theta\ B\ Q$$

[1]Note that $\frac{dQ^{others}}{dq}$ may not be a parameter. For instance, it might depend on the level of q. One of the assumptions, and of the limitations, of the CV approach is that it assumes that $\frac{dQ^{others}}{dq}$ is a constant parameter.

and θ can take the following values:

$$\begin{aligned} \text{PC / Bertrand:} &\quad \theta = 0; \quad MR = P \\ \text{Cournot:} &\quad \theta = \tfrac{1}{N}; \quad MR = P - \left(\tfrac{1}{N}\right)\ B\ Q \\ \text{Cartel } n < N \text{ firms:} &\quad \theta = \tfrac{n}{N}; \quad MR = P - \left(\tfrac{n}{N}\right)\ B\ Q \\ \text{Cartel all firms:} &\quad \theta = 1; \quad MR = P - B\ Q \end{aligned}$$

Note that the parameter θ is an inverse index of the degree of competition. θ is between between 0 and 1. $\theta = 0$ if firms behave as perfectly competitive firms or as Bertrand competitors in an homogeneous product industry. In the other extreme, $\theta = 1$ when all the firms collude such that the industry outcome is the monopoly outcome. In the middle, with Cournot competition $\theta = \frac{1}{N}$, and with collusion of a subgroup of n firms $\theta = \frac{n}{N}$. The weaker the degree of competition, the higher θ is.

4.2. Model: Differentiated product. *** Nevo. Econ Letters

4.3. Estimation. In the conjectural variation model, the condition marginal revenue equals marginal cost implies:

$$P_t - MC_t = \theta\ B_t\ Q_t$$

Now, the estimation of the demand does not provide a direct estimate of the price-cost margin and of the marginal cost. We need to estimate the parameter θ that measures the degree of competition.

To estimate θ, we combine the condition of marginal revenue equal to marginal cost ($P_t - MC_t = \theta\ B_t\ Q_t$) with our specification of the marginal cost function. Remember that:

$$MC_t = m_0 + m_1\ W_t + m_2\ \frac{Q_t}{K_t} + e_t^{MC}$$

Therefore, combining the two equations we have that:

$$P_t = m_0 + m_1\ W_t + m_2\ \frac{Q_t}{K_t} + \theta\ \left(\hat{B}_t Q_t\right) + e_t^{MC}$$

Note that $\hat{B}_t$ is the estimate of the slope of the demand at period t that comes from our estimation of the demand function. Therefore, $\hat{B}_t Q_t$ is a known regressor. This equation is a linear regression model with regressors or explanatory variables W_t, $\frac{Q_t}{K_t}$, and $\hat{B}_t Q_t$, and parameters m_0, m_1, m_2, and θ.

As before, the estimation of this equation by OLS will give us biased estimates of the parameters because Q_t is endogenous: that is, it is correlated with the error term e_t^{MC}. We need instrumental variables. As in the previous estimation of the marginal cost function, the demand "shifters" POP_t and I_t are valid instruments.

There are two other conditions that are important for the identification/estimation of the parameter θ: (1) the slope of the demand $\hat{B}_t$ changes over time; and (2) the marginal cost increases with the capacity utilization $\frac{Q_t}{K_t}$ and not just with total output Q_t. Suppose that these conditions do not hold. That is, suppose that the marginal cost function is $MC_t = m_0 + m_1\ W_t + m_2\ Q_t + e_t^{MC}$, and that the demand curve is $P_t = A_t - B\ Q_t$. Then, the equation that comes from the condition $MR_t = MC_t$ becomes:

$$P_t = m_0 + m_1\ W_t + (m_2 + \theta \hat{B})\ Q_t + e_t^{MC}$$

Now, the regressors are W_t and Q_t, instead of W_t, $\frac{Q_t}{K_t}$, and $\hat{B}_t Q_t$, as before. This means that we can identify/estimate the parameters m_0, m_1, and m_2^* where $m_2^* = (m_2 + \theta \hat{B})$. But given an estimate of m_2^*, we cannot estimate separately m_2 and θ. Suppose that the estimate of m_2^* is 0.06 and $\hat{B} = 0.02$. Then, we know $0.06 = m_2 + 0.02 * \theta$, but there are infinite combinations of m_2 and θ that satisfy this equation. For instance, both $\{\theta = 0$ and $m_2 = 0.06\}$ and $\{\theta = 1$ and $m_2 = 0.04\}$ satisfy that equation, but they have very different implications for the estimates of marginal cost and price cost margins.

