EMPIRICAL ANALYSIS OF INNOVATION IN OLIGOPOLY INDUSTRIES

Lecture 1:
Innovation and productivity growth: Production functions

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Objectives of the course

- This course deals with **empirical studies of firms' innovation**.

- We will cover state-of-the-art structural models and econometric methods to measure:

  1. The productivity effects of innovations.
  2. Consumer valuation of product innovations.
  3. Value of patents.
  4. Dynamic strategic behavior in firms' innovation.

- We will examine data through the eyes of three classes of structural models which are workhorses in empirical industrial organization:
  - production functions;
  - demand of differentiated products;
  - dynamic games

Lecture 1: Innovation and productivity growth: Production functions

- 1. What Determines Productivity?
- 2. Review of Production Function estimation
- 3. Measuring the productivity effects of R&D

1. What determines productivity?

Total Factor Productivity (TFP)

- Production function:

$$Y_{it} = A_{it} \; F(K_{it}, L_{it}, M_{it})$$

- A_{it} is denoted Total Factor Productivity (TFP).
- It is a factor-neutral shifter that captures variations in output not explained by observable inputs.
- TFP is a residual.

Large and persistent differences in TFP across firms

- Ubiquitous, even within narrowly defined industries and products.
- **Large:** 90th to 10th percentile TFP ratios: $\frac{A_{90th}}{A_{10th}}$
  - U.S. manufacturing, **average** within 4-digit SIC industries = **1.92** (Syverson, 2004)
  - Denmark: average = **3.75** (Fox and Smeets, 2011)
  - China or India, **average** > **5** (Hsieh & Klenow, 2009).
- **Persistent:**
  - AR(1) of log-TFP with annual frequency: autoregressive coefficients between 0.6 to 0.8.
- **It matters:** Higher productivity producers are more likely to survive.

Why firms differ in their productivity levels?

- What supports such large productivity differences in equilibrium?
- Can producers control the factors that influence productivity or are they purely external effects of the environment?
- If firms can partly control their TFP, what type of choices increase it?

Why dispersion is possible in equilibrium?

- Let the profit of a firm be:

$$\pi_i = R(q_i, A_i, d) - C(q_i, A_i, w) - F$$

$R(q_i, A_i, d) =$ Revenue function. $d =$ State of the industry
$C(q_i, A_i, w) =$ Cost function. $w =$ Input prices. $F =$ fixed costs.

- **Key condition:** either $R(.)$ is strictly concave in q_i, or $C(.)$ is strictly convex in q_i. [The variable profit function is strictly concave].

- **Example: Perfect competition [or Bertrand competition with homogeneous product]**. $R = P\ q_i$ is linear in q_i. We need $C(.)$ to be strictly convex. i.e., DRS in variable inputs.

- **Example: Cournot competition or Bertrand competition with differentiated product**. R is strictly concave in q_i (downward sloping demand). So we can have either CRS or DRS.

Why dispersion is possible in equilibrium? [2]

- Equilibrium can be described by two types of conditions.
- At the **intensive margin**, optimal $q_i^* = q^*[A_i, d, w]$ is such that:

$$MR_i \equiv \frac{\partial R(q_i, A_i, d)}{\partial q_i} = \frac{\partial C(q_i, A_i, w)}{\partial q_i} \equiv MC_i$$

- At the **extensive margin**, a firm is active in the market iff:

$$R(q^*[A_i, d, w], A_i, d) - C(q^*[A_i, d, w], A_i, w) - F \geq 0$$

- If variable profit is strictly concave, this equilibrium can support firms with different TFPs, A_i.
- It is not optimal for the firm with highest TFP to provide all the output in the industry.
- Firms with different TFPs (above a certain threshold value) operate in the same market.

How can a firm affect its TFP?

- (HR) Managerial Practices. (Bloom & Van Reenen, 2007; Ichniowski and Shaw, 2003)
- Learning-by-Doing (Benkard, 2000).
- Organizational structure (vertical integration vs outsourcing).
- Higher-Quality (Labor and Capital) inputs.
- **Adoption of new (IT) technologies.**(Brynjolfsson et al., 2008).
- **Investment in R&D.** Long literature linking R&D investment and productivity.
- **Innovation**. Many firms undertake both process and product innovation without formally reporting R&D spending.

Innovation, R&D, and TFP

- Multiple studies show evidence that R&D and innovation are very (the most?) important factors to explain firm heterogeneity in TFP level and growth.
- As usual, the main difficulty in these studies comes from separating causation from correlation.
- For the rest of this lecture, we review models, methods, and datasets in different empirical applications dealing with the causal effect of R&D and innovations on TFP.

Innovation, R&D, and TFP [2]

- But still we will have to address the question **"why firms have different propensities to innovate / invest in R&D?"**, e.g., managerial talent, competition, spillovers, ...
- What factors determine how large innovative activity will be?
- Can we predict whether product or process innovation will dominate, based on market features?

2. Review of Production Function estimation

Outline

Main References

- Olley & Pakes (1996) was a break-through.
- Levinshon & Petrin (2003) also very influential
- Ackerberg, Caves & Frazer (2015) on identification issues and interpretation in OP & LP.
- Blundell & Bond (2000) dynamic panel data approach to PF
- Bond & Soderbom (2005) interesting insights on identification.

Introduction

- Production functions (PF) are important elements in economics.
- Estimation of PFs plays a key role in empirical questions such as:

  - Productivity and its growth: measurement, heterogeneity (dispersion).
  - Missallocation of inputs. How allocation of capital and labor relates to TFP.
  - Estimation of Marginal Productivity of inputs; Estimation of Marginal Costs.
  - Technological change over time or across industries. Capital intensity. Skill labor intensity.
  - Evaluating the effects of adopting new technologies
  - Measuring learning-by-doing.

Measurement and Econometric Issues

- **(1) Measurement errors:** M.e. in output (e.g., deflated revenue but not output); M.e. in capital; differences in quality of labor.
- **(2) Specification:** Functional form: Cobb-Douglas, CES, Translog, Leontief, NP? Constant vs Random coefficients?
- **(3) Simultaneity:** Observed inputs may be correlated with unobserved inputs or productivity shocks (e.g., TFP, managerial ability, capacity utilization, quality of land). This correlation introduces biases in some estimators.
- **(4) Multicollinearity:** Some inputs may be highly correlated if they are highly complementary.
- **(5) Endogenous Exit/Selection:** Firm exit from the sample is not exogenous. Endogenous exit can introduce selection bias in estimation of PF parameters.

Measurement and Econometric Issues (2)

- First, we focus on **Endogeneity problems** and different solutions that have been proposed to deal with these problems.
- We discuss these issues in the context of a simple **Cobb-Douglas PF**. Some arguments and results can be extended to more general specifications of the PF.
- Second, we will study other issues:
  - Distinguishing Quantity-TFP and Revenue-TFP
  - Multiproduct firms
  - More flexible functional forms
  - Heterogeneous PF parameters

Model and Data

- Panel data of N firms over T periods with information on output, labor, and capital (in logs):

$$\{\ y_{it},\ \ell_{it},\ k_{it} : i = 1, 2, ..., N\ ;\quad t = 1, 2, ... T\ \}$$

- We are interested in the estimation of the Cobb-Douglas PF (in logs) [Simple extensions: other inputs; different technologies]:

$$y_{it} = \beta_L\ \ell_{it} + \beta_K\ k_{it} + \omega_{it} + e_{it}$$

ω_{it} = *unobserved inputs (for the econometrician) which are known to the firm when it decides K and L (e.g., managerial ability, quality of land, different technologies).*

e_{it} = *measurement error in output or shock affecting output that is unknown to the firm when it decides K and L.*

Simultaneity problem

- Marshack and Andrews (Ectca, 1944) presented one of the first descriptions of the simultaneity problem when estimating PF.

$$y_{it} = \beta_L \; \ell_{it} + \beta_K \; k_{it} + \omega_{it} + e_{it}$$

- If ω_{it} is known to the firm when he decides (k_{it}, ℓ_{it}), the observed inputs are correlated with the unobserved ω_{it} and the OLS estimators of β_L and β_K are biased.

Measurement error in inputs

- Inputs, and especially capital, can be measured with error.
- Perpetual inventory method: depreciation and initial value of capital stock are difficult to measure: $k_{it}^{obs} = k_{it} + e_{it}^{k}$.
- Measurement error: attenuation bias in estimated coefficients. Absolute bias increases with the noise-to-signal ratio $\frac{Var(e_{it}^{k})}{Var(k_{it}^{obs})}$.
- Noise-to-signal ratio for capital can be substantially larger than in first differences than in levels: $\frac{Var(\Delta e_{it}^{k})}{Var(\Delta k_{it}^{obs})} >>> \frac{Var(e_{it}^{k})}{Var(k_{it}^{obs})}$. OLS in first-differences often generates very small (or even negative) capital coefficients (e.g., Griliches and Hausman, 1986, JoE).