In particular, the sample variation in the slope of the inverse demand, $\hat{B}_t$, plays a very important role in the identification of the CV parameter θ. The intuition is simple. Suppose that from week $t = 1$ to week $t = 2$ there is an important increase in the slope of the inverse demand: $\hat{B}_2 >> \hat{B}_1$. An increase in $\hat{B}_t$ means that the demand becomes less price sensitive, more inelastic. For a monopolist, when the demand becomes more inelastic, the optimal price should increase. In general, for a firm with high level of market power (high θ), we should observe an important increase in prices associated with an increase in the slope. On the contrary, if the industry is characterized by very low market power (low θ) the increases in prices should be practically zero. For any value of θ, the "ceteris paribus" change in price is $(P_2 - P_1) = \theta(\hat{B}_2 - \hat{B}_1)\hat{Q}_1$. Therefore, the response of prices to an exogenous change in the slope of the demand (i.e., $(P_2 - P_1)/(\hat{B}_2 - \hat{B}_1)\hat{Q}_1$) contains key information for the estimation of θ.

4.4. An application. Genesove and Mullin (GM) study competition in the US sugar industry during the period 1890-1914. Why this period? The reason is that for this period they can collect high quality information on the value of marginal costs. Two aspects play are important in the collection of information on marginal costs. First, the production technology of refined sugar during this period was very simple and the marginal cost function can be characterized in terms of a simple linear function of the cost of raw sugar, the main intermediate input in the production of refined sugar. Most importantly, during this period there was an important investigation of the industry by the US anti-trust authority. As a result of that investigation, there are multiple reports from expert witnesses that provide estimates about the structure and magnitude of production costs in this industry.

As we describe below, GM use this information on marginal costs to test the validity of the standard conjectural variation approach for estimation of price cost margins and marginal costs. Here I describe briefly the main idea for this approach.

Let $P_t = P(Q_t)$ be the inverse demand function in the industry. Under the conjectural variation approach, the marginal revenue at period t is:

$$MR_t = P_t - \theta_t \; Q_t \; \frac{dP(Q_t)}{dQ_t}$$

where $dP(Q_t)/dQ_t$ is the derivative of the inverse demand function, and θ_t is the conjectural variation $\theta_t = \frac{1}{N_t}\left(1 + \frac{dQ_t^{others}}{dq_t}\right)$. The condition for profit maximization (marginal revenue equals marginal cost) is $P_t - \theta_t \; Q_t \; \frac{dP(Q_t)}{dQ_t} = MC_t$, and it implies the following condition for the Lerner Index $(P_t - MC_t)/P_t$:

$$\frac{P_t - MC_t}{P_t} = \theta_t \; \frac{Q_t}{P_t}\frac{dP(Q_t)}{dQ_t}$$

or given that price elasticity of demand is $\varepsilon_t = \frac{P_t}{Q_t}\frac{dQ_t}{dP_t}$, we have:

$$\frac{P_t - MC_t}{P_t} = \frac{\theta_t}{\varepsilon_t}$$

According to this expression, market power, as measured by the Lerner Index, depends on the elasticity of demand and on the "degree of competition", as measured by the conjectural variation. Solving for θ_t in this expression, we have:

$$\theta_t = \left(\frac{P_t - MC_t}{P_t}\right) \varepsilon_t$$

Therefore, if we can estimate the demand elasticity ε_t, and we observe marginal cost MC_t, then we have a simple and direct estimate of the conjectural variation θ_t. Without information on MCs, the estimation of θ_t should be based: (a) on our estimation of demand, and in particular, on exclusion restrictions that permit the identification of demand parameters; and (b) on our estimation of the MC function, on exclusion restrictions that permit the identification of this function. If assumptions (a) or (b) are not correct, our estimation of θ_t, and therefore of the Lerner Index, will be biased. GM evaluate these assumptions by comparing the estimates of θ_t using information on MCs and not using that information.

The rest of these notes briefly describe and discuss the following points in GM paper: (a) The industry; (b) The data; (c) Estimates of demand parameters; and (d) Estimation of θ.

NOTE / QUESTION: Suppose that you have data on MCs such that you can obtain a direct estimate of the market power as measured by the price-cost margin or the Lerner index. Would you still be interested in the estimation of θ? In general, the answer is "yes". The reason is that there are many empirical questions that we may want to answer using our model for which we need to know the value of θ. For instance, we may be interested in the following predictions: what will the PCM be if the elasticity of the demand increases/declines? what will the PCM be if the MC increases/declines? To answer these questions we need to know the value of θ.

4.4.1. *The industry.* Homogeneous product industry. Highly concentrated during the sample period, 1890-1914. The industry leader, American Sugar Refining Company (ASRC), had more than 65% of the market share during most of these years.

Production technology. Refined sugar companies buy "raw sugar" from suppliers in national or international markets, transformed it into refined sugar, and sell it to grocers. They sent sugar to grocers in barrels, without any product differentiation. Raw sugar is 96% sucrose and 4% water. Refined sugar is 100% sucrose. The process of transforming raw sugar into refined sugar is called "melting", and it consists of eliminating the 4% of water in raw sugar. Industry experts reported that the industry is a "fixed coefficient" production technology:[2]

$$Q^{refined} = \lambda \ Q^{raw}$$

where $Q^{refined}$ is refined sugar output, Q^{raw} is the input of raw sugar, and $\lambda \in (0,1)$ is a technological parameter. That is, 1 ton of raw sugar generates λ tons units of refined sugar.

Marginal cost function. Given this production technology, the marginal cost function is:

$$MC = c_0 + \frac{1}{\lambda} \ P^{raw}$$

where P^{raw} is the price of the input raw sugar (in dollars per pound), and c_0 is a component of the marginal cost that depends on labor and energy. Industry experts unanimously report that the value of the parameter λ was close to 0.93, and c_0 was around \$0.26 per pound. Therefore, the marginal cost at period (quarter) t, in dollars per pound of sugar, was:

$$MC_t = 0.26 + 1.075 \ P_t^{raw}$$

4.4.2. *The data.* Quarterly US data for the period 1890-1914. The dataset contains 97 quarterly observations on industry output, price, price of raw sugar, imports of raw sugar, and a seasonal dummy.

$$\text{Data} = \{ \ Q_t, \ P_t, \ P_t^{raw}, \ IMP_t, \ S_t : t = 1, 2, ..., 97\}$$

IMP_t represents the imports of raw sugar from Cuba. And S_t is a dummy variable for the Summer season: $S_t = 1$ is observation t is a Summer quarter, and $S_t = 0$ otherwise.

[2]Actually, the fixed coefficient Leontieff production function is $Q^{refined} = \min\{\lambda \ Q^{raw} \ ; \ f(L,K)\}$ where $f(L,K)$ is a function of labor and capital inputs. However, cost minimization will generally imply that $Q^{refined} = \lambda \ Q^{raw} = f(L,K)$.

The summer was a high demand season for sugar because most the production of canned fruits was concentrated during that season, and the canned fruit industry accounted for an important fraction of the demand of sugar.

Based on this data, we can also obtain a measure of marginal cost as $MC_t = 0.26 + 1.075$ P_t^{raw}.

4.4.3. *Estimates of demand parameters.* GM estimate four different models of demand. The main results are consistent for the four models. Here I concentrate on the linear demand.

$$Q_t = \beta_t \ (\alpha_t - P_t)$$

And the inverse demand equation is:

$$P_t = \alpha_t - \frac{1}{\beta_t} \ Q_t$$

Therefore, using the inverse demand equation that we have used in class, $P_t = A_t - B_t Q_t$, we have that $\alpha_t = A_t$ and $\beta_t = \frac{1}{B_t}$. I will refer to β_t as the slope of the demand or the price sensitivity of the demand.