Endogenous Exit

- Firms panel datasets are unbalanced, with significant amount of exits.
- We estimate the PF under selection:

$$y_{it} = \beta_L \; \ell_{it} + \beta_K \; k_{it} + \omega_{it} + e_{it} \qquad\qquad \text{if } d_{it} = 1$$

where d_{it} is the indicator of the event "firm i is active in the market at period t".

- Surviving firms are not randomly chosen, e.g., they are more productivity and use more capital and labor than exiting firms.

Endogenous Exit (2)

- The optimal exit/stay decision is:

$$d_{it} = 1\{\ V(k_{it}, \omega_{it}) \geq 0\ \}$$

$V(k_{it}, \omega_{it})$ is the value of the firm. Strictly increasing in k_{it} and ω_{it}.

- Since $V(k_{it}, \omega_{it})$ is strictly increasing in the two arguments, there is a cut-off value of productivity, $\omega^*(k_{it})$, such that:

$$d_{it} = 1\{\ \omega_{it} \geq \omega^*(k_{it})\ \}$$

and the threshold function ω^* is **strictly decreasing** in capital.

- Exit introduces correlation between error term of the PF and k_{it}:

$$\begin{aligned} E(\omega_{it} \mid k_{it},\ d_{it} = 1) &= E(\omega_{it} \mid k_{it}, \omega_{it} \geq \omega^*(k_{it})) \\ &= \lambda(k_{it}) \end{aligned}$$

where $\lambda(k_{it})$ is decreasing in k_{it}.

Endogenous Exit (3)

- $\lambda\left(k_{it}\right)$ is the selection term. The PF can be written as:

$$y_{it} = \beta_L \; \ell_{it} + \beta_K \; k_{it} + \lambda\left(k_{it}\right) + \tilde{\omega}_{it} + e_{it}$$

  where $\tilde{\omega}_{it} \equiv \{\omega_{it} \mid d_{it} = 1\} - \lambda\left(k_{it}\right)$ that, by construction, is mean-independent of k_{it}.

- Ignoring the selection term $\lambda\left(k_{it}\right)$ introduces bias in our estimates of the PF parameters.

- k_{it} is negatively correlated with the selection term $\lambda\left(k_{it}\right)$, and the selection problem tends to bias downward the estimate of the capital coefficient.

Estimation Methods: Input prices as IVs

- If input prices r_{it} are observable, and under the assumption that $cov\left(\omega_{it}, r_{it}\right) = 0$, we can use them as instruments.

- This approach has several **limitations/problems**:

  (a) Input prices are not always observable.

  (b) If there is only one competitive input market in the population under study, there is not any cross-sectional variation in r. Time-series variation is not enough for identification.

  (c) When input prices have cross-sectional variation, it could be because inputs markets are not competitive and firms with higher productivity pay higher prices, i.e., $cov\left(\omega_{it}, r_{it}\right) \neq 0$, making input prices not a valid instrument.

First order conditions for flexible inputs

- Suppose that labor is a perfectly flexible input and the firm is a price-taker in output and labor markets. Then, F.O.C. imply:

$$P_{it} \frac{\partial Y_{it}}{\partial L_{it}} = W_{it}$$

- For the Cobb-Douglas PF, this condition becomes:

$$\beta_L = \frac{W_{it} \; L_{it}}{P_{it} \; Y_{it}}$$

  i.e., β_L is identified by the wage bill-to-revenue ratio.

- In fact, this condition rejects this simple version of the model. Substantial sample variation in $\frac{W_{it} \; L_{it}}{P_{it} \; Y_{it}}$. Either $\beta_{L,it}$, or unobserved heterogeneity in cost of labor, or other assumptions do not hold.

- We will come back to this approach in Gandhi–Navarro-Rivers (2013).

Panel Data: Fixed Effects [Mundlak, 1966]

- Suppose that we have firm level panel data with information on output, capital and labor:

$$y_{it} = \beta_L \; \ell_{it} + \beta_K \; k_{it} + \omega_{it}$$

- The Fixed Effects estimator (i.e., its consistency) is based on the following assumptions:

$$\begin{array}{ll} (FE-1) & \omega_{it} = \eta_i + \delta_t + \omega_{it}^* \\ (FE-2) & var\left(\ell_{it} - \bar{\ell}_i\right) \text{ and } var\left(k_{it} - \bar{k}_i\right) \text{ are } > 0 \\ (FE-3) & \omega_{it}^* \text{ is not serially correlated} \\ (FE-4) & \omega_{it}^* \text{ is NOT known to the firm when} \\ & \text{it chooses inputs at period } t \end{array}$$

- η_i is interpreted as managerial ability, or a different technology that is constant over time.
- ω_{it}^* represents weather or any other random, unpredictable shock.

Panel Data: Fixed Effects (2)

- Under assumptions (PD-1) to (PD-4), the Fixed Effects estimator is consistent, i.e., OLS in the equation:

$$(y_{it} - \bar{y}_i) = \beta_L \ (\ell_{it} - \ell_i) + \beta_K \ (k_{it} - \bar{k}_i) + (\omega_{it} - \bar{\omega}_i)$$

- Consistency of the FE (with fixed T) requires the regressors $x_{it} = (\ell_{it}, k_{it})$ to be strictly exogenous. That is, for any (t, s):

$$cov\left(x_{it}, \omega^*_{is}\right) = cov\left(x_{it}, e_{is}\right) = 0$$

- Otherwise, the FE-transformed regressors $(\ell_{it} - \ell_i)$ and $(k_{it} - \bar{k}_i)$ would be correlated with the error $(\omega_{it} - \bar{\omega}_i)$.
- This is why Assumptions (FE-3) and (FE-4) are necessary for the consistency of the OLS estimator.
- In most applications, these are very restrictive conditions.

Dynamic Panel Data

- We can relax the assumption of strictly exogenous regressors, and estimate the PF using GMM in the equation in first differences (Arellano and Bond, 1993).

$$\Delta y_{it} = \beta_L \; \Delta \ell_{it} + \beta_K \; \Delta k_{it} + \Delta \delta_t + \Delta \omega^*_{it}$$

- We replace Assumption FE-4 with DPD-4

$(DPD - 4)$ Labor and capital are dynamic inputs:
$\ell_{it} = f_L(\ell_{i,t-1}, k_{i,t-1}, \omega_{it})$ and $k_{it} = f_K(\ell_{i,t-1}, k_{i,t-1}, \omega_{it})$

- Under these assumptions $\{\ell_{i,t-j}, k_{i,t-j}, y_{i,t-j} : j \geq 2\}$ are valid and useful instruments in the equation in first differences.

Dynamic Panel Data (2)

- Moment conditions for the GMM estimation of β_L, β_K, and $\Delta\delta_t$:

$$E\left(\ell_{i,t-j}\ \Delta\omega_{it}^{*}\right) = 0 \quad \text{for } t = 3, ..., T; \text{ and } j \leq t-2$$

$$E\left(k_{i,t-j}\ \Delta\omega_{it}^{*}\right) = 0 \quad \text{for } t = 3, ..., T; \text{ and } j \leq t-2$$

$$E\left(y_{i,t-j}\ \Delta\omega_{it}^{*}\right) = 0 \quad \text{for } t = 3, ..., T; \text{ and } j \leq t-2$$

Dynamic Panel Data (3)

- **Limitations of this approach:**

  (a) It provides downward biased and imprecise estimates of β_L and β_K (see Blundell and Bond, 1999, 2001). Overidentifying restrictions are typically rejected.

  (b) The i.i.d. assumption on ω^*_{it} is typically rejected: $\{x_{i,t-2}, y_{i,t-2}\}$ are not valid instruments.

  (c) Weak instruments problem: Arellano-Bond GMM estimator suffers of this problem in dynamic models where regressors in first differences are weakly autocorrelated.

  (d) First difference transformation also eliminates the cross-sectional variation and it is subject to the problem of measurement error in inputs.