GM consider the following specification for α_t and β_t:

$$\begin{aligned} \alpha_t &= \alpha_L \ (1 - S_t) + \alpha_H \ S_t + e_t^D \\ \beta_t &= \beta_L \ (1 - S_t) + \beta_H \ S_t \end{aligned}$$

α_L, α_H, β_L, and β_H are parameters. α_L and β_L are the intercept and the slope of the demand during the "Low Season" (when $S_t = 0$). And α_H and β_H are the intercept and the slope of the demand during the "High Season" (when $S_t = 1$). e_t^D is an error term that represents all the other variables that affect demand and that we do not observe.

Therefore, we can write the following inverse demand equation:

$$\begin{aligned} P_t &= \left[\alpha_L \ (1 - S_t) + \alpha_H \ S_t + e_t^D\right] - \left[\frac{1}{\beta_L}(1 - S_t) + \frac{1}{\beta_H} S_t\right] Q_t \\ & \text{or} \\ P_t &= \alpha_L + (\alpha_H - \alpha_L) S_t + \frac{1}{\beta_L}(-Q_t) + \left(\frac{1}{\beta_H} - \frac{1}{\beta_L}\right)(-S_t Q_t) + e_t^D \end{aligned}$$

This is a regression equation where the explanatory variables are a constant term, S_t, Q_t, and $S_t Q_t$, and the parameters are α_L, $(\alpha_H - \alpha_L)$, $\frac{1}{\beta_L}$, and $\left(\frac{1}{\beta_H} - \frac{1}{\beta_L}\right)$. From the estimation of these parameters, we can recover α_L, α_H, β_L, and β_H.

As we have discussed before, Q_t is an endogenous regressor in this regression equation. We need to use IV to deal with this endogeneity problem. In principle, it seems that we could we P_t^{raw} as an instrument. However, GM have a reasonable concern about the validity of this instrument. The demand of raw sugar from the US accounts for a significant fraction of the world demand of raw sugar. Therefore, exogenous shocks in the demand of refined sugar (e_t^D) might generate an increase if the world demand of raw sugar and in P_t^{raw} such that $Cov(e_t^D, P_t^{raw}) \neq 0$. Instead they use imports of raw sugar from Cuba as an instrument: almost 100% of the production of raw sugar in Cuba was exported to US, and the authors claim that variations in Cuban production of raw sugar was driven by supply/weather conditions and not by the demand from US ... Definitely, the validity of this instrument is also arguable.

These are the parameter estimates.

Demand Estimates		
Parameter	Estimate	Standard Error
α_L	5.81	(1.90)
α_H	7.90	(1.57)
β_L	2.30	(0.48)
β_H	1.36	(0.36)

According to these estimates, in the high season the demand shifts upwards but it also becomes more inelastic. The estimated price elasticities of demand in the low and the high season are $\varepsilon_L = 2.24$ and $\varepsilon_H = 1.04$, respectively. According to this, any model of oligopoly competition where firms have some market power predicts that the price cost margin should increase during the price season due to the lower price sensitivity of demand.

Before we discuss the estimates of the conjectural variation parameter, θ, it is interesting to illustrate the errors that researchers can make if in the absence of information about marginal costs they estimate price cost margins by making an adhoc assumption about the value of θ in the industry. As mentioned above, the industry was highly concentrated during this period. Though there were approximately 6 firms active during most of the sample period, one of the firms accounted for more than two-thirds of total output. Suppose 3 different researchers of this industry, researcher M, researcher C, and researcher S. Researcher M considers that the industry was basically a Monopoly/Cartel during this period (in fact, there was anti-trust investigation, so there may be some suspicions of collusive behavior). Therefore, he assumes that $\theta = 1$. Researcher C considers that the industry can be characterized by Cournot competition between the 6 firms, such that $\theta = 1/6$. Finally, researcher S thinks that this industry can be better described by a Stackelberg model with 1 leader and 5 Cournot followers, and therefore $\theta = 1/(2 * 6 - 1) = 1/11$. What are the respective predictions of these researchers about market power as measured by the Lerner index? The following table presents the researchers' predictions and also the actual value of the Lerner index based on our information on marginal costs (that we assume is not available for these 3 researchers). Remember that $Lerner = \frac{P-MC}{P} = \frac{\theta}{\varepsilon}$.

Predicted Market Power Based on Different Assumptions on θ				
Assumed θ	Predicted Lerner Low Season: $\frac{\theta}{\varepsilon_L}$	Actual Lerner Low Season: $\frac{P_L - MC}{P_L}$	Predicted Lerner High Season	Actual Lerner High Season: $\frac{P_H - MC}{P_H}$
Monopoly: $\theta = 1$	$\frac{1}{2.24} = 44.6\%$	3.8%	$\frac{1}{1.04} = 96.1\%$	6.5%
Cournot: $\theta = 1/6$	$\frac{1/6}{2.24} = 7.4\%$	3.8%	$\frac{1/6}{1.04} = 16.0\%$	6.5%
Stackelberg: $\theta = 1/11$	$\frac{1/11}{2.24} = 4.0\%$	3.8%	$\frac{1/11}{1.04} = 8.7\%$	6.5%

This table shows that the researcher M will make a very seriously biased prediction of market power in the industry. Since the elasticity of demand is quite low in this industry, especially during the high season, the assumption of Cartel implies a very high Lerner index, much higher than the actual one. Researcher C also over-estimates the actual Lerner index. The estimates of researcher S are only slightly upward biased.

Consider the judge of an anti-trust case where there is very little reliable information on the actual value of MCs. The picture of industry competition that this judge gets from the three researchers is very different. This judge would be interested in measures of market power in this industry that do not depend on an adhoc assumption about the value of θ.

4.4.4. *Estimation of θ.* Suppose that we do not observe the MC and we use the approach described Section 2 to estimate θ and then the lerner index. The condition marginal revenue equal to marginal cost implies the following equation:

$$P_t = c_0 + c_1 \; P_t^{raw} + \theta \; \frac{Q_t}{\beta_t} + e_t^{MC}$$

We treat c_0 and c_1 (the parameters in the marginal cost function) as parameter to estimate because we do not know that $c_0 = 0.26$ and $c_0 = 1.075$. We interpret e_t^{MC} as an error term in the marginal cost. After the estimation of the demand equation, we have $\hat{\beta}_t = 2.30(1 - S_t) + 1.36 S_t$. Therefore, we estimate the equation:

$$P_t = c_0 + c_1 \; P_t^{raw} + \theta \; \frac{Q_t}{\hat{\beta}_t} + e_t^{MC}$$

Since Q_t is endogeneously determined, it should be correlated with e_t^{MC}. To deal with this endogeneity problem, GM use instrumental variables. Again, the use imports from Cuba as an instrument for Q_t. In principle, they might have considered the seasonal dummy S_t as an instrument, but they were probably concerned that there may be also seasonality in the marginal cost such that e_t^{MC} and S_t might be correlated (e.g., wages of seasonal workers). The following table presents these IV estimates of c_0, c_1 and θ, their standard errors (in parentheses) and the "true" values of these parameters based on the information on marginal costs.

Estimates of Marginal Costs and θ			
Parameter	Estimate	(s.e.)	"True" value$^{(Note)}$
θ	0.038	(0.024)	0.10
c_0	0.466	(0.285)	0.26
c_1	1.052	(0.085)	1.075

Note: The "true" value of θ using information if MC is obtained using the relationship $\frac{P-MC}{P} = \frac{\theta}{\varepsilon}$, or $\theta = (\frac{P-MC}{P})\varepsilon$. Then, $\hat{\theta}_{\text{"}true\text{"}} = (\frac{\bar{P}-\overline{MC}}{\bar{P}})\bar{\varepsilon}$, where $\bar{P}$, $\overline{MC}$, $\bar{\varepsilon}$ are the sample means of price, marginal cost, and estimated demand elasticity, respectively.

The estimates of θ, c_0, and c_1, are not too far from their "true" values. This seems a validation of the CV approach for this particular industry. Based on this estimate of θ, the predicted values for the Lerner index in the low and in the high season are:

$$\text{Predicted Lerner Index in low season} = \frac{\theta}{\varepsilon_L} = \frac{0.038}{2.24} = 1.7\%$$

$$\text{Predicted Lerner Index in high season} = \frac{\theta}{\varepsilon_H} = \frac{0.038}{1.04} = 3.6\%$$

Remember that the true values of the Lerner index using information on marginal costs were 3.8% in the low season and 6.5% in the high season. Therefore, the estimates using the CV approach under-estimate the actual market power in the industry, but by a relatively small magnitude.