Blundell and Bond (2001)

- BB propose two important extensions to the previous approach.
- First, ω_{it}^* follows an AR(1) process: $\omega_{it}^* = \rho\ \omega_{it-1}^* + u_{it}$. The PF equation can be represented in semi-first differences as:

$$(y_{it} - \rho\ y_{it-1}) = \beta_L\,(\ell_{it} - \rho\ \ell_{it-1}) + \beta_K\,(k_{it} - \rho\ k_{it-1}) + \eta_i^* + \delta_t^* + u_{it}$$

- Second, the estimator is based not only on moment conditions in first differences (Arellano-Bond) but also on moment conditions in levels (Blundell-Bond).
- Under a stationarity condition, for $j \geq 1$:

$$\begin{aligned} E\left(\Delta\ell_{it-j}\ \ [\eta_i^* + u_{it}]\right) &= 0 \\ E\left(\Delta k_{it-j}\ \ [\eta_i^* + u_{it}]\right) &= 0 \\ E\left(\Delta y_{it-j}\ \ [\eta_i^* + u_{it}]\right) &= 0 \end{aligned}$$

Blundell and Bond (2001)

- In the equation in levels, $(\ell_{it-j}, k_{it-j}, y_{it-j})$ for $j \geq 1$ are valid instruments:

$$(y_{it} - \rho \ y_{it-1}) = \beta_L \left(\ell_{it} - \rho \ \ell_{it-1}\right) + \beta_K \left(k_{it} - \rho \ k_{it-1}\right) + \eta_i^* + u_{it}$$

- In the equation in first differences, $(\ell_{it-j}, k_{it-j}, y_{it-j})$ for $j \geq 2$ are valid instruments:

$$(\Delta y_{it} - \rho \ \Delta y_{it-1}) = \beta_L \left(\Delta \ell_{it} - \rho \ \Delta \ell_{it-1}\right) + \beta_K \left(\Delta k_{it} - \rho \ \Delta k_{it-1}\right) + \Delta u_{it}$$

Blundell and Bond (2001): Results

509 manufacturing firms; 1982-89				
Parameter	**OLS-Levels**	**WG**	**AB-GMM**	**SYS-GMM**
β_L	0.538	0.488	0.515	0.479
	(0.025)	(0.030)	(0.099)	(0.098)
β_K	0.266	0.199	0.225	0.492
	(0.032)	(0.033)	(0.126)	(0.074)
ρ	0.964	0.512	0.448	0.565
	(0.006)	(0.022)	(0.073)	(0.078)
Sargan (p-value)	-	-	0.073	0.032
m2	-	-	-0.69	-0.35
Constant RS (p-v)	0.000	0.000	0.006	0.641

Bond and Soderbom (2005) Monte Carlo experiments

- Bond and Soderbom perform Monte Carlo experiments to study the actual identification power of ACs.
- They consider both deterministic and stochastic ACs. They simulate data from this model and estimate the PF using Blundell and Bond GMM method.
- (a) With deterministic ACs identification is weak when: (1) ACs are too low; or (2) ACs are too high; or (3) ACs of the two inputs are two similar (collinearity).
- With stochastic ACs identification results improve considerably.

Control Function Methods

- Olley & Pakes (1996; OP) and Levinsohn & Petrin (2003; LP) are **control function methods**.
- Instead of looking for instruments for K and L, we look for observable variables that can "control for" (or proxy) unobserved TFP.
- The control variables should come from a model of firm behavior.
- Note: Both OP and LP assume that labor is perfectly flexible input. This assumption is completely innocuous for their results. To emphasize this point, I present here versions of OP and LP that treat labor as a potentially dynamic input.

Olley and Pakes (OP)

- Consider the following model of simultaneous equations:

$$\boxed{\begin{array}{ll} (PF) & y_{it} = \beta_L \ \ell_{it} + \beta_K \ k_{it} + \omega_{it} + e_{it} \\ \\ (LD) & \ell_{it} = f_L\left(\ell_{i,t-1}, k_{it}, \omega_{it}, r_{it}\right) \\ \\ (ID) & i_{it} = f_K\left(\ell_{i,t-1}, k_{it}, \omega_{it}, r_{it}\right) \end{array}}$$

  (LD) & (ID): firms' optimal labor and investment given state variables $(\ell_{i,t-1}, k_{it}, \omega_{it}, r_{it})$; r_{it} = input prices.

- OP consider the following assumptions:

$$\begin{array}{ll} (OP-1) & f_K\left(\ell_{i,t-1}, k_{it}, \omega_{it}, r_{it}\right) \text{ is invertible in } \omega_{it} \\ (OP-2) & \text{No cross-sectional variation in } r_{it}\text{: } r_{it} = r_t. \\ (OP-3) & \omega_{it} \text{ follows a first order Markov process.} \\ (OP-4) & k_{it} \text{ is decided at } t-1\text{: } k_{it} = (1-\delta) k_{i,t-1} + i_{i,t-1} \end{array}$$

Olley and Pakes (2)

- OP method deals both with the simultaneity problem and with the selection problem due to endogenous exit.
- It doesn't deal with potential measurement error in inputs.
- OP method proceeds in **two stages**.
- **First stage:** estimates β_L [Assumptions (OP-1) and (OP-2) are key]; and the **second stage** estimates β_K [Assumptions (OP-3) and (OP-4) are key].

Olley and Pakes First Stage

- Assumptions (OP-1) and (OP-2) imply that the investment equation is invertible in ω_{it}:

$$\omega_{it} = f_K^{-1}(\ell_{i,t-1}, k_{it}, i_{it}, r_t)$$

- Solving this equation in the PF we have:

$$\begin{aligned} y_{it} &= \beta_L \; \ell_{it} + \beta_K \; k_{it} + f_K^{-1}(\ell_{i,t-1}, k_{it}, i_{it}, r_t) + e_{it} \\ &= \beta_L \; \ell_{it} + \phi_t(\ell_{i,t-1}, k_{it}, i_{it}) + e_{it} \end{aligned}$$

- This is a **partially linear model**. Parameter β_L and functions $\phi_1(.)$, ..., $\phi_T(.)$ can be estimated using **semiparametric methods**.
- A possible method is Robinson's method (1988). OP use an $n - th$ order polynomial to approximate the ϕ_t functions.

Olley and Pakes First Stage

- This first stage is a "Control Function" method: instead of instrumenting the endogenous regressors, we include additional regressors that capture the endogenous part of the error term.
- We are controlling for endogeneity by including $(\ell_{i,t-1}, k_{it}, i_{it})$ as "proxies" of ω_{it}.
- Key assumptions for the identification of β_L:

  (a) Invertibility of $f_K(\ell_{i,t-1}, k_{it}, \omega_{it}, r_t)$ *w.r.t* ω_{it}.

  (b) $r_{it} = r_t$, *i.e., no cross-sectional variability in unobservables, other than* ω_{it}, *affecting investment.*

  (c) Given $(\ell_{i,t-1}, k_{it}, i_{it}, r_t)$, *labor* ℓ_{it} *still has sample variability.*

- Assumption (c) is key, and it is the base for Ackerberg-Caves-Frazer criticism/extension of Olley-Pakes approach.

Olley and Pakes First Stage

- **Example (with lparametric linear investment func.):**

$$\boxed{\begin{array}{rl} (PF) & y_{it} = \beta_L \ \ell_{it} + \beta_K \ k_{it} + \omega_{it} + e_{it} \\ \\ (Inverse\ ID) & \omega_{it} = \gamma_1 \ i_{it} + \gamma_2 \ \ell_{i,t-1} + \gamma_3 \ k_{it} + \gamma_4 \ r_{it} \end{array}}$$

- Then,

$$y_{it} = \beta_L \ \ell_{it} + (\beta_K + \gamma_3) \ k_{it} + \gamma_1 \ i_{it} + \gamma_2 \ \ell_{i,t-1} + (\gamma_4 r_{it} + e_{it})$$

- Note that ℓ_{it} is correlated with r_{it}. Therefore, we need $r_{it} = r_t$ and include time dummies to control for r_t in order to have consistency of the OLS estimator in this regression.
- Note also that to identify ℓ_{it} with enough precision we need not high collinearity between this variable and $(k_{it}, i_{it}, \ell_{i,t-1})$.

Olley and Pakes Second Stage

- **Estimation of** β_K. It is based on the other two assumptions:

$$\begin{array}{ll} (OP-3) & \omega_{it} \text{ follows a first order Markov process.} \\ (OP-4) & k_{it} \text{ is decided at } t-1\text{: } k_{it} = (1-\delta)k_{i,t-1} + i_{i,t-1} \end{array}$$

- Since ω_{it} is first order Markov, we can write:

$$\omega_{it} = E[\omega_{it} \mid \omega_{i,t-1}] + \xi_{it} = h\left(\omega_{i,t-1}\right) + \xi_{it}$$

where ξ_{it} is an innovation which is mean independent of any information at $t-1$ or before. And $h(.)$ is some unknown function.