4.5. Criticisms and limitations of the conjectural variation approach. - Corts (Journal of Econometrics, 1999)

- TBW

5. Competition and Collusion in the American Automobile Industry (Bresnahan, 1987)

TBW

6. Cartel stability (Porter, 1983)

TBW

APPENDIX: Stackelberg equilibrium with N firms (1 leader and N-1 Cournot followers)

The inverse demand is $P = A - BQ$, there are N firms (1 leader and $N-1$ followers), and all the firms have the same marginal cost MC. Let q_L be the quantity produced by the leader and Q_F the quantity of the $N-1$ followers. Given q_L the followers compete a la Cournot. The residual demand for the followers is $P = A - Bq_L - BQ_F$, that for notational simplicity we represent as $P = A_F - BQ_F$, where $A_F = A - Bq_L$. Given the residual demand $P = A_F - BQ_F$, followers compete a la Cournot. The marginal revenue of a (Cournot) follower is:

$$MR_F = A_F - BQ_F^{others} - 2Bq_F$$

where q_F represents output of a single follower, and Q_F^{others} represents total output of the other followers. The condition marginal revenue equals marginal cost is $A_F - BQ_F^{others} - 2Bq_F = MC$, and this implies that:

$$A_F - BQ_F - B\frac{Q_F}{N-1} = MC$$

Solving for Q_F, we have that:

$$Q_F = \left(\frac{N-1}{N}\right)\left(\frac{A_F - MC}{B}\right) = \left(\frac{N-1}{N}\right)\left(\frac{A - Bq_L - MC}{B}\right)$$

The leader takes into account how the followers will respond to his own choice of output. That is, he takes into account that $Q_F = \left(\frac{N-1}{N}\right)\left(\frac{A-Bq_L-MC}{B}\right)$. Solving this expression into the inverse demand equation, we have that:

$$\begin{aligned} P &= A - B\left(\frac{N-1}{N}\right)\left(\frac{A - Bq_L - MC}{B}\right) - Bq_L \\ &= A - \left(\frac{N-1}{N}\right)(A - Bq_L - MC) - Bq_L \\ &= \frac{A - MC}{N} + MC - \frac{B}{N}q_L \end{aligned}$$

The marginal revenue function for the leader is:

$$MR_L = \frac{A - MC}{N} + MC - 2\frac{B}{N}q_L$$

And the profit maximization condition, $MR_L = MC$, implies:

$$\frac{A - MC}{N} - 2\frac{B}{N}q_L = 0$$

Solving for q_L, we have:

$$q_L = \frac{A - MC}{2B}$$

Interestingly, note that the amount of output of a Stackelberg leader does not depend on the number of (Cournot) followers in the market, and it is equal to the output of a monopolist. Solving the expression of the equilibrium quantity q_L into the formula for the equilibrium output of the followers, we can get:

$$\begin{aligned} Q_F &= \left(\frac{N-1}{N}\right)\left(\frac{A - B\frac{A-MC}{2B} - MC}{B}\right) \\ &= \left(\frac{N-1}{N}\right)\left(\frac{A-MC}{2B}\right) \end{aligned}$$

Summing up q_L and Q_F, we obtain the equilibrium output of the industry:

$$\begin{aligned} Q &= q_L + Q_F = \left(\frac{A-MC}{2B}\right) + \left(\frac{N-1}{N}\right)\left(\frac{A-MC}{2B}\right) \\ &= \left(\frac{2N-1}{N}\right)\left(\frac{A-MC}{2B}\right) \end{aligned}$$

Finally, the equilibrium price-cost margin is:

$$\begin{aligned} P - MC &= A - B\left(\frac{2N-1}{N}\right)\left(\frac{A-MC}{2B}\right) - MC \\ &= A\left(1 - \frac{2N-1}{2N}\right) + \left(\frac{2N-1}{2N}\right) MC - MC \\ &= \frac{A-MC}{2N} \end{aligned}$$

To obtain the expression, $P - MC = \left(\frac{1}{2N-1}\right) B\ Q$, note that the equilibrium quantity is $Q = \left(\frac{2N-1}{N}\right)\left(\frac{A-MC}{2B}\right)$, and therefore $\frac{A-MC}{2N} = \left(\frac{1}{2N-1}\right) B\ Q$.