- ϕ_{it} is identified from 1st step; and $\phi_{it} = \beta_K\ k_{it} + \omega_{it}$. Then,

$$\phi_{it} = \beta_K\ k_{it} + h\left(\phi_{i,t-1} - \beta_K\ k_{i,t-1}\right) + \xi_{it}$$

Olley and Pakes — Second Stage

- We estimate $h(.)$ and β_K by applying recursively the same type of semiparametric method as in the first stage of OP.

$$\phi_{it} = \beta_K \ k_{it} + h\left(\phi_{i,t-1} - \beta_K \ k_{i,t-1}\right) + \xi_{it}$$

- Suppose that we consider a quadratic function for $h(.)$: i.e., $h(\omega) = \pi_1 \omega + \pi_2 \omega^2$. Then:

$$\phi_{it} = \beta_K \ k_{it} + \pi_1 \left(\phi_{i,t-1} - \beta_K \ k_{i,t-1}\right) + \pi_2 \left(\phi_{i,t-1} - \beta_K \ k_{i,t-1}\right)^2 + \xi_{it}$$

- It is clear that β_K, π_1 and π_2 are identified in this equation.

Olley and Pakes Second Stage

- Time-to build is a key assumption for the consistency of this method. If investment at period t is productive, then the equation becomes:

$$\phi_{it} = \beta_K \ k_{i,t+1} + h\left(\phi_{i,t-1} - \beta_K \ k_{it}\right) + \xi_{it}$$

- $k_{i,t+1}$ depends on investment at period t and therefore it is correlated with the innovation ξ_{it}.

OP: Empirical Application

- US Telecom. equipment industry: 1974-1987.
- Technological change and deregulation.
  - Elimination of barriers to entry;
  - Antitrust decisions against AT&T: The Consent Decree (implemented in 1984) –> divestiture of AT&T.
  - Substantial entry/exit of plants.
- Data: US Census of manufacturers.

OP: Empirical Application

TABLE VI

ALTERNATIVE ESTIMATES OF PRODUCTION FUNCTION PARAMETERS[a]
(STANDARD ERRORS IN PARENTHESES)

Sample:	Balanced Panel		Full Sample[c, d]					Nonparametric F_ω	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Estimation Procedure	Total	Within	Total	Within	OLS	Only P	Only h	Series	Kernel
Labor	.851	.728	.693	.629	.628		.608		
	(.039)	(.049)	(.019)	(.026)	(.020)		(.027)		
Capital	.173	.067	.304	.150	.219	.355	.339	.342	.355
	(.034)	(.049)	(.018)	(.026)	(.018)	(.02)	(.03)	(.035)	(.058)
Age	.002	−.006	−.0046	−.008	−.001	−.003	.000	−.001	.010
	(.003)	(.016)	(.0026)	(.017)	(.002)	(.002)	(.004)	(.004)	(.013)
Time	.024	.042	.016	.026	.012	.034	.011	.044	.020
	(.006)	(.017)	(.004)	(.017)	(.004)	(.005)	(.01)	(.019)	(.046)
Investment	—	—	—	—	.13	—	—	—	—
					(.01)				
Other Variables	—	—	—	—	—	Powers of P	Powers of h	Full Polynomial in P and h	Kernel in P and h
# Obs.[b]	896	896	2592	2592	2592	1758	1758	1758	1758

OP: Empirical Application

- Going from OLS balanced panel to OLS full sample almost doubles β_K and reduces β_L by 20%. [Importance of endogenous exit].
- Controlling for simultaneity further increases β_K and reduces β_L.

OP: Empirical Application

TABLE XI

DECOMPOSITION OF PRODUCTIVITY[a]
(EQUATION (16))

Year	p_t	$\bar{p}_t$	$\Sigma_i \Delta s_{it} \Delta p_{it}$	$\rho(p_t, k_t)$
1974	1.00	0.90	0.01	−0.07
1975	0.72	0.66	0.06	−0.11
1976	0.77	0.69	0.07	−0.12
1977	0.75	0.72	0.03	−0.09
1978	0.92	0.80	0.12	−0.05
1979	0.95	0.84	0.12	−0.05
1980	1.12	0.84	0.28	−0.02
1981	1.11	0.76	0.35	0.02
1982	1.08	0.77	0.31	−0.01
1983	0.84	0.76	0.08	−0.07
1984	0.90	0.83	0.07	−0.09
1985	0.99	0.72	0.26	0.02
1986	0.92	0.72	0.20	0.03
1987	0.97	0.66	0.32	0.10

Levinshon & Petrin (2003)

- The main difference with OP method is that LP use the demand function for intermediate inputs instead of the investment equation to invert out unobserved productivity.
- Two main motivations:
  - Investment can be responsive to more persistent shocks in TFP; materials is responsive to every shock in TFP.
  - In some datasets **Zero Investment** accounts for a large fraction of the data. At $i_{it} = 0$ (corner solution / extensive margin) there is not invertibility between i_{it} and ω_{it}. Problems: loss of efficiency; missing estimates of TFP for many observations.

Levinshon & Petrin (2003)

- They consider a Cobb-Douglas production function in terms of labor, capital, and intermediate inputs (materials):

$$y_{it} = \beta_L \ \ell_{it} + \beta_K \ k_{it} + \beta_M \ m_{it} + \omega_{it} + e_{it}$$

- Investment equation is replaced with demand for materials:

$$m_{it} = f_M \left(\ell_{i,t-1}, k_{it}, \omega_{it}, r_{it} \right)$$

- **Assumption LP-1**: $f_M \left(\ell_{i,t-1}, k_{it}, \omega_{it}, r_{it} \right)$ is invertible in ω_{it}.
- They maintain OP-2 [No other unobservables; $r_{it} = r_t$], OP-3 [Markov TFP], and OP-4 [Time-to-build].

Levinshon & Petrin: First Step

- Least squares estimation of parameter β_L and the nonparametric functions $\{\phi_t(.) : t = 1, 2, ..., T\}$ in regression equation:

$$y_{it} \quad = \quad \beta_L \ \ell_{it} + \phi_t(\ell_{i,t-1}, k_{it}, m_{it}) + e_{it}$$

- $\phi_t(\ell_{i,t-1}, k_{it}, m_{it}) = \beta_K \ k_{it} + \beta_M m_{it} + f_M^{-1}(\ell_{i,t-1}, k_{it}, m_{it}, r_t)$ and f_M^{-1} is the inverse function of f_M with respect to ω_{it}.

Levinshon & Petrin: Second Step

- The second step is also similar to OP's second step but in the model with the intermediate input.

- ϕ_{it} is estimated in 1st step; and $\phi_{it} = \beta_K \ k_{it} + \beta_M \ m_{it} + \omega_{it}$. Then,

$$\phi_{it} \quad = \quad \beta_K \ k_{it} + \beta_M \ m_{it} + h\left(\phi_{i,t-1} - \beta_K \ k_{i,t-1} - \beta_M \ m_{i,t-1}\right) + \xi_{it}$$

- Important difference with OP: In this second step $E(m_{it} \ \xi_{it}) \neq 0$, i.e., materials m_{it} is endogenous.

- LP propose two approaches:
  - **"unrestricted method"**: instrument m_{it} with its lagged values [see GNR (2013) criticism];
  - **"restricted method"**: under statis input, price-taking: $\beta_M =$ Cost of materials/Revenue.

LP: Empirical application

- Plant-level data from 8 different Chilean manufacturing industries: 1979-1985 [Pinochet period].

LP: Empirical Application. Var input shares

TABLE 3

Average Nominal Revenue Shares (Percentage), 1979-85

Industry	Unskilled	Skilled	Materials	Fuels	Electricity
Metals	15.2	8.3	44.9	1.6	1.7
Textiles	13.8	6.0	48.2	1.0	1.6
Food Products	12.1	3.5	60.3	2.1	1.3
Beverages	11.3	6.8	45.6	1.8	1.5
Other Chemicals	18.9	10.1	37.8	1.7	0.7
Printing & Pub.	19.8	10.7	40.1	0.5	1.3
Wood Products	20.6	5.3	47.0	3.0	2.4
Apparel	14.0	4.9	52.4	0.9	0.3

LP: Empirical Application: Zeroes

TABLE 2

Percent of Usable Observations, 1979-85

Industry	Investment	Fuels	Materials	Electricity
Metals	44.8	63.1	99.9	96.5
Textiles	41.2	51.2	99.9	97.0
Food Products	42.7	78.0	99.8	88.3
Beverages	44.0	73.9	99.8	94.1
Other Chemicals	65.3	78.4	100	96.5
Printing & Pub.	39.0	46.4	99.9	96.8
Wood Products	35.9	59.3	99.7	93.8
Apparel	35.2	34.5	99.9	97.2

LP: Empirical Application: Zeroes

TABLE 4
Unrestricted and Restricted Parameter Estimates for 8 Industries
(Bootstrapped Standard Errors in Parentheses)

Input	Industry (ISIC Code) 311	381	321	331	352	322	342	313
Unskilled labor								
	0.138	0.164	0.138	0.206	0.137	0.163	0.192	0.087
	(0.010)	(0.032)	(0.027)	(0.035)	(0.039)	(0.044)	(0.048)	(0.082)
Skilled labor								
	0.053	0.185	0.139	0.136	0.254	0.125	0.161	0.164
	(0.008)	(0.017)	(0.030)	(0.032)	(0.036)	(0.038)	(0.036)	(0.087)
Materials								
	0.703	0.587	0.679	0.617	0.567	0.621	0.483	0.626
	(0.013)	(0.017)	(0.019)	(0.022)	(0.045)	(0.020)	(0.028)	(0.075)
Fuels								
	0.023	0.024	0.041	0.018	0.004	0.0162	0.053	0.087
	(0.004)	(0.008)	(0.012)	(0.018)	(0.020)	(0.016)	(0.014)	(0.027)
Capital								
unrestricted	0.13	0.09	0.08	0.18	0.17	0.10	0.21	0.08
	(0.032)	(0.027)	(0.054)	(0.029)	(0.034)	(0.024)	(0.042)	(0.050)
restricted	0.14	0.09	0.06	0.11	0.15	0.09	0.21	0.07
	(0.011)	(0.02)	(0.019)	(0.025)	(0.034)	(0.039)	(0.045)	(0.11)
Electricity								
unrestricted	0.038	0.020	0.017	0.032	0.017	0.022	0.020	0.012
	(0.021)	(0.010)	(0.024)	(0.028)	(0.032)	(0.014)	(0.024)	(0.022)
restricted	0.011	0.015	0.014	0.021	0.005	0.008	0.011	0.012
No. Obs.	6051	1394	1129	1032	758	674	507	465

Ackerberg-Caves-Frazer critique

- Their criticism applies both to OP and LP. Here we present it in the context of OP.

- **Main point:**

[1] If OP (or LP) assumptions are correct, then conditional on (k_{it}, i_{it}) in OP [or (k_{it}, m_{it}) in LP] there should not be any cross-sectional variation left in ℓ_{it}. Perfect collinearity between ℓ_{it} and $\phi_t(k_{it}, i_{it})$.

[2] In the data, we find that conditional on (k_{it}, i_{it}) [or to (k_{it}, m_{it}), or even to $(\ell_{i,t-1}, k_{it}, m_{it})$] there is still cross-sectional variation left in ℓ_{it}. This should be because the assumptions of the model do not hold.

- Identification may be spurious; estimation can be inconsistent ... unless there are alternative assumptions that explain/allow for the not perfect collinearity between ℓ_{it} and $\phi_t(k_{it}, i_{it})$ and imply consistency of these control function methods.

Ackerberg-Caves-Frazer critique

- The state variables of the firm's problem are (k_{it}, i_{it}, r_t), then the firm's labor demand is:

$$\ell_{it} = f_L \left(k_{it}, \omega_{it}, r_t \right)$$

- And given that $\omega_{it} = f_K^{-1}(k_{it}, i_{it}, r_t)$, we have that:

$$\begin{aligned} \ell_{it} &= f_L \left(k_{it},\ f_K^{-1}(k_{it}, i_{it}, r_t),\ r_t \right) \\ &= G \left(k_{it}, i_{it}, r_t \right) \end{aligned}$$

- Therefore, for (k_{it}, i_{it}, r_t) fixed, current labor ℓ_{it} should not have any sample variability.

- That is, if the assumptions in OP model hold, then there should be a deterministic relationship between ℓ_{it} and $\phi_t(k_{it}, i_{it})$ and it should not

Ackerberg-Caves-Frazer critique (2)

- In the data, we observe that there is not perfect collinearity between ℓ_{it} and $\phi_t(k_{it}, i_{it})$. Two possible explanations:

[1] Unobservable $r_{it} \neq r_t$ that enter in the demand of both labor and investment $\Rightarrow$ OP and LP are **inconsistent estimation methods**.

[2] Unobservable $r_{it} \neq r_t$ that enter in the demand of labor but NOT in the investment decision $\Rightarrow$ OP and LP are **consistent estimation methods**.

- ACF consider arguments/assumptions that can generate scenario [2] and save OP / LP methods.

- The also propose an alternative method.

ACF: Saving OP & LP

- Consider the model:

$$\boxed{\begin{array}{ll} (PF) & y_{it} = \beta_L \ \ell_{it} + \beta_K \ k_{it} + \omega_{it} + e_{it} \\ \\ (LD) & \ell_{it} = f_L \left(\ell_{i,t-1}, k_{it}, \omega_{it}, r_{it}^L\right) \\ \\ (ID) & i_{it} = f_K \left(\ell_{i,t-1}, k_{it}, \omega_{it}, r_{it}^K\right) \end{array}}$$

where:

$$\begin{array}{ll} (OP-1) & f_K \left(\ell_{i,t-1}, k_{it}, \omega_{it}, r_{it}^K\right) \text{ is invertible in } \omega_{it} \\ (OP-2) & \text{No cross-sectional variation in } r_{it}^K\text{: } r_{it}^K = r_t^K. \\ (OP-3) & \omega_{it} \text{ follows a first order Markov process.} \\ (OP-4) & k_{it} \text{ is decided at } t-1\text{: } k_{it} = (1-\delta)k_{i,t-1} + i_{i,t-1} \end{array}$$

$$(ACF-1) \quad var(r_{it}^L \mid t, i_{it}, \ell_{i,t-1}, k_{it}) > 0$$

ACF: Saving OP & LP

$$(ACF-1) \quad var(r_{it}^{L} \mid t, i_{it}, \ell_{i,t-1}, k_{it}) > 0$$

- Some economic assumptions that generate (OP-2) and (ACF-1):

  ** Idiosyncratic shock in the price of labor that is i.i.d. over time occurs after the investment decision.*

ACF: A new method. Quasi-fixed inputs

- Consider a CD-PF with labor and capital as only inputs. Suppose that OP assumptions hold such that ℓ_{it} is perfectly collinear with $\phi_t(\ell_{i,t-1}, k_{it}, i_{it})$.
- If both capital and labor are quasi-fixed inputs, then it is possible to use a control function method in the spirit of OP or LP to identify/estimate β_L and β_K.
- Or in other words, this model has moment conditions that identify β_L and β_K (Wooldridge, EL 2009).

ACF: A new method. Quasi-fixed inputs

- In the first step we have:

$$
\begin{aligned}
y_{it} &= \beta_L \, \ell_{it} + \phi_t(\ell_{i,t-1}, k_{it}, i_{it}) + e_{it} \\
&= \beta_L \, g_t(\ell_{i,t-1}, k_{it}, i_{it}) + \phi_t(\ell_{i,t-1}, k_{it}, i_{it}) + e_{it} \\
&= \psi_t(\ell_{i,t-1}, k_{it}, i_{it}) + e_{it}
\end{aligned}
$$

- In this first step, we estimate $\psi_t(\ell_{i,t-1}, k_{it}, i_{it})$ nonparametrically.

ACF: A new method. Quasi-fixed inputs

- In the second step, given ψ_{it}, and taking into account that $\psi_{it} = \beta_L \ \ell_{it} + \beta_K \ k_{it} + \omega_{it}$, and $\omega_{it} = h\left(\omega_{i,t-1}\right) + \xi_{it}$, we have that:

$$\psi_{it} = \beta_L \ \ell_{it} + \beta_K \ k_{it} + h\left(\psi_{it} - \beta_L \ \ell_{it-1} + \beta_K \ k_{it-1}\right) + \xi_{it}$$

- In this second step, ℓ_{it} is correlated with ξ_{it}, but $\left(k_{it}, \psi_{it}, \ell_{it-1}, \ k_{it-1}\right)$ are not, and $\left(\ell_{it-2}, k_{it-2}\right)$ can be used to instrument ℓ_{it}.
- This approach is in the same spirit as the Dynamic Panel Data (DPD) methods of Arellano-Bond and Blundell-Bond.

Other identifying conditions: Quasi-fixed inputs [4]

- This approach cannot be applied if some inputs (e.g., materials) are perfectly flexible.
- The PF coefficient parameter of the flexible inputs cannot be identified from the moment conditions in the second step.

Other identifying conditions: F.O.C. for flexible inputs

- Klette & Grilliches (1996), Doraszelski & Jaumandreu (2013), and Gandhi, Navarro, & Rivers (2013) propose combining conditions from the PF with conditions from the demand of variable inputs.
- This approach requires the price of the variable input to be observable to the researcher, though this price may not have cross-sectional variation across firms.
- Note that in the LP method, the function that relates m_{it} with the state variables is just the condition "VMP of materials equal to price of materials". The parameters in this condition are the same as in the PF. This approach takes these restrictions into account.