[**Equilibrium uniqueness**] Is equilibrium uniqueness a common feature of this class of models of market structure? Is it multiple equilibria a relevant/important issue for estimation? Is it important for predictions and counterfactual experiments?

Is equilibrium uniqueness a common feature of this class of models of market structure? The short answer is no. There are at least three assumptions in our simple model that are playing an important role in the uniqueness of the equilibrium: (a) linearity assumptions, i.e., linear demand, constant marginal costs, treating number of firms as a continuous variable; (b) homogeneous firms, i.e., homogeneous product and the same costs; and (c) no dynamics.

Once we relax any of these assumptions, multiple equilibria is the rule more than the exception. In general, richer models of market structure have multiple equilibria for a wide range of values of structural parameters and exogenous variables.

Is multiplicity of equilibria a relevant/important issue for estimation? It may or may not, depending on the structure of the model and on the estimation method that we choose. We will study this issue in detail during the course, but let me provide here some intuition for why sometimes multiple equilibria is not a serious issue for estimation and other times it is an issue.

For the sake of illustration, consider a very simple (and perhaps not so plausible) example of multiplicity of equilibria in the context of our model. Suppose that the fixed cost of operating a plant in the market FC_{mt} is a decreasing function of the number of firms in the local market. For instance, there are positive synergies between firms in terms of attracting skill labor, etc. Then, $FC_{mt} = \alpha_{mt} - \delta \ N_{mt}$, where δ is a positive parameter. Then, the equilibrium condition for market entry becomes:

$$\left(\frac{Q_{mt}}{N_{mt}}\right)^2 = \frac{1}{B_{mt}} \left(\alpha_{mt} - \delta \ N_{mt}\right)$$

This equilibrium equation can imply multiple equilibria for the number of firms in the market. Basically, the entry decision is a coordination game. There is an stable equilibrium where many firms enter in the market; there is other stable equilibrium where very few firms enter; and there is intermediate equilibrium that is not stable.

Is this an issue for estimation? Not necessarily. The equilibrium condition can be written as a regression equation:

$$\left(\frac{Q_{mt}}{N_{mt}}\right)^2 = \alpha \ S_{mt} + \delta \ (S_{mt} \, N_{mt}) + e_{mt}$$

The number of firms is an endogenous variable because it is correlated with the unobservables in the error term e_{mt}. However, if we have instruments to estimate this equation, we can estimate this equation using Instrumental Variables (IV) regardless of the multiple equilibria in the model. In fact, multiple equilibria may help for identification. For instance, if there is multiple equilibria in the data and equilibrium selection is random and independent of e_{mt}, then multiple equilibria helps for identification because it generates additional sample variation in the number of firms that is independent of the error term.

However, multiplicity of equilibria can be also a nuisance for the identification and estimation of these models. Suppose that we want to estimate the model using the maximum likelihood method. There are for using Maximum Likelihood (ML) instead of a GMM approach, and we will see them during the course. For instance, for nonlinear models with endogenous variables and non-additive unobservables there are not IV or GMM estimation methods. To use the ML method we need to derive the distribution of the endogenous variables conditional on the exogenous variables and the parameters of the model. However, in a model with multiple equilibria there is not such a thing as "the" distribution of the endogenous variables. There are multiple distributions, one for each equilibrium type. Therefore, we do not have a likelihood function but a likelihood correspondence. Is the MLE well define in this case? How to compute it? Is it computationally feasible? Are there alternative methods that are computationally simpler? We will address all these questions later in the course.

Is multiplicity of equilibria an issue for predictions and counterfactual experiments using the estimated model? Yes, of course. But it doesn't mean that we cannot make predictions or counterfactual experiments using these models. We will see different approaches and discuss their relative advantages and limitations.

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