Other identifying conditions: F.O.C. flexible inputs [2]

- For the CD-PF, with materials as flexible input, we have that:

$$\boxed{\begin{array}{ll} (PF) & y_{it} = \beta_L \; \ell_{it} + \beta_K \; k_{it} + \beta_M \; m_{it} + \omega_{it} + e_{it} \\ \\ (FOC) & p_t - p_t^M = \ln(\beta_M) + \beta_L \; \ell_{it} + \beta_K \; k_{it} + (\beta_M - 1) m_{it} + \omega_{it} \end{array}}$$

- The difference between these two equations is:

$$\ln(s_{it}^M) = \ln(\beta_M) + e_{it}$$

where $s_{it}^M \equiv \dfrac{P_t^M M_{it}}{P_t Y_{it}}$ is the ratio between material expenditures and revenue.

Other identifying conditions: F.O.C. flexible inputs [3]

- The parameter(s) of the flexible inputs are identified from the expensiture-share equations.

- The parameter(s) of the quasi-fixed inputs are identified using the dynamic conditions described above.

- Gandhi, Navarro, & Rivers (2013) show that this approach can be extended in two important way:
  - To a nonparametric specification of the production function: $y_{it} = f(\ell_{it}, k_{it}, m_{it}) + \omega_{it} + e_{it}$.
  - To a model with monopolistic competition (instead of perfect competition) with and isoelastic product demand.

Other identifying conditions: F.O.C. flexible inputs [4]

- This approach relies on two strong assumptions.
- There is not any bias or missing parameter in the MC of the flexible input. Suppose that $MC_{Mt} = P_t^M \ \tau$, then our estimate of β_M will actually estimate $\beta_M \ \tau$.
- In its current version, this method cannot incorporate oligopoly competition in the product market, only a limited form of monopolistic competition.

3. Measuring the productivity effects of R&D

Measuring the returns to R&D investment

- Investment in R&D and innovation is expensive. Investors (e.g., firms, policy makers) are interested in measuring its returns, private and social.
- **Process R&D:** Directed towards invention of new methods of production.
- **Product R&D:** Directed towards creation of new and improved goods.
- Both can increase the firm's TFP.
- It can have also **spillover effects in other firms**: competition spillovers, and/or knowledge spillovers.

Knowledge capital model (Grilliches, 1979)

- Most studies measuring returns to R&D are based on the estimation production function, i.e., effect of R&D on TFP.

- Cobb-Douglas in logs:

$$y_{it} = \beta_L \ \ell_{it} + \beta_K \ k_{it} + \beta_M \ m_{it} + \beta_R \ k_{it}^R + \omega_{it} + e_{it}$$

k_{it} = log of **stock of physical capital;**
k_{it}^R = log of **stock of knowledge capital**

- A major difficulty is the **measurement of the stock of knowledge capital**.

Measurement of knowledge capital

- We observe firms' R&D expenses, R_{it}. How to construct K_{it}^{R}?
- **Perpetual inventory method.** Given $\{R_{it} : t = 1, 2, ... T_i\}$, the transition rule:

$$K_{it}^{R} = (1 - \delta_R) \ K_{i,t-1}^{R} + R_{it}$$

  and values for δ_R and K_{i0} we can construct $\{K_{it}^{R} : t = 1, 2, ... T_i\}$.
- How to choose δ_R and K_{i0}?
- It is very difficult to know the true value of the rate of technological obsolescence, δ_R: it can be endogenous, vary across industries and firms, ...

Measurement of knowledge capital (2)

- Different studies using patent renewal data (Pakes and Schankerman, 1984; Pakes, 1986) or Tobin's Q model (Hall, 2005) estimate depreciation rates ranging between 10% and 35%.
- Different authors (e.g., Grilliches and Mairesse, 1984) have performed sensitivity analysis on the estimates of β_R for different value of δ_R.
- They report small differences, if any, in the estimate of β_R when δ_R varies between 8% and 25%.

Doraszelski & Jaumandreu (REStud, 2013)

- In their model, TFP and Knowledge capital (KC) are unobservables to the researcher.
- They follow a stochastic process that is endogenous and depends on (observable) R&D investments.
- The model accounts for **uncertainty and heterogeneity** across firms in the link between R&D and TFP.
- It takes into account that the outcome of R&D investments is subject to a high degree of uncertainty.
- For the estimation of the structural parameters in PF and stochastic process of KC, they exploit first order conditions for variable inputs.

D&J: Model

- The PF (in logs) is:

$$y_{it} = \beta_L \; \ell_{it} + \beta_K \; k_{it} + \beta_M \; m_{it} + \omega_{it} + e_{it}$$

- log-TFP ω_{it} follows a stochastic process with transition probability:

$$p\left(\omega_{it+1} \mid \omega_{it}, \; r_{it}\right)$$

where r_{it} is log-R&D expenditure.

- Every period t a firm chooses static inputs (ℓ_{it}, m_{it}) and investment in physical capital and R&D (i_{it}, r_{it}) to maximize its value.

$$V(s_{it}) = \max_{i_{it}, r_{it}} \left\{ \pi(s_{it}) - c^{(1)}(i_{it}) - c^{(2)}(r_{it}) + \rho \mathbb{E}\left[V(s_{it+1}) | s_{it}, i_{it}, r_{it} \right] \right\}$$

with $s_{it} = (k_{it}, \omega_{it}$, input prices $[w_{it}]$, demand shifters $[d_{it}])$.

D&J: TFP stochastic process

- The Markov structure of log-TFP implies:

$$\omega_{it} = \mathbb{E}\left[\omega_{it} \mid \omega_{it-1},\ r_{it-1}\right] + \xi_{it} = g\left(\omega_{it-1},\ r_{it-1}\right) + \xi_{it}$$

  where $\mathbb{E}\left[\xi_{it} \mid \omega_{it-1},\ r_{it-1}\right] = 0$.

- The *productivity innovation* ξ_{it} captures to sources of uncertainty for the firm:
  - the naturally linked to the evolution of TFP;
  - the uncertainty inherent to R&D (e.g., chance of discovery, degree of applicability, success in implementation).

D&J: Marginal Revenue (Market Power)

- D&J identification approach exploits marginal conditions (MR = MC) for variable inputs. This requires an assumption about competition/market power.

- They assume:

$$MR_{it} = P_{it} \left(1 - \frac{1}{\eta(p_{it}, d_{it})} \right)$$

where $\eta(p_{it}, d_{it})$ is price elasticity of demand for firm i, i.e., monopolisitc competition.

D&J: VMP of labor = wage

- This marginal condition of optimality for labor provides a closed-form expression for labor demand.

- Solving for log-TFP in the labor demand equation, we get:

$$\begin{aligned} \omega_{it} &= \lambda - \beta_K \; k_{it} + (1 - \beta_L - \beta_M) \; \ell_{it} + (1 - \beta_M) \; (w_{it} - p_{it}) \\ &+\beta_M \; (p_{Mit} - p_{it}) - \ln \left(1 - \frac{1}{\eta(p_{it}, d_{it})} \right) \end{aligned}$$

- We represent the RHS as $h(x_{it}, \beta)$, such that $\omega_{it} = h(x_{it}, \beta)$, with:

$$x_{it} = (k_{it}, \; \ell_{it}, \; w_{it}, \; p_{Mit}, \; p_{it}, \; d_{it})$$

D&J: Estimation

- Combining the PF equation with the stochastic process for TFP, and the marginal condition for optima labor, we have the equation:

$$y_{it} = \beta_L \ \ell_{it} + \beta_K \ k_{it} + \beta_M \ m_{it} + g\left[h(x_{it-1}, \beta), \ r_{it-1}\right] + \xi_{it} + e_{it}$$

- And form the marginal condition for labor we have:

$$h(x_{it}, \beta) = g\left[h(x_{it-1}, \beta), \ r_{it-1}\right] + \xi_{it}$$

- The "parameters" in this system of equations are: β_L, β_K, β_M, $g(.)$, and $\eta(.)$.
- The unobservables ξ_{it} and e_{it} is mean independent of any observable variable at period $t-1$ or before.
- Therefore, x_{it-1} and r_{it-1} are exogenous w.r.t. $\xi_{it} + e_{it}$. Capital stock k_{it} is also exogenous because time-to-build. **But we need to instrument ℓ_{it} and m_{it}.**

D&J: Identification

- To see that the parameters of the model are identified, it is convenient to consider a simplified version with: $\beta_K = \beta_M = 1/\eta(.) = 0$ and $g\left[\omega_{t-1},\ r_{t-1}\right] = \rho_\omega\ \omega_{t-1} + \rho_r\ r_{t-1}$. Them we have:

$$y_{it} = \beta_L\ \ell_{it} + \rho_\omega\left[(1-\beta_L)\,\ell_{it-1} + w_{it-1} - p_{it-1}\right] + \rho_r\ r_{it-1} + \xi_{it} + e_{it}$$

- Using as instruments $Z_{it} = \left(y_{it-1},\ \ell_{it-1},\ w_{it-1} - p_{it-1},\ r_{it-1}\right)$, moment conditions $\mathbb{E}\left[Z_{it}\ \left(\xi_{it} + e_{it}\right)\right] = 0$ identify β_L, ρ_ω, ρ_r.

- Given the identification of these parameters, we know $\omega_{it} = h(x_{it}, \beta) = (1-\beta_L)\ell_{it} + (w_{it} - p_{it})$. The model implies, that:

$$\xi_{it} = h(x_{it}, \beta) - \rho_\omega\ h(x_{it}, \beta) - \rho_r\ r_{it-1}$$

such that ξ_{it} is identified, and so its variance $Var(\xi_{it})$ that represents uncertainty in the link between R&D and TFP.

D&J: Identification (2)

- The instrument $w_{it-1} - p_{it-1}$ plays a very important role in the identification of the model.
- Without variation in lagged (real) input prices the model is NOT identified.
- But note that the model does not use contemporaneous input prices as instruments because they can be correlated with the innovation ξ_{it}.

D&J: Data

- Panel of Spanish manufacturing firms ($N = 1,870$). Annual data for period $1990 - 1999$ (max $T_i = 10$).
- 10 industries (SIC 2-digits).
- Period of rapid growth in output and physical capital, coupled with stagnant employment.
- **R&D intensity = R&D expenditure / Sales.** Average among all firms is 0.6% (smaller than in France, Germany, or UK, $> 2\%$).
- **R&D intensity** among performers (column 13) is between 1% and 3.5%.

TABLE 1

Descriptive statistics

Industry	Obs.[a]	Firms[a]	Entry[a] (%)	Exit[a] (%)	Rates of growth[a]: Output (s. d.)	Capital (s. d.)	Labour (s. d.)	Materials (s. d.)	Price (s. d.)	With R&D[b]: Obs. (%)	Stable (%)	Occas. (%)	R&D inten. (s. d.)
	(1)	(2)	(3)	(4)	(5)	(7)	(6)	(8)	(9)	(10)	(11)	(12)	(13)
1. Metals and metal products	1235	289	88	17	0.050	0.086	0.010	0.038	0.012	420	63	72	0.0126
			(30.4)	(5.9)	(0.238)	(0.278)	(0.183)	(0.346)	(0.055)	(34.0)	(21.8)	(24.9)	(0.0144)
2. Non-metallic minerals	621	131	20	15	0.037	0.062	−0.001	0.039	0.010	186	16	41	0.0100
			(15.3)	(11.5)	(0.208)	(0.238)	(0.141)	(0.308)	(0.059)	(30.0)	(12.2)	(31.3)	(0.0211)
3. Chemical products	1218	275	64	15	0.068	0.093	0.007	0.054	0.007	672	124	55	0.0268
			(23.3)	(5.5)	(0.196)	(0.238)	(0.146)	(0.254)	(0.061)	(55.2)	(45.1)	(20.0)	(0.0353)
4. Agric. and ind. machinery	576	132	36	6	0.059	0.078	0.010	0.046	0.013	322	52	35	0.0219
			(27.3)	(4.5)	(0.275)	(0.247)	(0.170)	(0.371)	(0.032)	(55.9)	(39.4)	(26.5)	(0.0275)
6. Transport equipment	637	148	39	10	0.087	0.114	0.011	0.087	0.007	361	62	35	0.0224
			(26.4)	(6.8)	(0.354)	(0.255)	(0.207)	(0.431)	(0.037)	(56.7)	(41.9)	(23.6)	(0.0345)
7. Food, drink, and tobacco	1408	304	47	22	0.025	0.094	−0.003	0.019	0.022	386	56	64	0.0071
			(15.5)	(7.2)	(0.224)	(0.271)	(0.186)	(0.305)	(0.065)	(27.4)	(18.4)	(21.1)	(0.0281)
8. Textile, leather, and shoes	1278	293	77	49	0.020	0.059	−0.007	0.012	0.016	378	39	66	0.0152
			(26.3)	(16.7)	(0.233)	(0.235)	(0.192)	(0.356)	(0.040)	(29.6)	(13.3)	(22.5)	(0.0219)
9. Timber and furniture	569	138	52	18	0.038	0.077	0.014	0.029	0.020	66	7	18	0.0138
			(37.7)	(13.0)	(0.278)	(0.257)	(0.210)	(0.379)	(0.035)	(12.6)	(5.1)	(13.8)	(0.0326)
10. Paper and printing products	665	160	42	10	0.035	0.099	−0.000	0.026	0.019	113	21	25	0.0143
			(26.3)	(6.3)	(0.183)	(0.303)	(0.140)	(0.265)	(0.089)	(17.0)	(13.1)	(13.8)	(0.0250)

D&J: Production Function Estimates

- Comparing GMM and OLS estimates, correcting for endogeneity has the expected implications, e.g., β_L and β_M decline, and $\beta_{K.}$ increases.
- There are not big differences in the β estimates across industries.
- Test of OIR from instruments: Cannot be rejected at 5% level.
- Test of parameter restrictions (in the two equations): Rejected at 5% level only in 2 out of 10 industries.

D&J: Production Function Estimates

TABLE 2
Production function estimates and specification tests

Industry	OLS[a]			GMM[a]			Overidentifying restrictions test		Parameter restrictions test	
	Capital (std. err.)	Labour (std. err.)	Materials (std. err.)	Capital (std. err.)	Labour (std. err.)	Materials (std. err.)	$\chi^2(df)$	p val.	$\chi^2(3)$	p val.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1. Metals and metal products	0.109	0.252	0.642	0.106	0.111	0.684	62.553	0.129	11.666	0.009
	(0.013)	(0.022)	(0.020)	(0.014)	(0.031)	(0.011)	(51)			
2. Non-metallic minerals	0.096	0.275	0.655	0.227	0.137	0.633	50.730	0.329	6.047	0.109
	(0.021)	(0.034)	(0.028)	(0.014)	(0.016)	(0.014)	(47)			
3. Chemical products	0.060	0.239	0.730	0.132	0.122	0.713	48.754	0.402	0.105	0.991
	(0.010)	(0.021)	(0.020)	(0.015)	(0.026)	(0.011)	(47)			
4. Agric. and ind. machinery	0.051	0.284	0.671	0.079	0.281	0.642	45.833	0.396	1.798	0.615
	(0.017)	(0.038)	(0.027)	(0.015)	(0.029)	(0.013)	(44)			
6. Transport equipment	0.080	0.289	0.636	0.117	0.158	0.675	40.296	0.745	0.414	0.937
	(0.023)	(0.033)	(0.046)	(0.015)	(0.023)	(0.016)	(47)			
7. Food, drink, and tobacco	0.094	0.177	0.739	0.068	0.129	0.766	61.070	0.068	8.866	0.031
	(0.014)	(0.016)	(0.016)	(0.014)	(0.024)	(0.008)	(46)			
8. Textile, leather, and shoes	0.059	0.335	0.605	0.057	0.313	0.593	66.143	0.075	4.749	0.191
	(0.010)	(0.024)	(0.019)	(0.011)	(0.016)	(0.013)	(51)			
9. Timber and furniture	0.079	0.283	0.670	0.131	0.176	0.697	44.951	0.390	0.618	0.892
	(0.019)	(0.029)	(0.029)	(0.009)	(0.017)	(0.011)	(43)			
10. Paper and printing products	0.092	0.321	0.621	0.121	0.249	0.617	51.371	0.152	5.920	0.118
	(0.016)	(0.029)	(0.025)	(0.013)	(0.025)	(0.014)	(42)			

D&J: Stochastic Process for TFP

- The model where TFP is exogenous (doesn't depend on R&D) is clearly rejected.
- Models with linear effects or without complementarity between ω_{t-1} and r_{t-1} are rejected.
- $Var(e)$ is approx. equal to $Var(\omega)$ in most industries.
- $Var(\xi) / Var(\omega)$ is between 30% and 75%. Very significant uncertainty of the effect of R&D on TFP.
- Significant differences across industries.

D&J: Stochastic Process for TFP

Industry	Exogeneity test		Separability test		$\frac{Var(e_{jt})}{Var(\omega_{jt})}$	$\frac{Var(\xi_{jt})}{Var(\omega_{jt})}$
	$\chi^2(10)$	p val.	$\chi^2(3)$	p val.		
	(1)	(2)	(3)	(4)	(5)	(6)
1. Metals and metal products	65.55	0.000	16.360	0.001	0.735	0.407
2. Non-metallic minerals	92.65	0.000	13.027	0.005	0.842	0.410
3. Chemical products	40.79	0.000	8.647	0.034	0.749	0.244
4. Agric. and ind. machinery	51.88	0.000	11.605	0.009	1.410	0.505
6. Transport equipment	56.85	0.000	18.940	0.000	1.626	0.524
7. Food, drink, and tobacco	38.29	0.000	7.186	0.066	1.526	0.300
8. Textile, leather, and shoes	29.91	0.001	18.417	0.000	1.121	0.750
9. Timber and furniture	118.17	0.000	32.260	0.000	1.417	0.515
10. Paper and printing products	59.73	0.000	23.249	0.000	0.713	0.433

D&J: Testing the standard Knowledge Capital model

- Testing three versions of the Knowledge capital model
- **Basic model:** $\omega_{it} + e_{it} = \beta_R \ k_{it}^R + e_{it}$. Rejected for all industries.
- **Hall & Hayashi (1989) and Klette (1996) KC model.** $K_{it}^R = \left[K_{it-1}^R\right]^{\sigma} \left[1 + R_{it-1}^R\right]^{1-\sigma} \exp\{\xi_{it}\}$. Using D&J notation:

$$\omega_{it} = \sigma \ \omega_{it-1} + (1 - \sigma) \ r_{it-1} + \xi_{it}$$

  Rejected at 5% in 8 industries, and at 7% in all industries.
- **Model with:** $\beta_R \ k_{it}^R + \omega_{it} + e_{it}$, and ω_{it} with exogenous Markov process. Rejected at 5% in 2 industries, and at 10% in 6 industries.

D&J: Testing the standard Knowledge Capital model

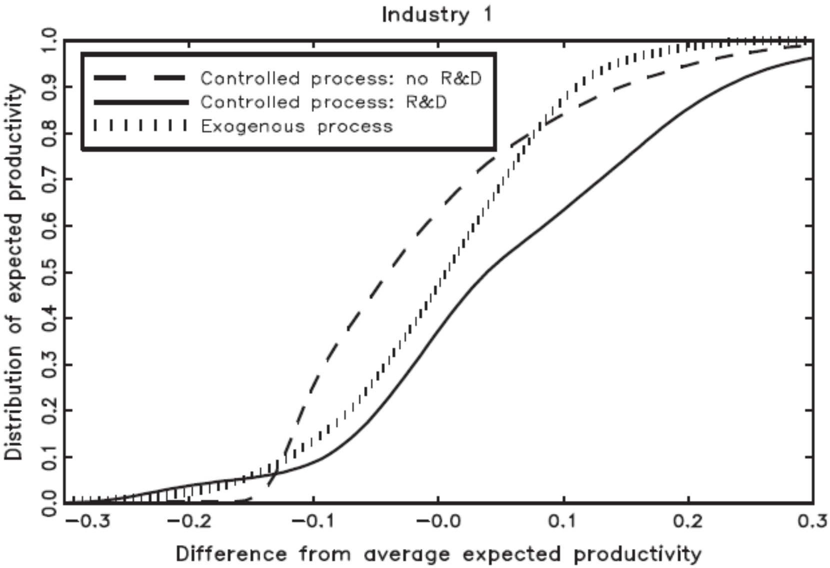
Knowledge capital model tests

Basic		Generalization 1		Generalization 2	
$N(0,1)$	p val.	$N(0,1)$	p val.	$N(0,1)$	p val.
(7)	(8)	(9)	(10)	(11)	(12)
−2.815	0.002	−2.431	0.008	−1.987	0.023
−2.041	0.021	−1.541	0.062	−0.784	0.216
−3.239	0.001	−2.090	0.018	−1.400	0.081
−2.693	0.004	−1.588	0.056	−1.493	0.068
−2.317	0.010	−2.042	0.021	−1.821	0.034
−3.263	0.001	−2.499	0.006	−0.901	0.184
−2.770	0.003	−1.788	0.037	−1.488	0.068
−2.510	0.006	−2.097	0.018	−1.028	0.152
−3.076	0.001	−2.210	0.014	−1.595	0.055

D&J: R&D and TFP (Counterfactuals)

- Distribution of TFP with R&D stochastically dominates distribution without R&D.
- Differences in means are **between 3% and 5%** for all industries and firm sizes, except for small firms in industries with low observed R&D intensity.

D&J: R&D and TFP (Counterfactuals)



D&J: Elasticities of TFP w.r.t. R&D and lagged TFP

- **Elasticity w.r.t. R&D:**
  - Considerable variation between and within industries.
  - Average is 0.015.
- **Degree of persistence:**
  - Considerable between and within industries.
  - Non-performers have a higher degree of persistence than performers.
  - Persistence is negatively related to the degree of uncertainty

D&J: Elasticities of TFP w.r.t. R&D

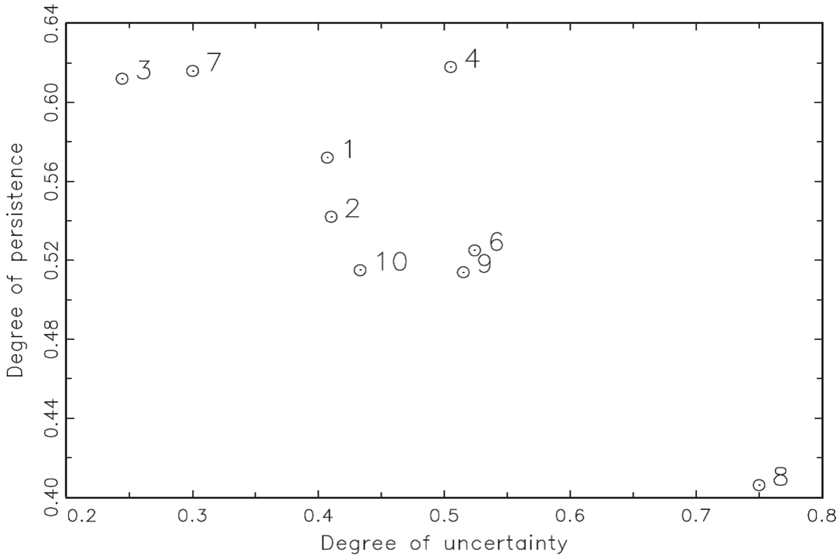
Industry	Elasticity wrt. R_{jt-1} [a] Q_1	Q_2	Q_3	Mean
	(1)	(2)	(3)	(4)
1. Metals and metal products	−0.013	0.007	0.021	0.022
2. Non-metallic minerals	−0.018	−0.012	0.000	−0.006
3. Chemical products	0.009	0.011	0.014	0.013
4. Agric. and ind. machinery	−0.017	−0.009	0.021	0.005
6. Transport equipment	−0.034	−0.008	0.010	0.020
7. Food, drink, and tobacco	−0.008	0.010	0.026	0.020
8. Textile, leather, and shoes	−0.003	0.014	0.051	0.046
9. Timber and furniture	−0.031	0.005	0.048	0.004
10. Paper and printing products	−0.036	0.022	0.049	0.013

D&J: Elasticities of TFP w.r.t. lagged TFP

Elasticity wrt. ω_{jt-1}[b]

Performers			Non-performers		
Q_1	Q_2	Q_3	Q_1	Q_2	Q_3
(5)	(6)	(7)	(8)	(9)	(10)
0.504	0.619	0.755	0.441	0.759	0.901
0.433	0.477	0.575	0.377	0.646	0.878
0.459	0.523	0.634	0.547	0.815	0.947
0.434	0.721	0.791	0.729	0.894	0.979
0.404	0.615	0.727	0.423	0.513	0.646
0.445	0.705	0.867	0.822	0.930	0.965
0.090	0.325	0.626	0.491	0.605	0.689
0.458	0.585	0.814	0.303	0.430	0.641
0.405	0.676	0.812	0.569	0.644	0.670

D&J: TFP persistence and Uncertainty (industry)



D&J: Summary of results

- They model TFP growth as the consequence of R&D expenditures with uncertain outcomes.
- Results show that this model can explain better the relationship between TFP and R&D than standard Knowledge Capital models without uncertainty and nonlinearity.
- R&D is a major determinant of the differences in TFP across firms and of their evolution.
- They also find that firm-level uncertainty in the outcome of R&D is considerable.
- Their estimates suggest that engaging in R&D roughly doubles the degree of uncertainty in the evolution of a producer's TFP.

Aw, Roberts, and Xu (AER, 2011)

- They highlight the bidirectional causality between R&D and productivity in the context of Taiwanese electronics exporters.
- They find that firms that select into exporting tend to already be more productive than their domestic counterparts, but the decision to export is often accompanied by large R&D investments.
- These investments raise exporters' productivity levels further in turn, highlighting both selection and causal effects tying productivity to R&D.
- Exporters are more willing to innovate on the margin because they can spread the potential gains of productivity growth across a larger market.